Computer algebra independent integration tests
1_Algebraic_functions/1.2_Trinomial_products/1.2.1Quadratic/1.2.1.6(g+hx)^m(a+bx+cx

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Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

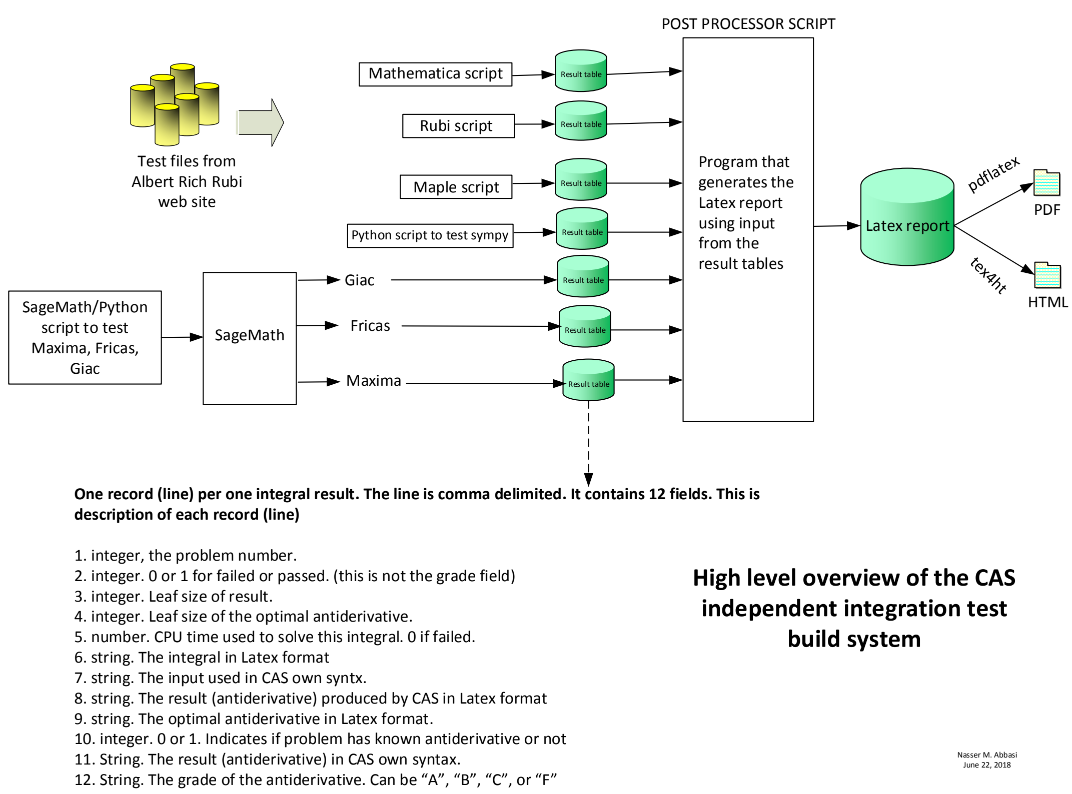
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (143)	% 0. (0)
Rubi in Sympy	% 56.64 (81)	% 43.36 (62)
Mathematica	% 99.3 (142)	% 0.7 (1)
Maple	% 98.6 (141)	% 1.4 (2)
Maxima	% 8.39 (12)	% 91.61 (131)
Fricas	% 48.95 (70)	% 51.05 (73)
Sympy	% 6.99 (10)	% 93.01 (133)
Giac	% 37.06 (53)	% 62.94 (90)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

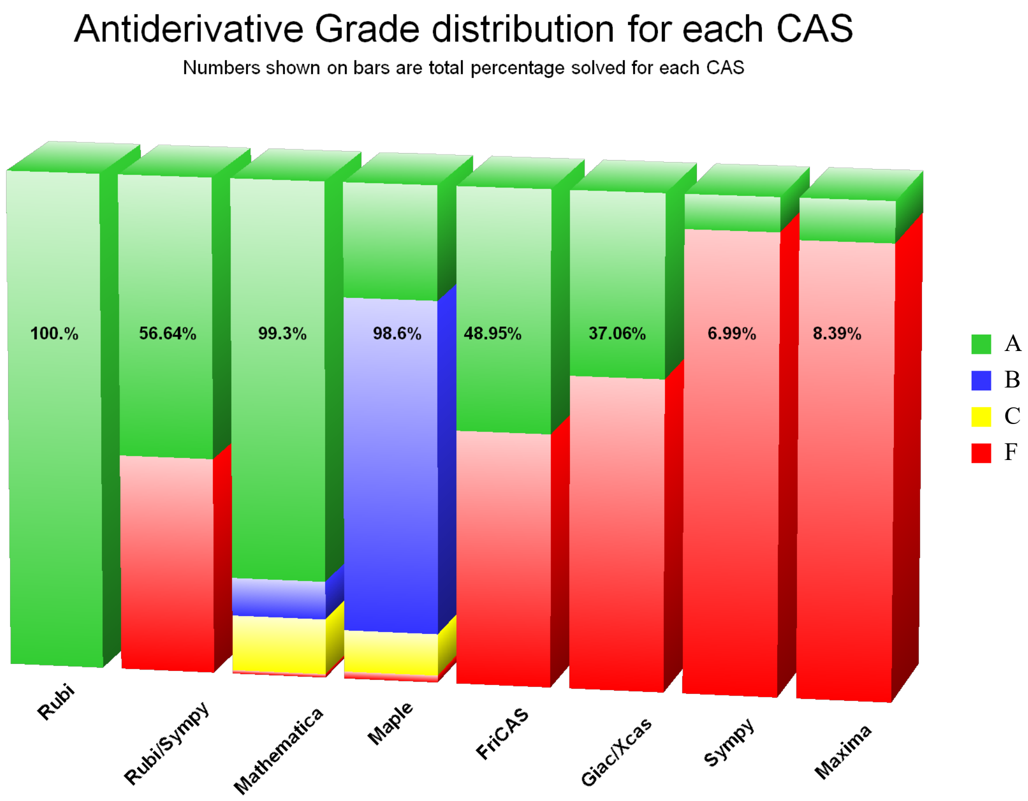
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

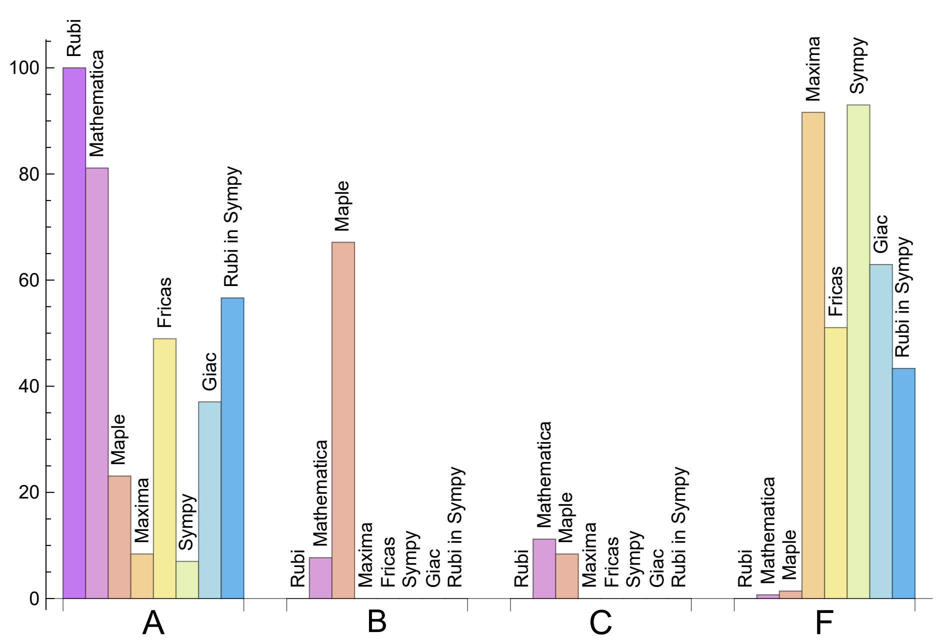
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Rubi in Sympy	56.64	0.	0.	43.36
Mathematica	81.12	7.69	11.19	0.7
Maple	23.08	67.13	8.39	1.4
Maxima	8.39	0.	0.	91.61
Fricas	48.95	0.	0.	51.05
Sympy	6.99	0.	0.	93.01
Giac	37.06	0.	0.	62.94

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	3.23	334.17	1.	302.	1.
Rubi in Sympy	74.32	193.99	0.96	170.	0.95
Mathematica	2.23	552.23	2.3	445.5	1.42
Maple	0.03	52029.2	107.04	1172.	3.85
Maxima	0.79	568.	3.58	348.	2.56
Fricas	10.66	1574.01	5.81	85.5	1.51
Sympy	10.57	599.4	3.67	506.	3.8
Giac	0.41	169.6	1.39	117.	1.31

1.8 list of integrals that has no closed form antiderivative

{}

1.9 list of integrals not solved by each system

Not solved by Rubi {}

Not solved by Rubi in Sympy {1, 2, 3, 4, 5, 6, 9, 12, 13, 14, 15, 16, 19, 20, 22, 35, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 69, 71, 74, 75, 76, 77, 78, 81, 83, 84, 85, 86, 87, 88, 89, 90, 102, 103, 107, 108, 109, 110, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 143}

Not solved by Mathematica {143}

Not solved by Maple {142, 143}

Not solved by Maxima {1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 135, 136, 137, 140, 141, 142, 143}

Not solved by Fricas {4, 5, 6, 8, 9, 15, 16, 19, 20, 21, 35, 39, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 142, 143}

Not solved by Sympy {4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 140, 141, 142, 143}

Not solved by Giac {6, 7, 8, 9, 11, 12, 15, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 33, 34, 35, 39, 49, 50, 51, 53, 54, 55, 56, 60, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 140, 141, 142, 143}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {19, 20, 21, 109, 110, 112, 113, 114, 115, 118, 119, 120, 121, 125}

Mathematica {20, 25, 26, 30, 125, 130, 142}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	86	133	0	1	332	117	0
normalized size	1	1.	0.91	1.41	0.	0.01	3.53	1.24	0.
time (sec)	N/A	0.233	0.143	0.01	0.	0.271	3.848	0.264	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	204	373	0	1	928	355	0
normalized size	1	1.	0.89	1.64	0.	0.	4.07	1.56	0.
time (sec)	N/A	0.739	0.536	0.009	0.	0.278	12.608	0.265	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	441	422	822	0	1	1940	841	0
normalized size	1	1.	0.96	1.86	0.	0.	4.4	1.91	0.
time (sec)	N/A	1.581	1.816	0.012	0.	0.282	30.066	0.267	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	212	745	0	0	0	359	0
normalized size	1	1.	0.77	2.72	0.	0.	0.	1.31	0.
time (sec)	N/A	0.651	0.765	0.012	0.	0.	0.	0.267	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	596	523	10537	0	0	0	1	0
normalized size	1	1.	0.88	17.68	0.	0.	0.	0.	0.
time (sec)	N/A	3.941	4.053	0.04	0.	0.	0.	0.272	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	450	3358	0	0	0	0	0
normalized size	1	1.	1.36	10.15	0.	0.	0.	0.	0.
time (sec)	N/A	1.322	3.	0.075	0.	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	370	714	0	8253	0	0	238
normalized size	1	1.	1.49	2.87	0.	33.14	0.	0.	0.96
time (sec)	N/A	0.492	2.255	0.029	0.	53.151	0.	0.	64.376

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	380	517	2758	0	0	0	0	439
normalized size	1	1.	1.36	7.24	0.	0.	0.	0.	1.15
time (sec)	N/A	1.922	2.781	0.034	0.	0.	0.	0.	167.217

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	797	796	1847	6422	0	0	0	0	0
normalized size	1	1.	2.32	8.06	0.	0.	0.	0.	0.
time (sec)	N/A	4.318	7.371	0.04	0.	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	55	46	88	62	0	65	44
normalized size	1	1.	1.17	0.98	1.87	1.32	0.	1.38	0.94
time (sec)	N/A	0.093	0.026	0.015	0.756	0.286	0.	0.27	15.265

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	394	637	0	1227	0	0	138
normalized size	1	1.	3.37	5.44	0.	10.49	0.	0.	1.18
time (sec)	N/A	0.38	0.646	0.171	0.	0.304	0.	0.	29.971

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	182	6871419	0	16226	0	0	0
normalized size	1	1.	0.38	14197.2	0.	33.52	0.	0.	0.
time (sec)	N/A	25.519	0.505	0.623	0.	0.649	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	182	175	510	0	1	1260	258	0
normalized size	1	0.99	0.95	2.77	0.	0.01	6.85	1.4	0.
time (sec)	N/A	0.7	0.363	0.01	0.	0.296	29.302	0.268	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	542	535	1672	0	1	0	996	0
normalized size	1	1.	0.99	3.08	0.	0.	0.	1.84	0.
time (sec)	N/A	2.961	1.853	0.013	0.	0.426	0.	0.278	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	398	267	1698	0	0	0	0	0
normalized size	1	0.98	0.66	4.18	0.	0.	0.	0.	0.
time (sec)	N/A	1.127	1.062	0.014	0.	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1075	1067	1376	54204	0	0	0	1	0
normalized size	1	0.99	1.28	50.42	0.	0.	0.	0.	0.
time (sec)	N/A	11.087	8.889	0.054	0.	0.	0.	0.388	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	131	340	0	1	709	296	131
normalized size	1	1.	0.94	2.43	0.	0.01	5.06	2.11	0.94
time (sec)	N/A	0.308	0.263	0.007	0.	0.306	4.818	0.277	49.73

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	131	340	0	1	680	279	126
normalized size	1	1.	0.94	2.43	0.	0.01	4.86	1.99	0.9
time (sec)	N/A	0.241	0.049	0.006	0.	0.302	4.752	0.276	50.043

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	617	615	1629	16209	0	0	0	0	0
normalized size	1	1.	2.64	26.27	0.	0.	0.	0.	0.
time (sec)	N/A	18.536	6.249	0.026	0.	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1092	1092	3733	59465	0	0	0	0	0
normalized size	1	1.	3.42	54.46	0.	0.	0.	0.	0.
time (sec)	N/A	32.056	6.52	0.04	0.	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	696	2269	0	0	0	0	394
normalized size	1	1.	1.67	5.45	0.	0.	0.	0.	0.95
time (sec)	N/A	6.323	1.727	0.079	0.	0.	0.	0.	141.626

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	780	780	411	784	0	9262	0	0	0
normalized size	1	1.	0.53	1.01	0.	11.87	0.	0.	0.
time (sec)	N/A	10.709	2.732	0.065	0.	45.857	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	512	1771	0	12119	0	0	308
normalized size	1	1.	1.7	5.86	0.	40.13	0.	0.	1.02
time (sec)	N/A	2.143	1.207	0.067	0.	56.029	0.	0.	64.421

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	282	608	0	2045	0	1	155
normalized size	1	1.	2.79	6.02	0.	20.25	0.	0.01	1.53
time (sec)	N/A	0.318	0.774	0.044	0.	0.4	0.	1.286	28.905

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	161	324	487	466	0	0	144
normalized size	1	1.	1.16	2.33	3.5	3.35	0.	0.	1.04
time (sec)	N/A	0.485	0.675	0.079	0.8	0.301	0.	0.	39.843

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	263	760	915	586	0	0	168
normalized size	1	1.	1.58	4.58	5.51	3.53	0.	0.	1.01
time (sec)	N/A	0.581	1.521	0.033	0.807	0.294	0.	0.	50.533

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	304	1560	1723	872	0	0	189
normalized size	1	1.	1.58	8.08	8.93	4.52	0.	0.	0.98
time (sec)	N/A	0.709	0.936	0.027	0.853	0.299	0.	0.	84.182

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	220	186	490	431	0	0	150
normalized size	1	1.	1.46	1.23	3.25	2.85	0.	0.	0.99
time (sec)	N/A	0.601	1.253	0.06	0.8	0.285	0.	0.	40.304

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	252	466	902	763	0	0	170
normalized size	1	1.	1.45	2.68	5.18	4.39	0.	0.	0.98
time (sec)	N/A	0.725	1.792	0.026	0.805	0.288	0.	0.	53.005

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	283	878	1723	1088	0	0	194
normalized size	1	1.	1.44	4.46	8.75	5.52	0.	0.	0.98
time (sec)	N/A	0.816	3.365	0.023	0.837	0.29	0.	0.	79.285

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	41	14	0	66	36	78	14
normalized size	1	1.	2.73	0.93	0.	4.4	2.4	5.2	0.93
time (sec)	N/A	0.052	0.018	0.011	0.	0.273	2.397	0.277	13.311

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	109	40	0	134	0	146	42
normalized size	1	1.	2.48	0.91	0.	3.05	0.	3.32	0.95
time (sec)	N/A	0.145	0.153	0.013	0.	0.276	0.	0.277	54.53

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	20	0	180	68	0	24
normalized size	1	1.	1.	0.83	0.	7.5	2.83	0.	1.
time (sec)	N/A	0.057	0.019	0.018	0.	0.272	2.308	0.	14.814

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	126	45	0	433	0	0	56
normalized size	1	1.	2.25	0.8	0.	7.73	0.	0.	1.
time (sec)	N/A	0.136	0.212	0.014	0.	0.293	0.	0.	34.288

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	767	3606	0	0	0	0	0
normalized size	1	1.	3.08	14.48	0.	0.	0.	0.	0.
time (sec)	N/A	1.944	1.85	0.028	0.	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	46	48	0	115	0	109	46
normalized size	1	1.	0.96	1.	0.	2.4	0.	2.27	0.96
time (sec)	N/A	0.148	0.068	0.007	0.	0.456	0.	0.275	33.254

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	1057	94	0	76	0	132	15
normalized size	1	1.	62.18	5.53	0.	4.47	0.	7.76	0.88
time (sec)	N/A	0.067	6.283	0.015	0.	0.288	0.	0.274	18.129

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	1079	123	0	167	0	220	83
normalized size	1	1.	12.55	1.43	0.	1.94	0.	2.56	0.97
time (sec)	N/A	0.438	6.27	0.011	0.	0.293	0.	0.273	61.225

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	108	121	0	0	0	0	129
normalized size	1	1.	0.79	0.89	0.	0.	0.	0.	0.95
time (sec)	N/A	0.299	0.182	0.025	0.	0.	0.	0.	42.465

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	108	105	0	1	0	158	196
normalized size	1	1.	0.51	0.5	0.	0.	0.	0.75	0.92
time (sec)	N/A	0.393	0.129	0.043	0.	0.324	0.	0.27	40.439

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	96	84	0	1	0	132	146
normalized size	1	1.	0.6	0.52	0.	0.01	0.	0.82	0.91
time (sec)	N/A	0.226	0.095	0.013	0.	0.325	0.	0.272	28.43

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	85	65	0	1	0	107	136
normalized size	1	1.	0.57	0.44	0.	0.01	0.	0.72	0.92
time (sec)	N/A	0.149	0.075	0.013	0.	0.34	0.	0.272	26.19

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	139	94	0	1	0	138	150
normalized size	1	1.	0.87	0.59	0.	0.01	0.	0.86	0.94
time (sec)	N/A	0.377	0.106	0.014	0.	0.342	0.	0.275	42.121

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	120	108	0	1	0	170	146
normalized size	1	1.	0.77	0.69	0.	0.01	0.	1.09	0.94
time (sec)	N/A	0.371	0.079	0.017	0.	0.336	0.	0.277	38.292

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	125	141	0	1	0	269	151
normalized size	1	1.	0.78	0.88	0.	0.01	0.	1.67	0.94
time (sec)	N/A	0.38	0.179	0.018	0.	0.335	0.	0.282	41.578

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	236	532	0	1	0	497	313
normalized size	1	1.	0.74	1.68	0.	0.	0.	1.57	0.99
time (sec)	N/A	0.833	0.431	0.022	0.	0.354	0.	0.289	71.228

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	176	383	0	1	0	362	221
normalized size	1	1.	0.78	1.69	0.	0.	0.	1.59	0.97
time (sec)	N/A	0.376	0.36	0.016	0.	0.336	0.	0.288	47.435

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	132	259	0	1	0	250	180
normalized size	1	1.	0.67	1.31	0.	0.01	0.	1.26	0.91
time (sec)	N/A	0.258	0.247	0.014	0.	0.329	0.	0.282	44.775

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	246	214	0	1	0	0	197
normalized size	1	1.	1.17	1.01	0.	0.	0.	0.	0.93
time (sec)	N/A	0.537	0.274	0.015	0.	0.995	0.	0.	68.453

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	168	259	0	1	0	0	189
normalized size	1	1.	0.83	1.28	0.	0.	0.	0.	0.94
time (sec)	N/A	0.501	0.39	0.018	0.	0.461	0.	0.	63.063

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	177	345	0	1	0	0	201
normalized size	1	1.	0.82	1.6	0.	0.	0.	0.	0.93
time (sec)	N/A	0.537	0.447	0.019	0.	0.6	0.	0.	66.684

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	882	7739	0	0	0	1	0
normalized size	1	1.	1.95	17.12	0.	0.	0.	0.	0.
time (sec)	N/A	4.096	3.326	0.077	0.	0.	0.	0.667	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	785	5581	0	0	0	0	0
normalized size	1	1.	1.99	14.13	0.	0.	0.	0.	0.
time (sec)	N/A	3.701	1.684	0.02	0.	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	472	3249	0	0	0	0	296
normalized size	1	1.	1.58	10.9	0.	0.	0.	0.	0.99
time (sec)	N/A	0.894	2.38	0.019	0.	0.	0.	0.	77.585

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	667	3544	0	1	0	0	0
normalized size	1	1.	1.86	9.9	0.	0.	0.	0.	0.
time (sec)	N/A	2.709	1.207	0.022	0.	39.914	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	600	3703	0	0	0	0	0
normalized size	1	1.	1.57	9.69	0.	0.	0.	0.	0.
time (sec)	N/A	3.126	1.159	0.051	0.	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	507	507	767	3993	0	0	0	1	0
normalized size	1	1.	1.51	7.88	0.	0.	0.	0.	0.
time (sec)	N/A	4.11	2.691	0.026	0.	0.	0.	1.345	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	795	795	1565	19148	0	0	0	1	0
normalized size	1	1.	1.97	24.09	0.	0.	0.	0.	0.
time (sec)	N/A	9.888	6.225	0.043	0.	0.	0.	0.918	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	591	591	1176	14709	0	0	0	1	0
normalized size	1	1.	1.99	24.89	0.	0.	0.	0.	0.
time (sec)	N/A	7.026	4.085	0.024	0.	0.	0.	0.826	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	482	932	8954	0	0	0	0	0
normalized size	1	1.	1.93	18.5	0.	0.	0.	0.	0.
time (sec)	N/A	8.827	2.716	0.022	0.	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	956	9728	0	0	0	1	0
normalized size	1	1.	1.93	19.61	0.	0.	0.	0.	0.
time (sec)	N/A	5.342	2.071	0.027	0.	0.	0.	1.315	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	604	604	880	9912	0	0	0	1	0
normalized size	1	1.	1.46	16.41	0.	0.	0.	0.	0.
time (sec)	N/A	5.933	1.498	0.029	0.	0.	0.	0.664	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	668	668	949	10298	0	0	0	1	0
normalized size	1	1.	1.42	15.42	0.	0.	0.	0.	0.
time (sec)	N/A	6.973	4.534	0.03	0.	0.	0.	1.619	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	696	2397	0	0	0	0	371
normalized size	1	1.	1.83	6.31	0.	0.	0.	0.	0.98
time (sec)	N/A	2.429	1.235	0.037	0.	0.	0.	0.	145.077

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	602	1796	0	0	0	0	337
normalized size	1	1.	1.75	5.22	0.	0.	0.	0.	0.98
time (sec)	N/A	1.311	0.842	0.022	0.	0.	0.	0.	96.086

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	484	1172	0	6865	0	0	294
normalized size	1	1.	1.65	3.99	0.	23.35	0.	0.	1.
time (sec)	N/A	0.682	4.295	0.018	0.	0.875	0.	0.	53.238

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	425	589	0	6849	0	0	267
normalized size	1	1.	1.6	2.21	0.	25.75	0.	0.	1.
time (sec)	N/A	0.453	2.574	0.017	0.	0.865	0.	0.	45.554

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	513	681	0	0	0	0	325
normalized size	1	1.	1.55	2.06	0.	0.	0.	0.	0.98
time (sec)	N/A	1.856	3.422	0.021	0.	0.	0.	0.	131.601

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	567	736	0	0	0	0	0
normalized size	1	1.	1.54	2.01	0.	0.	0.	0.	0.
time (sec)	N/A	2.678	1.141	0.023	0.	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	678	911	0	0	0	0	439
normalized size	1	1.	1.48	1.99	0.	0.	0.	0.	0.96
time (sec)	N/A	4.068	1.895	0.025	0.	0.	0.	0.	177.497

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	499	499	727	6124	0	0	0	0	0
normalized size	1	1.	1.46	12.27	0.	0.	0.	0.	0.
time (sec)	N/A	4.639	1.826	0.039	0.	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	596	4752	0	0	0	0	398
normalized size	1	1.	1.45	11.59	0.	0.	0.	0.	0.97
time (sec)	N/A	1.707	1.427	0.023	0.	0.	0.	0.	154.803

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	617	3000	0	0	0	0	400
normalized size	1	1.	1.5	7.3	0.	0.	0.	0.	0.97
time (sec)	N/A	1.96	1.752	0.02	0.	0.	0.	0.	158.234

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	598	1713	0	0	0	0	0
normalized size	1	1.	1.44	4.12	0.	0.	0.	0.	0.
time (sec)	N/A	1.512	1.403	0.02	0.	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	526	526	889	1945	0	0	0	0	0
normalized size	1	1.	1.69	3.7	0.	0.	0.	0.	0.
time (sec)	N/A	4.828	2.179	0.023	0.	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	618	618	996	2046	0	0	0	0	0
normalized size	1	1.	1.61	3.31	0.	0.	0.	0.	0.
time (sec)	N/A	5.442	3.16	0.025	0.	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	419	1817	0	0	0	0	0
normalized size	1	1.	1.07	4.64	0.	0.	0.	0.	0.
time (sec)	N/A	1.875	2.742	0.025	0.	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	397	1810	0	0	0	0	0
normalized size	1	1.	1.26	5.73	0.	0.	0.	0.	0.
time (sec)	N/A	1.106	2.395	0.021	0.	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	448	1667	0	0	0	0	257
normalized size	1	1.	1.59	5.91	0.	0.	0.	0.	0.91
time (sec)	N/A	0.735	2.172	0.019	0.	0.	0.	0.	138.653

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	343	1669	0	1	0	0	240
normalized size	1	1.	1.29	6.27	0.	0.	0.	0.	0.9
time (sec)	N/A	0.532	0.717	0.019	0.	100.791	0.	0.	89.738

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	473	1764	0	1	0	1	0
normalized size	1	1.	1.77	6.61	0.	0.	0.	0.	0.
time (sec)	N/A	1.693	0.525	0.023	0.	17.468	0.	0.293	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	379	1819	0	1	0	0	257
normalized size	1	1.	1.33	6.36	0.	0.	0.	0.	0.9
time (sec)	N/A	1.545	1.395	0.032	0.	25.181	0.	0.	176.368

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	505	1953	0	1	0	0	0
normalized size	1	1.	1.43	5.53	0.	0.	0.	0.	0.
time (sec)	N/A	1.939	0.79	0.026	0.	136.041	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	501	501	536	4884	0	0	0	0	0
normalized size	1	1.	1.07	9.75	0.	0.	0.	0.	0.
time (sec)	N/A	2.857	1.575	0.04	0.	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	482	4900	0	0	0	0	0
normalized size	1	1.	1.16	11.75	0.	0.	0.	0.	0.
time (sec)	N/A	2.215	1.263	0.025	0.	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	416	4567	0	0	0	0	0
normalized size	1	1.	1.19	13.09	0.	0.	0.	0.	0.
time (sec)	N/A	1.345	0.97	0.022	0.	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	396	4574	0	0	0	0	0
normalized size	1	1.	1.26	14.52	0.	0.	0.	0.	0.
time (sec)	N/A	1.156	1.107	0.022	0.	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	443	4765	0	0	0	0	0
normalized size	1	1.	0.94	10.16	0.	0.	0.	0.	0.
time (sec)	N/A	2.778	0.866	0.026	0.	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	433	4799	0	0	0	0	0
normalized size	1	1.	0.94	10.37	0.	0.	0.	0.	0.
time (sec)	N/A	2.6	0.78	0.027	0.	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	614	614	429	5056	0	0	0	0	0
normalized size	1	1.	0.7	8.23	0.	0.	0.	0.	0.
time (sec)	N/A	3.195	0.974	0.029	0.	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	207	1346	0	1	0	0	170
normalized size	1	1.	1.1	7.12	0.	0.01	0.	0.	0.9
time (sec)	N/A	0.67	0.694	0.027	0.	166.762	0.	0.	98.292

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	79	102	112	95	0	99	56
normalized size	1	1.	1.05	1.36	1.49	1.27	0.	1.32	0.75
time (sec)	N/A	0.143	0.021	0.02	0.795	0.299	0.	0.275	37.494

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	124	789	0	1806	0	0	141
normalized size	1	1.	0.95	6.07	0.	13.89	0.	0.	1.08
time (sec)	N/A	0.382	0.2	0.08	0.	0.336	0.	0.	39.476

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	399	516	0	0	0	0	338
normalized size	1	1.	1.08	1.4	0.	0.	0.	0.	0.92
time (sec)	N/A	1.576	4.762	0.025	0.	0.	0.	0.	144.552

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	404	410	0	0	0	0	260
normalized size	1	1.	1.41	1.43	0.	0.	0.	0.	0.91
time (sec)	N/A	1.271	2.784	0.021	0.	0.	0.	0.	124.668

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	353	399	0	0	0	0	241
normalized size	1	1.	1.33	1.5	0.	0.	0.	0.	0.91
time (sec)	N/A	0.533	0.788	0.02	0.	0.	0.	0.	78.789

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	289	354	0	3717	0	0	202
normalized size	1	1.	1.31	1.61	0.	16.9	0.	0.	0.92
time (sec)	N/A	0.341	0.486	0.019	0.	0.641	0.	0.	57.718

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	301	358	0	3565	0	0	202
normalized size	1	1.	1.37	1.63	0.	16.2	0.	0.	0.92
time (sec)	N/A	0.334	0.455	0.02	0.	0.702	0.	0.	55.761

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	351	391	0	1	0	1	241
normalized size	1	1.	1.31	1.46	0.	0.	0.	0.	0.9
time (sec)	N/A	1.319	1.031	0.022	0.	71.979	0.	0.281	124.81

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F(-1)	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	432	427	0	0	0	4	262
normalized size	1	1.	1.48	1.47	0.	0.	0.	0.01	0.9
time (sec)	N/A	1.364	2.273	0.024	0.	0.	0.	0.658	131.872

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F(-1)	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	418	519	0	0	0	4	342
normalized size	1	1.	1.11	1.38	0.	0.	0.	0.01	0.91
time (sec)	N/A	1.593	2.714	0.026	0.	0.	0.	0.677	159.291

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	466	466	480	1648	0	0	0	0	0
normalized size	1	1.	1.03	3.54	0.	0.	0.	0.	0.
time (sec)	N/A	2.904	5.486	0.025	0.	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	414	1480	0	0	0	0	0
normalized size	1	1.	1.21	4.34	0.	0.	0.	0.	0.
time (sec)	N/A	2.181	3.658	0.023	0.	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	583	1427	0	0	0	0	338
normalized size	1	1.	1.96	4.8	0.	0.	0.	0.	1.14
time (sec)	N/A	0.935	1.373	0.023	0.	0.	0.	0.	167.761

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	407	1360	0	0	0	0	342
normalized size	1	1.	1.36	4.55	0.	0.	0.	0.	1.14
time (sec)	N/A	0.862	2.447	0.021	0.	0.	0.	0.	156.897

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	584	1376	0	0	0	0	352
normalized size	1	1.	1.88	4.44	0.	0.	0.	0.	1.14
time (sec)	N/A	0.927	1.403	0.02	0.	0.	0.	0.	151.232

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	742	1518	0	0	0	1	0
normalized size	1	1.	1.88	3.85	0.	0.	0.	0.	0.
time (sec)	N/A	2.334	6.481	0.024	0.	0.	0.	0.295	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	454	537	1656	0	0	0	0	0
normalized size	1	1.	1.18	3.65	0.	0.	0.	0.	0.
time (sec)	N/A	2.75	3.392	0.025	0.	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	761	761	1546	14815	0	0	0	0	0
normalized size	1	1.	2.03	19.47	0.	0.	0.	0.	0.
time (sec)	N/A	8.377	6.305	0.028	0.	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	549	549	1112	10138	0	0	0	0	0
normalized size	1	1.	2.03	18.47	0.	0.	0.	0.	0.
time (sec)	N/A	14.046	2.863	0.022	0.	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	699	6019	0	0	0	0	420
normalized size	1	1.	1.62	13.97	0.	0.	0.	0.	0.97
time (sec)	N/A	1.636	4.703	0.	0.	0.	0.	0.	157.719

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	523	521	903	6460	0	0	0	0	0
normalized size	1	1.	1.73	12.35	0.	0.	0.	0.	0.
time (sec)	N/A	7.14	1.868	0.025	0.	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	736	736	972	6765	0	0	0	0	0
normalized size	1	1.	1.32	9.19	0.	0.	0.	0.	0.
time (sec)	N/A	8.042	3.755	0.028	0.	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	918	3131	0	0	0	0	0
normalized size	1	1.	1.68	5.74	0.	0.	0.	0.	0.
time (sec)	N/A	7.745	2.058	0.026	0.	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	815	2321	0	0	0	0	0
normalized size	1	1.	1.76	5.01	0.	0.	0.	0.	0.
time (sec)	N/A	7.394	1.39	0.023	0.	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	874	1516	0	15270	0	0	384
normalized size	1	1.	2.17	3.77	0.	37.99	0.	0.	0.96
time (sec)	N/A	2.555	6.179	0.02	0.	5.157	0.	0.	127.151

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	633	761	0	15237	0	0	357
normalized size	1	1.	1.69	2.03	0.	40.74	0.	0.	0.95
time (sec)	N/A	0.791	4.572	0.	0.	6.795	0.	0.	107.217

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	451	994	859	0	0	0	0	0
normalized size	1	1.	2.2	1.9	0.	0.	0.	0.	0.
time (sec)	N/A	5.868	6.246	0.022	0.	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	866	983	0	0	0	0	0
normalized size	1	1.	1.59	1.81	0.	0.	0.	0.	0.
time (sec)	N/A	10.059	1.872	0.026	0.	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	679	679	1008	1296	0	0	0	0	0
normalized size	1	1.	1.48	1.91	0.	0.	0.	0.	0.
time (sec)	N/A	17.187	3.369	0.029	0.	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	779	779	1066	13951	0	0	0	0	0
normalized size	1	1.	1.37	17.91	0.	0.	0.	0.	0.
time (sec)	N/A	24.266	5.97	0.034	0.	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	609	609	897	10781	0	0	0	0	0
normalized size	1	1.	1.47	17.7	0.	0.	0.	0.	0.
time (sec)	N/A	13.191	4.724	0.028	0.	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	609	609	868	6813	0	0	0	0	0
normalized size	1	1.	1.43	11.19	0.	0.	0.	0.	0.
time (sec)	N/A	12.776	3.99	0.024	0.	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	666	666	906	3889	0	0	0	0	0
normalized size	1	1.	1.36	5.84	0.	0.	0.	0.	0.
time (sec)	N/A	4.165	4.528	0.	0.	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	816	814	1331	4384	0	0	0	0	0
normalized size	1	1.	1.63	5.37	0.	0.	0.	0.	0.
time (sec)	N/A	24.85	5.879	0.026	0.	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	1101	159	0	242	0	254	153
normalized size	1	1.	7.86	1.14	0.	1.73	0.	1.81	1.09
time (sec)	N/A	1.034	6.294	0.024	0.	0.308	0.	0.29	121.949

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	1093	144	0	236	0	250	129
normalized size	1	1.	9.5	1.25	0.	2.05	0.	2.17	1.12
time (sec)	N/A	0.887	6.284	0.019	0.	0.3	0.	0.275	115.126

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	1087	130	0	213	0	231	116
normalized size	1	1.	11.09	1.33	0.	2.17	0.	2.36	1.18
time (sec)	N/A	0.484	6.269	0.009	0.	0.305	0.	0.275	83.245

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	69	891	92	0	54	0	92	71
normalized size	1	1.01	13.1	1.35	0.	0.79	0.	1.35	1.04
time (sec)	N/A	0.151	6.362	0.011	0.	0.292	0.	0.271	25.026

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	800	121	0	167	0	223	88
normalized size	1	1.	8.42	1.27	0.	1.76	0.	2.35	0.93
time (sec)	N/A	0.26	4.356	0.009	0.	0.294	0.	0.272	52.695

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	1068	152	0	221	0	269	129
normalized size	1	1.	8.22	1.17	0.	1.7	0.	2.07	0.99
time (sec)	N/A	0.865	6.25	0.023	0.	0.3	0.	0.272	124.364

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	1135	169	0	258	0	363	150
normalized size	1	1.	7.52	1.12	0.	1.71	0.	2.4	0.99
time (sec)	N/A	1.001	6.298	0.023	0.	0.306	0.	0.274	118.371

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	111	147	209	128	0	115	138
normalized size	1	1.	0.74	0.99	1.4	0.86	0.	0.77	0.93
time (sec)	N/A	0.232	0.157	0.022	0.789	0.287	0.	0.271	28.563

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	70	96	140	108	0	95	94
normalized size	1	1.	0.68	0.93	1.36	1.05	0.	0.92	0.91
time (sec)	N/A	0.113	0.084	0.009	0.78	0.283	0.	0.268	17.707

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	37	163	0	72	0	85	26
normalized size	1	1.	1.32	5.82	0.	2.57	0.	3.04	0.93
time (sec)	N/A	0.118	0.027	0.021	0.	0.295	0.	0.281	23.149

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	93	245	0	170	0	215	71
normalized size	1	1.	1.11	2.92	0.	2.02	0.	2.56	0.85
time (sec)	N/A	0.209	0.24	0.027	0.	0.29	0.	0.29	32.086

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	129	306	0	251	0	313	126
normalized size	1	1.	0.93	2.2	0.	1.81	0.	2.25	0.91
time (sec)	N/A	0.357	0.187	0.032	0.	0.295	0.	0.289	47.051

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	13	16	15	15	31	15	12
normalized size	1	1.	0.87	1.07	1.	1.	2.07	1.	0.8
time (sec)	N/A	0.008	0.017	0.005	0.715	0.276	0.407	0.271	2.594

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	13	14	12	15	10	15	10
normalized size	1	1.	0.81	0.88	0.75	0.94	0.62	0.94	0.62
time (sec)	N/A	0.008	0.009	0.004	0.702	0.274	15.185	0.271	1.904

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	13	20	0	15	0	0	12
normalized size	1	1.	0.87	1.33	0.	1.	0.	0.	0.8
time (sec)	N/A	0.045	0.014	0.005	0.	0.27	0.	0.	9.685

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	13	18	0	15	0	0	12
normalized size	1	1.	0.87	1.2	0.	1.	0.	0.	0.8
time (sec)	N/A	0.093	0.01	0.005	0.	0.279	0.	0.	11.966

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	331	0	0	0	0	0	224
normalized size	1	1.	1.37	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	0.344	1.038	0.314	0.	0.	0.	0.	29.789

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	488	488	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.378	0.93	0.668	0.	0.	0.	0.	0.

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [63] had the largest ratio of [0.5556]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.	25	0.16
2	A	5	4	1.	27	0.148
3	A	5	4	1.	27	0.148

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
4	A	8	7	1.	27	0.259
5	A	9	8	1.	27	0.296
6	A	9	6	1.	30	0.2
7	A	5	3	1.	30	0.1
8	A	6	4	1.	30	0.133
9	A	7	5	1.	30	0.167
10	A	5	4	1.	23	0.174
11	A	5	4	1.	23	0.174
12	A	5	4	1.	30	0.133
13	A	6	5	0.99	28	0.179
14	A	6	5	1.	30	0.167
15	A	9	5	0.98	30	0.167
16	A	10	6	0.99	30	0.2
17	A	5	5	1.	34	0.147
18	A	5	5	1.	34	0.147
19	A	9	6	1.	32	0.188
20	A	10	7	1.	32	0.219
21	A	5	3	1.	32	0.094
22	A	5	3	1.	29	0.103
23	A	5	3	1.	29	0.103
24	A	6	6	1.	26	0.231
25	A	5	4	1.	30	0.133
26	A	7	6	1.	30	0.2
27	A	7	6	1.	30	0.2
28	A	5	3	1.	30	0.1
29	A	6	4	1.	30	0.133
30	A	7	5	1.	30	0.167
31	A	2	2	1.	26	0.077
32	A	5	5	1.	26	0.192
33	A	2	2	1.	24	0.083
34	A	5	5	1.	20	0.25
35	A	6	5	1.	36	0.139
36	A	2	2	1.	36	0.056
37	A	2	2	1.	32	0.062
38	A	13	9	1.	32	0.281
39	A	5	5	1.	38	0.132
40	A	6	6	1.	35	0.171
41	A	5	5	1.	33	0.152
42	A	5	5	1.	32	0.156
43	A	8	8	1.	35	0.229
44	A	8	8	1.	35	0.229
45	A	8	8	1.	35	0.229
46	A	6	6	1.	38	0.158
47	A	5	5	1.	36	0.139
48	A	5	5	1.	35	0.143
49	A	7	6	1.	38	0.158
50	A	7	6	1.	38	0.158

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
51	A	7	6	1.	38	0.158
52	A	9	6	1.	27	0.222
53	A	11	7	1.	25	0.28
54	A	8	5	1.	24	0.208
55	A	12	9	1.	27	0.333
56	A	18	12	1.	27	0.444
57	A	22	13	1.	27	0.482
58	A	10	7	1.	27	0.259
59	A	16	9	1.	25	0.36
60	A	9	6	1.	24	0.25
61	A	17	11	1.	27	0.407
62	A	21	14	1.	27	0.518
63	A	26	15	1.	27	0.556
64	A	10	6	1.	27	0.222
65	A	8	5	1.	27	0.185
66	A	5	3	1.	25	0.12
67	A	5	3	1.	24	0.125
68	A	10	7	1.	27	0.259
69	A	11	8	1.	27	0.296
70	A	15	9	1.	27	0.333
71	A	10	7	1.	27	0.259
72	A	6	4	1.	27	0.148
73	A	6	4	1.	25	0.16
74	A	6	4	1.	24	0.167
75	A	12	9	1.	27	0.333
76	A	14	11	1.	27	0.407
77	A	15	9	1.	28	0.321
78	A	9	6	1.	28	0.214
79	A	9	6	1.	26	0.231
80	A	8	5	1.	25	0.2
81	A	17	9	1.	28	0.321
82	A	16	8	1.	28	0.286
83	A	20	10	1.	28	0.357
84	A	17	10	1.	28	0.357
85	A	10	7	1.	28	0.25
86	A	10	7	1.	26	0.269
87	A	9	6	1.	25	0.24
88	A	19	11	1.	28	0.393
89	A	18	10	1.	28	0.357
90	A	26	13	1.	28	0.464
91	A	9	6	1.	24	0.25
92	A	8	6	1.	22	0.273
93	A	10	9	1.	17	0.529
94	A	13	7	1.	28	0.25
95	A	10	6	1.	28	0.214
96	A	8	5	1.	28	0.179
97	A	5	3	1.	26	0.115

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
98	A	5	3	1.	25	0.12
99	A	9	4	1.	28	0.143
100	A	10	5	1.	28	0.179
101	A	13	6	1.	28	0.214
102	A	13	9	1.	28	0.321
103	A	9	6	1.	28	0.214
104	A	6	4	1.	28	0.143
105	A	6	4	1.	26	0.154
106	A	6	4	1.	25	0.16
107	A	12	7	1.	28	0.25
108	A	12	7	1.	28	0.25
109	A	9	6	1.	30	0.2
110	A	9	6	1.	28	0.214
111	A	8	5	1.	27	0.185
112	A	17	9	1.	30	0.3
113	A	23	10	1.	30	0.333
114	A	12	6	1.	30	0.2
115	A	8	5	1.	30	0.167
116	A	5	3	1.	28	0.107
117	A	5	3	1.	27	0.111
118	A	9	4	1.	30	0.133
119	A	12	5	1.	30	0.167
120	A	16	7	1.	30	0.233
121	A	10	7	1.	30	0.233
122	A	6	4	1.	30	0.133
123	A	6	4	1.	28	0.143
124	A	6	4	1.	27	0.148
125	A	12	7	1.	30	0.233
126	A	24	14	1.	30	0.467
127	A	20	13	1.	30	0.433
128	A	16	12	1.	30	0.4
129	A	6	4	1.01	28	0.143
130	A	10	8	1.	27	0.296
131	A	17	11	1.	30	0.367
132	A	20	12	1.	30	0.4
133	A	8	6	1.	34	0.176
134	A	6	5	1.	30	0.167
135	A	3	3	1.	34	0.088
136	A	5	5	1.	34	0.147
137	A	7	7	1.	34	0.206
138	A	1	1	1.	17	0.059
139	A	1	1	1.	15	0.067
140	A	2	2	1.	23	0.087
141	A	3	3	1.	21	0.143
142	A	2	2	1.	40	0.05
143	A	2	2	1.	104	0.019

3 Listing of integrals

3.1 $\int \frac{(A+Bx)(a+bx+cx^2)}{d+fx^2}\,dx$

Optimal. Leaf size=94

$$-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(-aAf+Acd+bBd)}{\sqrt{d}f^{3/2}}-\frac{\log\left(d+fx^2\right)(-aBf-Abf+Bcd)}{2f^2}+\frac{x(Ac+bB)}{f}+\frac{Bcx^2}{2f}$$

```
[Out] ((b*B + A*c)*x)/f + (B*c*x^2)/(2*f) - ((b*B*d + A*c*d - a*A*f)*Ar
cTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(3/2)) - ((B*c*d - A*b*f -
a*B*f)*Log[d + f*x^2])/(2*f^2)
```

Rubi [A] time = 0.233197, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(-aAf+Acd+bBd)}{\sqrt{d}f^{3/2}}-\frac{\log\left(d+fx^2\right)(-aBf-Abf+Bcd)}{2f^2}+\frac{x(Ac+bB)}{f}+\frac{Bcx^2}{2f}$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2),x]
```

```
[Out] ((b*B + A*c)*x)/f + (B*c*x^2)/(2*f) - ((b*B*d + A*c*d - a*A*f)*Ar
cTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(3/2)) - ((B*c*d - A*b*f -
a*B*f)*Log[d + f*x^2])/(2*f^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bc\int x\,dx}{f}+(Ac+Bb)\int\frac{1}{f}\,dx+\frac{(Abf+Baf-Bcd)\log\left(d+fx^2\right)}{2f^2}+\frac{(Aaf-Acd-Bbd)\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}f^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)*(c*x**2+b*x+a)/(f*x**2+d),x)
```

```
[Out] B*c*Integral(x, x)/f + (A*c + B*b)*Integral(1/f, x) + (A*b*f + B*
a*f - B*c*d)*log(d + f*x**2)/(2*f**2) + (A*a*f - A*c*d - B*b*d)*a
tan(sqrt(f)*x/sqrt(d))/(sqrt(d)*f**(3/2))
```

Mathematica [A] time = 0.143179, size = 86, normalized size = 0.91

$$\frac{\log\left(d+fx^2\right)(aBf+Abf-Bcd)-\frac{2\sqrt{f}\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(-aAf+Acd+bBd)}{\sqrt{d}}+fx(2Ac+2bB+Bcx)}{2f^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2),x]
```

```
[Out] (f*x*(2*b*B + 2*A*c + B*c*x) - (2*Sqrt[f]*(b*B*d + A*c*d - a*A*f)
*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/Sqrt[d] + (-(B*c*d) + A*b*f + a*B*f
)*Log[d + f*x^2])/(2*f^2)
```

Maple [A] time = 0.01, size = 133, normalized size = 1.4

$$\frac{Bcx^2}{2f} + \frac{Acx}{f} + \frac{bBx}{f} + \frac{\ln\left(fx^2+d\right)Ab}{2f} + \frac{\ln\left(fx^2+d\right)aB}{2f} - \frac{\ln\left(fx^2+d\right)Bcd}{2f^2} + Aa\arctan\left(fx\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}} - \frac{Acd}{f}\arctan\left(fx\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}} - \frac{Bbd}{f}\arctan\left(fx\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x)
```

```
[Out] 1/2*B*c*x^2/f+1/f*A*c*x+1/f*b*B*x+1/2/f*ln(f*x^2+d)*A*b+1/2/f*ln(
f*x^2+d)*a*B-1/2/f^2*ln(f*x^2+d)*B*c*d+1/(d*f)^(1/2)*arctan(x*f/(
d*f)^(1/2))*a*A-1/f/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*A*c*d-1/f
/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*B*b*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)*(B*x + A)/(f*x^2 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270519, size = 1, normalized size = 0.01

$$\left[\frac{\left(Aaf^2-(Bb+Ac)df\right)\log\left(\frac{2\,dfx+\left(fx^2-d\right)\sqrt{-df}}{fx^2+d}\right)+\left(Bcfx^2+2\,(Bb+Ac)fx-(Bcd-(Ba+Ab)f)\log\left(fx^2+d\right)\right)\sqrt{-df}}{2\sqrt{-df}\,f^2}, \frac{2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)*(B*x + A)/(f*x^2 + d),x, algorithm="fricas")
```

```
[Out] [1/2*((A*a*f^2 - (B*b + A*c)*d*f)*log((2*d*f*x + (f*x^2 - d)*sqrt
(-d*f))/(f*x^2 + d)) + (B*c*f*x^2 + 2*(B*b + A*c)*f*x - (B*c*d -
(B*a + A*b)*f)*log(f*x^2 + d))*sqrt(-d*f))/(sqrt(-d*f)*f^2), 1/2*
(2*(A*a*f^2 - (B*b + A*c)*d*f)*arctan(sqrt(d*f)*x/d) + (B*c*f*x^2
 + 2*(B*b + A*c)*f*x - (B*c*d - (B*a + A*b)*f)*log(f*x^2 + d))*sq
rt(d*f))/(sqrt(d*f)*f^2)]
```

Sympy [A] time = 3.84803, size = 332, normalized size = 3.53

$$\frac{Bcx^2}{2f} + \left(\frac{Abf + Baf - Bcd}{2f^2} - \frac{\sqrt{-df^5}\,(Aaf - Acd - Bbd)}{2df^4}\right)\log\left(x + \frac{-Abdf - Badf + Bcd^2 + 2df^2\left(\frac{Abf+Baf-Bcd}{2f^2} - \frac{\sqrt{-df^5}(Aaf-Acd-Bbd)}{2df^4}\right)}{Aaf^2 - Acdf - Bbdf}\right) + \left(\frac{Abf + Baf - Bcd}{2f^2} + \frac{\sqrt{-df^5}\,(Aaf - Acd - Bbd)}{2df^4}\right)\log\left(x + \frac{-Abdf - Badf + Bcd^2 + 2df^2\left(\frac{Abf+Baf-Bcd}{2f^2} + \frac{\sqrt{-df^5}(Aaf-Acd-Bbd)}{2df^4}\right)}{Aaf^2 - Acdf - Bbdf}\right) + \frac{x(Ac + Bb)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)*(c*x**2+b*x+a)/(f*x**2+d),x)
```

```
[Out] B*c*x**2/(2*f) + ((A*b*f + B*a*f - B*c*d)/(2*f**2) - sqrt(-d*f**5
)*(A*a*f - A*c*d - B*b*d)/(2*d*f**4))*log(x + (-A*b*d*f - B*a*d*f
 + B*c*d**2 + 2*d*f**2*((A*b*f + B*a*f - B*c*d)/(2*f**2) - sqrt(-
d*f**5)*(A*a*f - A*c*d - B*b*d)/(2*d*f**4)))/(A*a*f**2 - A*c*d*f
- B*b*d*f)) + ((A*b*f + B*a*f - B*c*d)/(2*f**2) + sqrt(-d*f**5)*(
A*a*f - A*c*d - B*b*d)/(2*d*f**4))*log(x + (-A*b*d*f - B*a*d*f +
B*c*d**2 + 2*d*f**2*((A*b*f + B*a*f - B*c*d)/(2*f**2) + sqrt(-d*f
**5)*(A*a*f - A*c*d - B*b*d)/(2*d*f**4)))/(A*a*f**2 - A*c*d*f - B
*b*d*f)) + x*(A*c + B*b)/f
```

GIAC/XCAS [A] time = 0.26355, size = 117, normalized size = 1.24

$$-\frac{(Bbd + Acd - Aaf)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f} - \frac{(Bcd - Baf - Abf)\ln\left(fx^2 + d\right)}{2\,f^2} + \frac{Bcfx^2 + 2\,Bbfx + 2\,Acfx}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)*(B*x + A)/(f*x^2 + d),x, algorithm="giac")
```

```
[Out] -(B*b*d + A*c*d - A*a*f)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f) - 1/
2*(B*c*d - B*a*f - A*b*f)*ln(f*x^2 + d)/f^2 + 1/2*(B*c*f*x^2 + 2*
B*b*f*x + 2*A*c*f*x)/f^2
```

3.2 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+fx^2}\,dx$

Optimal. Leaf size=228

$$-\frac{\log\left(d+fx^2\right)\left(2Abf(cd-af)-B\left(-f\left(b^2d-a^2f\right)-2acdf+c^2d^2\right)\right)}{2f^3}+\frac{x^2\left(2Abcf-B\left(-2acf+b^2(-f)+c^2d\right)\right)}{2f^2}-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(-A(cd-af)^2-2bBd(cd-af)+Ab^2df\right)}{\sqrt{d}f^{5/2}}+\frac{x\left(-Ac(cd-2af)-bB(2cd-2af)+Ab^2f\right)}{f^2}+\frac{cx^3(Ac+2bB)}{3f}+\frac{Bc^2x^4}{4f}$$

```
[Out] ((A*b^2*f - A*c*(c*d - 2*a*f) - b*B*(2*c*d - 2*a*f))*x)/f^2 + ((2
*A*b*c*f - B*(c^2*d - b^2*f - 2*a*c*f))*x^2)/(2*f^2) + (c*(2*b*B
+ A*c)*x^3)/(3*f) + (B*c^2*x^4)/(4*f) - ((A*b^2*d*f - 2*b*B*d*(c*
d - a*f) - A*(c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]
*f^(5/2)) - ((2*A*b*f*(c*d - a*f) - B*(c^2*d^2 - 2*a*c*d*f - f*(b
^2*d - a^2*f)))*Log[d + f*x^2])/(2*f^3)
```

Rubi [A] time = 0.739239, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148

$$-\frac{\log\left(d+fx^2\right)\left(2Abf(cd-af)-B\left(-f\left(b^2d-a^2f\right)-2acdf+c^2d^2\right)\right)}{2f^3}+\frac{x^2\left(2Abcf-B\left(-2acf+b^2(-f)+c^2d\right)\right)}{2f^2}-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(-A(cd-af)^2-2bBd(cd-af)+Ab^2df\right)}{\sqrt{d}f^{5/2}}+\frac{x\left(-Ac(cd-2af)-bB(2cd-2af)+Ab^2f\right)}{f^2}+\frac{cx^3(Ac+2bB)}{3f}+\frac{Bc^2x^4}{4f}$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2),x]
```

```
[Out] ((A*b^2*f - A*c*(c*d - 2*a*f) - b*B*(2*c*d - 2*a*f))*x)/f^2 + ((2
*A*b*c*f - B*(c^2*d - b^2*f - 2*a*c*f))*x^2)/(2*f^2) + (c*(2*b*B
+ A*c)*x^3)/(3*f) + (B*c^2*x^4)/(4*f) - ((A*b^2*d*f - 2*b*B*d*(c*
d - a*f) - A*(c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]
*f^(5/2)) - ((2*A*b*f*(c*d - a*f) - B*(c^2*d^2 - 2*a*c*d*f - f*(b
^2*d - a^2*f)))*Log[d + f*x^2])/(2*f^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bc^2x^4}{4f}+\frac{cx^3\left(Ac+2Bb\right)}{3f}+\left(2Aacf+Ab^2f-Ac^2d+2Babf-2Bbcd\right)\int\frac{1}{f^2}\,dx+\frac{\left(2Abcf+2Bacf+Bb^2f-Bc^2d\right)\int x\,dx}{f^2}+\frac{\left(2Aabf^2-2Abcdf+Ba^2f^2-2Bacdf-Bb^2df+Bc^2d^2\right)\log\left(d+fx^2\right)}{2f^3}+\frac{\left(Aa^2f^2-2Aacdf-Ab^2df+Ac^2d^2-2Babdf+2Bbcd^2\right)\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}f^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)*(c*x**2+b*x+a)**2/(f*x**2+d),x)
```

```
[Out] B*c**2*x**4/(4*f) + c*x**3*(A*c + 2*B*b)/(3*f) + (2*A*a*c*f + A*b
**2*f - A*c**2*d + 2*B*a*b*f - 2*B*b*c*d)*Integral(f**(-2), x) +
(2*A*b*c*f + 2*B*a*c*f + B*b**2*f - B*c**2*d)*Integral(x, x)/f**2
 + (2*A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b*
*2*d*f + B*c**2*d**2)*log(d + f*x**2)/(2*f**3) + (A*a**2*f**2 - 2
*A*a*c*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**
2)*atan(sqrt(f)*x/sqrt(d))/(sqrt(d)*f**(5/2))
```

Mathematica [A] time = 0.536077, size = 204, normalized size = 0.89

$$\frac{6\log\left(d+fx^2\right)\left(B\left(a^2f^2-2acdf+b^2(-d)f+c^2d^2\right)+2Abf(af-cd)\right)+fx\left(4Ac\left(6af-3cd+cfx^2\right)+4bB\left(6af-6cd+\right.\right.}{12f^3}$$
$$+\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(A(cd-af)^2+2bBd(cd-af)-Ab^2df\right)}{\sqrt{d}f^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2),x]
```

```
[Out] ((-(A*b^2*d*f) + 2*b*B*d*(c*d - a*f) + A*(c*d - a*f)^2)*ArcTan[(S
qrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(5/2)) + (f*x*(12*A*b*c*f*x + 6*b^
2*f*(2*A + B*x) + 3*B*c*x*(-2*c*d + 4*a*f + c*f*x^2) + 4*A*c*(-3*
c*d + 6*a*f + c*f*x^2) + 4*b*B*(-6*c*d + 6*a*f + 2*c*f*x^2)) + 6*
(2*A*b*f*(-(c*d) + a*f) + B*(c^2*d^2 - b^2*d*f - 2*a*c*d*f + a^2*
f^2))*Log[d + f*x^2])/(12*f^3)
```

Maple [A] time = 0.009, size = 373, normalized size = 1.6

$$\frac{Bc^2x^4}{4f}+\frac{Ax^3c^2}{3f}+\frac{2Bx^3bc}{3f}+\frac{Ax^2bc}{f}+\frac{Bx^2ac}{f}+\frac{Bx^2b^2}{2f}-\frac{Bx^2c^2d}{2f^2}+2\frac{Acax}{f}$$
$$+\frac{Ab^2x}{f}-\frac{Ac^2dx}{f^2}+2\frac{bBax}{f}-2\frac{Bbcdx}{f^2}+\frac{\ln\left(fx^2+d\right)Aab}{f}-\frac{\ln\left(fx^2+d\right)Abcd}{f^2}$$
$$+\frac{\ln\left(fx^2+d\right)Ba^2}{2f}-\frac{\ln\left(fx^2+d\right)Bacd}{f^2}-\frac{\ln\left(fx^2+d\right)Bb^2d}{2f^2}+\frac{\ln\left(fx^2+d\right)Bc^2d^2}{2f^3}$$
$$+Aa^2\arctan\left(fx\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}-2\frac{Acad}{f\sqrt{df}}\arctan\left(\frac{fx}{\sqrt{df}}\right)-\frac{Ab^2d}{f}\arctan\left(fx\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}$$
$$+\frac{Ac^2d^2}{f^2}\arctan\left(fx\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}-2\frac{bBad}{f\sqrt{df}}\arctan\left(\frac{fx}{\sqrt{df}}\right)+2\frac{Bbcd^2}{f^2\sqrt{df}}\arctan\left(\frac{fx}{\sqrt{df}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x)
```

```
[Out] 1/4*B*c^2*x^4/f+1/3/f*A*x^3*c^2+2/3/f*B*x^3*b*c+1/f*A*x^2*b*c+1/f
*B*x^2*a*c+1/2/f*B*x^2*b^2-1/2/f^2*B*x^2*c^2*d+2/f*A*c*a*x+1/f*A*
b^2*x-1/f^2*A*c^2*d*x+2/f*b*B*a*x-2/f^2*B*b*c*d*x+1/f*ln(f*x^2+d)
*A*a*b-1/f^2*ln(f*x^2+d)*A*b*c*d+1/2/f*ln(f*x^2+d)*B*a^2-1/f^2*ln
(f*x^2+d)*B*a*c*d-1/2/f^2*ln(f*x^2+d)*B*b^2*d+1/2/f^3*ln(f*x^2+d)
*B*c^2*d^2+1/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*A*a^2-2/f/(d*f)^
(1/2)*arctan(x*f/(d*f)^(1/2))*A*a*c*d-1/f/(d*f)^(1/2)*arctan(x*f/
(d*f)^(1/2))*A*b^2*d+1/f^2/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*A*
c^2*d^2-2/f/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*B*a*b*d+2/f^2/(d*
```

```
f)^(1/2)*arctan(x*f/(d*f)^(1/2))*B*b*c*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)^2*(B*x + A)/(f*x^2 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277998, size = 1, normalized size = 0.

$$\left[\frac{6\left(Aa^2f^3 + \left(2\,Bbc + Ac^2\right)d^2f - \left(2\,Bab + Ab^2 + 2\,Aac\right)df^2\right)\log\left(\frac{2\,dfx + \left(fx^2 - d\right)\sqrt{-df}}{fx^2 + d}\right) + \left(3\,Bc^2f^2x^4 + 4\left(2\,Bbc + Ac^2\right)f^2x^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)^2*(B*x + A)/(f*x^2 + d),x, algorithm="fricas")
```

```
[Out] [1/12*(6*(A*a^2*f^3 + (2*B*b*c + A*c^2)*d^2*f - (2*B*a*b + A*b^2
+ 2*A*a*c)*d*f^2)*log((2*d*f*x + (f*x^2 - d)*sqrt(-d*f))/(f*x^2 +
 d)) + (3*B*c^2*f^2*x^4 + 4*(2*B*b*c + A*c^2)*f^2*x^3 - 6*(B*c^2*
d*f - (B*b^2 + 2*(B*a + A*b)*c)*f^2)*x^2 - 12*((2*B*b*c + A*c^2)*
d*f - (2*B*a*b + A*b^2 + 2*A*a*c)*f^2)*x + 6*(B*c^2*d^2 - (B*b^2
+ 2*(B*a + A*b)*c)*d*f + (B*a^2 + 2*A*a*b)*f^2)*log(f*x^2 + d))*s
qrt(-d*f))/(sqrt(-d*f)*f^3), 1/12*(12*(A*a^2*f^3 + (2*B*b*c + A*c
^2)*d^2*f - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f^2)*arctan(sqrt(d*f)*x
/d) + (3*B*c^2*f^2*x^4 + 4*(2*B*b*c + A*c^2)*f^2*x^3 - 6*(B*c^2*d
*f - (B*b^2 + 2*(B*a + A*b)*c)*f^2)*x^2 - 12*((2*B*b*c + A*c^2)*d
*f - (2*B*a*b + A*b^2 + 2*A*a*c)*f^2)*x + 6*(B*c^2*d^2 - (B*b^2 +
 2*(B*a + A*b)*c)*d*f + (B*a^2 + 2*A*a*b)*f^2)*log(f*x^2 + d))*sq
rt(d*f))/(sqrt(d*f)*f^3)]
```

Sympy [A] time = 12.6081, size = 928, normalized size = 4.07

$$\frac{Bc^2x^4}{4f} + \left(\frac{2Aabf^2 - 2Abcdf + Ba^2f^2 - 2Bacdf - Bb^2df + Bc^2d^2}{2f^3}\right.$$
$$\left. - \frac{\sqrt{-df^7}\left(Aa^2f^2 - 2Aacdf - Ab^2df + Ac^2d^2 - 2Babdf + 2Bbcd^2\right)}{2df^6}\right)\log\left(x + \frac{-2Aabdf^2 + 2Abcd^2f - Ba^2df^2 + 2Bacd^2f}{}\right.$$
$$+ \left(\frac{2Aabf^2 - 2Abcdf + Ba^2f^2 - 2Bacdf - Bb^2df + Bc^2d^2}{2f^3}\right.$$
$$\left. + \frac{\sqrt{-df^7}\left(Aa^2f^2 - 2Aacdf - Ab^2df + Ac^2d^2 - 2Babdf + 2Bbcd^2\right)}{2df^6}\right)\log\left(x + \frac{-2Aabdf^2 + 2Abcd^2f - Ba^2df^2 + 2Bacd^2f}{}\right.$$
$$+ \frac{x^3\left(Ac^2 + 2Bbc\right)}{3f} + \frac{x^2\left(2Abcf + 2Bacf + Bb^2f - Bc^2d\right)}{2f^2} + \frac{x\left(2Aacf + Ab^2f - Ac^2d + 2Babf - 2Bbcd\right)}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)*(c*x**2+b*x+a)**2/(f*x**2+d),x)
```

```
[Out] B*c**2*x**4/(4*f) + ((2*A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 -
2*B*a*c*d*f - B*b**2*d*f + B*c**2*d**2)/(2*f**3) - sqrt(-d*f**7)*
(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d
*f + 2*B*b*c*d**2)/(2*d*f**6))*log(x + (-2*A*a*b*d*f**2 + 2*A*b*c
*d**2*f - B*a**2*d*f**2 + 2*B*a*c*d**2*f + B*b**2*d**2*f - B*c**2
*d**3 + 2*d*f**3*((2*A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B
*a*c*d*f - B*b**2*d*f + B*c**2*d**2)/(2*f**3) - sqrt(-d*f**7)*(A*
a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f
+ 2*B*b*c*d**2)/(2*d*f**6)))/(A*a**2*f**3 - 2*A*a*c*d*f**2 - A*b*
*2*d*f**2 + A*c**2*d**2*f - 2*B*a*b*d*f**2 + 2*B*b*c*d**2*f)) + (
(2*A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*
d*f + B*c**2*d**2)/(2*f**3) + sqrt(-d*f**7)*(A*a**2*f**2 - 2*A*a*
c*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2
*d*f**6))*log(x + (-2*A*a*b*d*f**2 + 2*A*b*c*d**2*f - B*a**2*d*f*
*2 + 2*B*a*c*d**2*f + B*b**2*d**2*f - B*c**2*d**3 + 2*d*f**3*((2*
A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f
 + B*c**2*d**2)/(2*f**3) + sqrt(-d*f**7)*(A*a**2*f**2 - 2*A*a*c*d
*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*
f**6)))/(A*a**2*f**3 - 2*A*a*c*d*f**2 - A*b**2*d*f**2 + A*c**2*d*
*2*f - 2*B*a*b*d*f**2 + 2*B*b*c*d**2*f)) + x**3*(A*c**2 + 2*B*b*c
)/(3*f) + x**2*(2*A*b*c*f + 2*B*a*c*f + B*b**2*f - B*c**2*d)/(2*f
**2) + x*(2*A*a*c*f + A*b**2*f - A*c**2*d + 2*B*a*b*f - 2*B*b*c*d
)/f**2
```

GIAC/XCAS [A] time = 0.265342, size = 355, normalized size = 1.56

$$\frac{\left(2\,Bbcd^2 + Ac^2d^2 - 2\,Babdf - Ab^2df - 2\,Aacdf + Aa^2f^2\right)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f^2} + \frac{\left(Bc^2d^2 - Bb^2df - 2\,Bacdf - 2\,Abcdf + Ba^2f^2 + 2\,Aabf^2\right)\ln\left(fx^2+d\right)}{2\,f^3} + \frac{3\,Bc^2f^3x^4 + 8\,Bbcf^3x^3 + 4\,Ac^2f^3x^3 - 6\,Bc^2df^2x^2 + 6\,Bb^2f^3x^2 + 12\,Bacf^3x^2 + 12\,Abcf^3x^2 - 24\,Bbcdf^2x - 12\,Ac^2df^2x + }{12\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)^2*(B*x + A)/(f*x^2 + d),x, algorithm="giac")
```

```
[Out] (2*B*b*c*d^2 + A*c^2*d^2 - 2*B*a*b*d*f - A*b^2*d*f - 2*A*a*c*d*f
+ A*a^2*f^2)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f^2) + 1/2*(B*c^2*d
^2 - B*b^2*d*f - 2*B*a*c*d*f - 2*A*b*c*d*f + B*a^2*f^2 + 2*A*a*b*
f^2)*ln(f*x^2 + d)/f^3 + 1/12*(3*B*c^2*f^3*x^4 + 8*B*b*c*f^3*x^3
+ 4*A*c^2*f^3*x^3 - 6*B*c^2*d*f^2*x^2 + 6*B*b^2*f^3*x^2 + 12*B*a*
c*f^3*x^2 + 12*A*b*c*f^3*x^2 - 24*B*b*c*d*f^2*x - 12*A*c^2*d*f^2*
x + 24*B*a*b*f^3*x + 12*A*b^2*f^3*x + 24*A*a*c*f^3*x)/f^4
```

3.3 $\int \frac{(A+Bx)(a+bx+cx^2)^3}{d+fx^2}\,dx$

Optimal. Leaf size=441

$$\frac{\log\left(d+fx^2\right)\left(Abf\left(-f\left(b^2d-3a^2f\right)-6acdf+3c^2d^2\right)-B(cd-af)\left(-f\left(3b^2d-a^2f\right)-2acdf+c^2d^2\right)\right)}{2f^4} - \frac{x^2\left(Abf\left(-6acf+b^2(-f)+3c^2d\right)-B\left(-3cf\left(b^2d-a^2f\right)+3ab^2f^2-3ac^2df+c^3d^2\right)\right)}{2f^3} - \frac{x\left(-Ac\left(3a^2f^2-3acdf+c^2d^2\right)+3Ab^2f(cd-af)-3bB(cd-af)^2+b^3Bdf\right)}{f^3} + \frac{cx^4\left(3Abcf-B\left(-3acf-3b^2f+c^2d\right)\right)}{4f^2} + \frac{x^3\left(-Ac^2(cd-3af)-3bBc(cd-2af)+3Ab^2cf+b^3Bf\right)}{3f^2} + \frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(3Ab^2df(cd-af)-A(cd-af)^3-3bBd(cd-af)^2+b^3Bd^2f\right)}{\sqrt{d}f^{7/2}} + \frac{c^2x^5(Ac+3bB)}{5f} + \frac{Bc^3x^6}{6f}$$

```
[Out] -(((b^3*B*d*f + 3*A*b^2*f*(c*d - a*f) - 3*b*B*(c*d - a*f)^2 - A*c
*(c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2))*x)/f^3) - ((A*b*f*(3*c^2*d -
b^2*f - 6*a*c*f) - B*(c^3*d^2 - 3*a*c^2*d*f + 3*a*b^2*f^2 - 3*c*f
*(b^2*d - a^2*f)))*x^2)/(2*f^3) + ((b^3*B*f + 3*A*b^2*c*f - A*c^2
*(c*d - 3*a*f) - 3*b*B*c*(c*d - 2*a*f))*x^3)/(3*f^2) + (c*(3*A*b*
c*f - B*(c^2*d - 3*b^2*f - 3*a*c*f))*x^4)/(4*f^2) + (c^2*(3*b*B +
 A*c)*x^5)/(5*f) + (B*c^3*x^6)/(6*f) + ((b^3*B*d^2*f + 3*A*b^2*d*
f*(c*d - a*f) - 3*b*B*d*(c*d - a*f)^2 - A*(c*d - a*f)^3)*ArcTan[(
Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(7/2)) + ((A*b*f*(3*c^2*d^2 - 6*a
*c*d*f - f*(b^2*d - 3*a^2*f)) - B*(c*d - a*f)*(c^2*d^2 - 2*a*c*d*
f - f*(3*b^2*d - a^2*f)))*Log[d + f*x^2])/(2*f^4)
```

Rubi [A] time = 1.58091, antiderivative size = 441, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\frac{\log\left(d+fx^2\right)\left(Abf\left(-f\left(b^2d-3a^2f\right)-6acdf+3c^2d^2\right)-B(cd-af)\left(-f\left(3b^2d-a^2f\right)-2acdf+c^2d^2\right)\right)}{2f^4} - \frac{x^2\left(Abf\left(-6acf+b^2(-f)+3c^2d\right)-B\left(-3cf\left(b^2d-a^2f\right)+3ab^2f^2-3ac^2df+c^3d^2\right)\right)}{2f^3} - \frac{x\left(-Ac\left(3a^2f^2-3acdf+c^2d^2\right)+3Ab^2f(cd-af)-3bB(cd-af)^2+b^3Bdf\right)}{f^3} + \frac{cx^4\left(3Abcf-B\left(-3acf-3b^2f+c^2d\right)\right)}{4f^2} + \frac{x^3\left(-Ac^2(cd-3af)-3bBc(cd-2af)+3Ab^2cf+b^3Bf\right)}{3f^2} + \frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(3Ab^2df(cd-af)-A(cd-af)^3-3bBd(cd-af)^2+b^3Bd^2f\right)}{\sqrt{d}f^{7/2}} + \frac{c^2x^5(Ac+3bB)}{5f} + \frac{Bc^3x^6}{6f}$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2),x]
```


```
[Out] -(((b^3*B*d*f + 3*A*b^2*f*(c*d - a*f) - 3*b*B*(c*d - a*f)^2 - A*c
*(c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2))*x)/f^3) - ((A*b*f*(3*c^2*d -
b^2*f - 6*a*c*f) - B*(c^3*d^2 - 3*a*c^2*d*f + 3*a*b^2*f^2 - 3*c*f
*(b^2*d - a^2*f)))*x^2)/(2*f^3) + ((b^3*B*f + 3*A*b^2*c*f - A*c^2
*(c*d - 3*a*f) - 3*b*B*c*(c*d - 2*a*f))*x^3)/(3*f^2) + (c*(3*A*b*
c*f - B*(c^2*d - 3*b^2*f - 3*a*c*f))*x^4)/(4*f^2) + (c^2*(3*b*B +
 A*c)*x^5)/(5*f) + (B*c^3*x^6)/(6*f) + ((b^3*B*d^2*f + 3*A*b^2*d*
f*(c*d - a*f) - 3*b*B*d*(c*d - a*f)^2 - A*(c*d - a*f)^3)*ArcTan[(
Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(7/2)) + ((A*b*f*(3*c^2*d^2 - 6*a
*c*d*f - f*(b^2*d - 3*a^2*f)) - B*(c*d - a*f)*(c^2*d^2 - 2*a*c*d*
f - f*(3*b^2*d - a^2*f)))*Log[d + f*x^2])/(2*f^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)*(c*x**2+b*x+a)**3/(f*x**2+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.81564, size = 422, normalized size = 0.96

$$\frac{fx\left(3b\left(4B\left(15a^2f^2+10acf\left(fx^2-3d\right)+c^2\left(15d^2-5dfx^2+3f^2x^4\right)\right)+15Acfx\left(4af-2cd+cfx^2\right)\right)+c\left(4A\left(45a^2f^2+\right.\right.}{}$$

$$+\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(3Ab^2df(cd-af)-A(cd-af)^3-3bBd(cd-af)^2+b^3Bd^2f\right)}{\sqrt{d}f^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2),x]
```

```
[Out] ((b^3*B*d^2*f + 3*A*b^2*d*f*(c*d - a*f) - 3*b*B*d*(c*d - a*f)^2 -
 A*(c*d - a*f)^3)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(7/2))
+ (f*x*(10*b^3*f*(-6*B*d + 3*A*f*x + 2*B*f*x^2) + 15*b^2*f*(3*B*x
*(-2*c*d + 2*a*f + c*f*x^2) + 4*A*(-3*c*d + 3*a*f + c*f*x^2)) + 3
*b*(15*A*c*f*x*(-2*c*d + 4*a*f + c*f*x^2) + 4*B*(15*a^2*f^2 + 10*
a*c*f*(-3*d + f*x^2) + c^2*(15*d^2 - 5*d*f*x^2 + 3*f^2*x^4))) + c
*(5*B*x*(18*a^2*f^2 + 9*a*c*f*(-2*d + f*x^2) + c^2*(6*d^2 - 3*d*f
*x^2 + 2*f^2*x^4)) + 4*A*(45*a^2*f^2 + 15*a*c*f*(-3*d + f*x^2) +
c^2*(15*d^2 - 5*d*f*x^2 + 3*f^2*x^4)))) - 30*(A*b*f*(-3*c^2*d^2 +
 b^2*d*f + 6*a*c*d*f - 3*a^2*f^2) + B*(c*d - a*f)*(c^2*d^2 - 3*b^
2*d*f - 2*a*c*d*f + a^2*f^2))*Log[d + f*x^2])/(60*f^4)
```

Maple [A] time = 0.012, size = 822, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x)
```

```
[Out] 6/f^2/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*B*a*b*c*d^2-3/f^2*A*b^2
*c*d*x-3/2/f^2*B*x^2*b^2*c*d-3/2/f^2*B*x^2*a*c^2*d-3/2/f^2*A*x^2*
b*c^2*d-3/f^2*A*a*c^2*d*x+2/f*B*x^3*a*b*c+3/f*A*x^2*a*b*c+3/f^3*B
*b*c^2*d^2*x-3/f^3/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*B*b*c^2*d^
3-6/f^2*B*a*b*c*d*x+3/f^2/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*A*a
*c^2*d^2+3/f^2/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*A*b^2*c*d^2-3/
f/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*B*a^2*b*d-3/f^2*ln(f*x^2+d)
*A*a*b*c*d-3/f/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*A*a^2*c*d-3/f/
(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*A*a*b^2*d-3/2/f^2*ln(f*x^2+d)
*B*a^2*c*d-3/2/f^2*ln(f*x^2+d)*B*a*b^2*d+3/2/f^3*ln(f*x^2+d)*B*a*
c^2*d^2-1/f^2*B*x^3*b*c^2*d+3/2/f^3*ln(f*x^2+d)*A*b*c^2*d^2+3/2/f
^3*ln(f*x^2+d)*B*b^2*c*d^2-1/f^3/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/
2))*A*c^3*d^3+1/f^2/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*b^3*B*d^2
+1/5/f*A*x^5*c^3+1/2/f*A*x^2*b^3+1/2/f*ln(f*x^2+d)*B*a^3+1/(d*f)^
(1/2)*arctan(x*f/(d*f)^(1/2))*A*a^3+1/3/f*B*x^3*b^3+1/f^3*A*c^3*d
```

```
^2*x+3/f*b*B*a^2*x+3/f*A*c*a^2*x+3/f*b^2*A*a*x+3/4/f*B*x^4*b^2*c-
1/4/f^2*B*x^4*c^3*d-1/3/f^2*A*x^3*c^3*d+3/2/f*B*x^2*a^2*c+3/2/f*B
*x^2*a*b^2-1/2/f^2*ln(f*x^2+d)*A*b^3*d-1/2/f^4*ln(f*x^2+d)*B*c^3*
d^3+3/2/f*ln(f*x^2+d)*A*a^2*b-1/f^2*b^3*B*d*x+3/4/f*B*x^4*a*c^2+3
/4/f*A*x^4*b*c^2+3/5/f*B*x^5*b*c^2+1/2/f^3*B*x^2*c^3*d^2+1/f*A*x^
3*b^2*c+1/f*A*x^3*a*c^2+1/6*B*c^3*x^6/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)^3*(B*x + A)/(f*x^2 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282428, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)^3*(B*x + A)/(f*x^2 + d),x, algorithm="fricas")
```

```
[Out] [1/60*(30*(A*a^3*f^4 - (3*B*b*c^2 + A*c^3)*d^3*f + (B*b^3 + 3*A*a
*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f^2 - 3*(B*a^2*b + A*a*b^2 + A*
a^2*c)*d*f^3)*log((2*d*f*x + (f*x^2 - d)*sqrt(-d*f))/(f*x^2 + d))
 + (10*B*c^3*f^3*x^6 + 12*(3*B*b*c^2 + A*c^3)*f^3*x^5 - 15*(B*c^3
*d*f^2 - 3*(B*b^2*c + (B*a + A*b)*c^2)*f^3)*x^4 - 20*((3*B*b*c^2
+ A*c^3)*d*f^2 - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*f^3)
*x^3 + 30*(B*c^3*d^2*f - 3*(B*b^2*c + (B*a + A*b)*c^2)*d*f^2 + (3
*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*f^3)*x^2 + 60*((3*B*b*c
^2 + A*c^3)*d^2*f - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d
*f^2 + 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*f^3)*x - 30*(B*c^3*d^3 - 3
*(B*b^2*c + (B*a + A*b)*c^2)*d^2*f + (3*B*a*b^2 + A*b^3 + 3*(B*a^
2 + 2*A*a*b)*c)*d*f^2 - (B*a^3 + 3*A*a^2*b)*f^3)*log(f*x^2 + d))*
sqrt(-d*f))/(sqrt(-d*f)*f^4), 1/60*(60*(A*a^3*f^4 - (3*B*b*c^2 +
A*c^3)*d^3*f + (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f^
2 - 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^3)*arctan(sqrt(d*f)*x/d)
+ (10*B*c^3*f^3*x^6 + 12*(3*B*b*c^2 + A*c^3)*f^3*x^5 - 15*(B*c^3*
d*f^2 - 3*(B*b^2*c + (B*a + A*b)*c^2)*f^3)*x^4 - 20*((3*B*b*c^2 +
 A*c^3)*d*f^2 - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*f^3)*
x^3 + 30*(B*c^3*d^2*f - 3*(B*b^2*c + (B*a + A*b)*c^2)*d*f^2 + (3*
B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*f^3)*x^2 + 60*((3*B*b*c^
2 + A*c^3)*d^2*f - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d*
f^2 + 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*f^3)*x - 30*(B*c^3*d^3 - 3*
(B*b^2*c + (B*a + A*b)*c^2)*d^2*f + (3*B*a*b^2 + A*b^3 + 3*(B*a^2
 + 2*A*a*b)*c)*d*f^2 - (B*a^3 + 3*A*a^2*b)*f^3)*log(f*x^2 + d))*s
qrt(d*f))/(sqrt(d*f)*f^4)]
```

Sympy [A] time = 30.0664, size = 1940, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)*(c*x**2+b*x+a)**3/(f*x**2+d),x)
```

```
[Out] B*c**3*x**6/(6*f) + ((3*A*a**2*b*f**3 - 6*A*a*b*c*d*f**2 - A*b**3
*d*f**2 + 3*A*b*c**2*d**2*f + B*a**3*f**3 - 3*B*a**2*c*d*f**2 - 3
*B*a*b**2*d*f**2 + 3*B*a*c**2*d**2*f + 3*B*b**2*c*d**2*f - B*c**3
*d**3)/(2*f**4) - sqrt(-d*f**9)*(A*a**3*f**3 - 3*A*a**2*c*d*f**2
- 3*A*a*b**2*d*f**2 + 3*A*a*c**2*d**2*f + 3*A*b**2*c*d**2*f - A*c
**3*d**3 - 3*B*a**2*b*d*f**2 + 6*B*a*b*c*d**2*f + B*b**3*d**2*f -
 3*B*b*c**2*d**3)/(2*d*f**8))*log(x + (-3*A*a**2*b*d*f**3 + 6*A*a
*b*c*d**2*f**2 + A*b**3*d**2*f**2 - 3*A*b*c**2*d**3*f - B*a**3*d*
f**3 + 3*B*a**2*c*d**2*f**2 + 3*B*a*b**2*d**2*f**2 - 3*B*a*c**2*d
**3*f - 3*B*b**2*c*d**3*f + B*c**3*d**4 + 2*d*f**4*((3*A*a**2*b*f
**3 - 6*A*a*b*c*d*f**2 - A*b**3*d*f**2 + 3*A*b*c**2*d**2*f + B*a*
*3*f**3 - 3*B*a**2*c*d*f**2 - 3*B*a*b**2*d*f**2 + 3*B*a*c**2*d**2
*f + 3*B*b**2*c*d**2*f - B*c**3*d**3)/(2*f**4) - sqrt(-d*f**9)*(A
*a**3*f**3 - 3*A*a**2*c*d*f**2 - 3*A*a*b**2*d*f**2 + 3*A*a*c**2*d
**2*f + 3*A*b**2*c*d**2*f - A*c**3*d**3 - 3*B*a**2*b*d*f**2 + 6*B
*a*b*c*d**2*f + B*b**3*d**2*f - 3*B*b*c**2*d**3)/(2*d*f**8)))/(A*
a**3*f**4 - 3*A*a**2*c*d*f**3 - 3*A*a*b**2*d*f**3 + 3*A*a*c**2*d*
*2*f**2 + 3*A*b**2*c*d**2*f**2 - A*c**3*d**3*f - 3*B*a**2*b*d*f**
3 + 6*B*a*b*c*d**2*f**2 + B*b**3*d**2*f**2 - 3*B*b*c**2*d**3*f))
+ ((3*A*a**2*b*f**3 - 6*A*a*b*c*d*f**2 - A*b**3*d*f**2 + 3*A*b*c*
*2*d**2*f + B*a**3*f**3 - 3*B*a**2*c*d*f**2 - 3*B*a*b**2*d*f**2 +
 3*B*a*c**2*d**2*f + 3*B*b**2*c*d**2*f - B*c**3*d**3)/(2*f**4) +
sqrt(-d*f**9)*(A*a**3*f**3 - 3*A*a**2*c*d*f**2 - 3*A*a*b**2*d*f**
2 + 3*A*a*c**2*d**2*f + 3*A*b**2*c*d**2*f - A*c**3*d**3 - 3*B*a**
2*b*d*f**2 + 6*B*a*b*c*d**2*f + B*b**3*d**2*f - 3*B*b*c**2*d**3)/
(2*d*f**8))*log(x + (-3*A*a**2*b*d*f**3 + 6*A*a*b*c*d**2*f**2 + A
*b**3*d**2*f**2 - 3*A*b*c**2*d**3*f - B*a**3*d*f**3 + 3*B*a**2*c*
d**2*f**2 + 3*B*a*b**2*d**2*f**2 - 3*B*a*c**2*d**3*f - 3*B*b**2*c
*d**3*f + B*c**3*d**4 + 2*d*f**4*((3*A*a**2*b*f**3 - 6*A*a*b*c*d*
f**2 - A*b**3*d*f**2 + 3*A*b*c**2*d**2*f + B*a**3*f**3 - 3*B*a**2
*c*d*f**2 - 3*B*a*b**2*d*f**2 + 3*B*a*c**2*d**2*f + 3*B*b**2*c*d*
*2*f - B*c**3*d**3)/(2*f**4) + sqrt(-d*f**9)*(A*a**3*f**3 - 3*A*a
**2*c*d*f**2 - 3*A*a*b**2*d*f**2 + 3*A*a*c**2*d**2*f + 3*A*b**2*c
*d**2*f - A*c**3*d**3 - 3*B*a**2*b*d*f**2 + 6*B*a*b*c*d**2*f + B*
b**3*d**2*f - 3*B*b*c**2*d**3)/(2*d*f**8)))/(A*a**3*f**4 - 3*A*a*
*2*c*d*f**3 - 3*A*a*b**2*d*f**3 + 3*A*a*c**2*d**2*f**2 + 3*A*b**2
*c*d**2*f**2 - A*c**3*d**3*f - 3*B*a**2*b*d*f**3 + 6*B*a*b*c*d**2
*f**2 + B*b**3*d**2*f**2 - 3*B*b*c**2*d**3*f)) + x**5*(A*c**3 + 3
*B*b*c**2)/(5*f) + x**4*(3*A*b*c**2*f + 3*B*a*c**2*f + 3*B*b**2*c
*f - B*c**3*d)/(4*f**2) + x**3*(3*A*a*c**2*f + 3*A*b**2*c*f - A*c
**3*d + 6*B*a*b*c*f + B*b**3*f - 3*B*b*c**2*d)/(3*f**2) + x**2*(6
*A*a*b*c*f**2 + A*b**3*f**2 - 3*A*b*c**2*d*f + 3*B*a**2*c*f**2 +
3*B*a*b**2*f**2 - 3*B*a*c**2*d*f - 3*B*b**2*c*d*f + B*c**3*d**2)/
(2*f**3) + x*(3*A*a**2*c*f**2 + 3*A*a*b**2*f**2 - 3*A*a*c**2*d*f
- 3*A*b**2*c*d*f + A*c**3*d**2 + 3*B*a**2*b*f**2 - 6*B*a*b*c*d*f
- B*b**3*d*f + 3*B*b*c**2*d**2)/f**3
```

GIAC/XCAS [A] time = 0.266579, size = 841, normalized size = 1.91

$$-\frac{\left(3\,Bbc^2d^3 + Ac^3d^3 - Bb^3d^2f - 6\,Babcd^2f - 3\,Ab^2cd^2f - 3\,Aac^2d^2f + 3\,Ba^2bdf^2 + 3\,Aab^2df^2 + 3\,Aa^2cdf^2 - Aa^3f^3\right)\,\mathrm{arc}}{\sqrt{df}\,f^3}$$

$$-\frac{\left(Bc^3d^3 - 3\,Bb^2cd^2f - 3\,Bac^2d^2f - 3\,Abc^2d^2f + 3\,Bab^2df^2 + Ab^3df^2 + 3\,Ba^2cdf^2 + 6\,Aabcdf^2 - Ba^3f^3 - 3\,Aa^2bf^3\right)\ln\,(}{2\,f^4}$$

$$+\frac{10\,Bc^3f^5x^6 + 36\,Bbc^2f^5x^5 + 12\,Ac^3f^5x^5 - 15\,Bc^3df^4x^4 + 45\,Bb^2cf^5x^4 + 45\,Bac^2f^5x^4 + 45\,Abc^2f^5x^4 - 60\,Bbc^2df^4x^3 - 2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)^3*(B*x + A)/(f*x^2 + d),x, algorithm="giac")
```

```
[Out] -(3*B*b*c^2*d^3 + A*c^3*d^3 - B*b^3*d^2*f - 6*B*a*b*c*d^2*f - 3*A
*b^2*c*d^2*f - 3*A*a*c^2*d^2*f + 3*B*a^2*b*d*f^2 + 3*A*a*b^2*d*f^
2 + 3*A*a^2*c*d*f^2 - A*a^3*f^3)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)
*f^3) - 1/2*(B*c^3*d^3 - 3*B*b^2*c*d^2*f - 3*B*a*c^2*d^2*f - 3*A*
b*c^2*d^2*f + 3*B*a*b^2*d*f^2 + A*b^3*d*f^2 + 3*B*a^2*c*d*f^2 + 6
```

```
*A*a*b*c*d*f^2 - B*a^3*f^3 - 3*A*a^2*b*f^3)*ln(f*x^2 + d)/f^4 + 1
/60*(10*B*c^3*f^5*x^6 + 36*B*b*c^2*f^5*x^5 + 12*A*c^3*f^5*x^5 - 1
5*B*c^3*d*f^4*x^4 + 45*B*b^2*c*f^5*x^4 + 45*B*a*c^2*f^5*x^4 + 45*
A*b*c^2*f^5*x^4 - 60*B*b*c^2*d*f^4*x^3 - 20*A*c^3*d*f^4*x^3 + 20*
B*b^3*f^5*x^3 + 120*B*a*b*c*f^5*x^3 + 60*A*b^2*c*f^5*x^3 + 60*A*a
*c^2*f^5*x^3 + 30*B*c^3*d^2*f^3*x^2 - 90*B*b^2*c*d*f^4*x^2 - 90*B
*a*c^2*d*f^4*x^2 - 90*A*b*c^2*d*f^4*x^2 + 90*B*a*b^2*f^5*x^2 + 30
*A*b^3*f^5*x^2 + 90*B*a^2*c*f^5*x^2 + 180*A*a*b*c*f^5*x^2 + 180*B
*b*c^2*d^2*f^3*x + 60*A*c^3*d^2*f^3*x - 60*B*b^3*d*f^4*x - 360*B*
a*b*c*d*f^4*x - 180*A*b^2*c*d*f^4*x - 180*A*a*c^2*d*f^4*x + 180*B
*a^2*b*f^5*x + 180*A*a*b^2*f^5*x + 180*A*a^2*c*f^5*x)/f^6
```

3.4 $\int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)}\,dx$

Optimal. Leaf size=274

$$\frac{\log\left(a+bx+cx^2\right)\left(-aBf+Abf+Bcd\right)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}-\frac{\log\left(d+fx^2\right)\left(-aBf+Abf+Bcd\right)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}+\frac{\sqrt{f}\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(aAf-Acd+bBd\right)}{\sqrt{d}\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)}{\sqrt{b^2-4ac}\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}$$

```
[Out] (Sqrt[f]*(b*B*d - A*c*d + a*A*f)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sq
rt[d]*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) - ((A*b^2*f + 2*
A*c*(c*d - a*f) - b*B*(c*d + a*f))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^
2*f))) + ((B*c*d + A*b*f - a*B*f)*Log[a + b*x + c*x^2])/(2*(c^2*d
^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) - ((B*c*d + A*b*f - a*B*f)*L
og[d + f*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f)))
```

Rubi [A] time = 0.651443, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259

$$\frac{\log\left(a+bx+cx^2\right)\left(-aBf+Abf+Bcd\right)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}-\frac{\log\left(d+fx^2\right)\left(-aBf+Abf+Bcd\right)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}+\frac{\sqrt{f}\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(aAf-Acd+bBd\right)}{\sqrt{d}\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)}{\sqrt{b^2-4ac}\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/((a + b*x + c*x^2)*(d + f*x^2)),x]
```

```
[Out] (Sqrt[f]*(b*B*d - A*c*d + a*A*f)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sq
rt[d]*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) - ((A*b^2*f + 2*
A*c*(c*d - a*f) - b*B*(c*d + a*f))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^
2*f))) + ((B*c*d + A*b*f - a*B*f)*Log[a + b*x + c*x^2])/(2*(c^2*d
^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) - ((B*c*d + A*b*f - a*B*f)*L
og[d + f*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f)))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.76498, size = 212, normalized size = 0.77

$$\frac{\sqrt{d}\left(\sqrt{4ac-b^2}(-aBf+Abf+Bcd)\left(\log(a+x(b+cx))-\log\left(d+fx^2\right)\right)+2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(2Ac(cd-af)-bB(af+cd)\right.\right.}{2\sqrt{d}\sqrt{4ac-b^2}\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/((a + b*x + c*x^2)*(d + f*x^2)),x]
```

```
[Out] (2*Sqrt[-b^2 + 4*a*c]*Sqrt[f]*(b*B*d - A*c*d + a*A*f)*ArcTan[(Sqr
t[f]*x)/Sqrt[d]] + Sqrt[d]*(2*(A*b^2*f + 2*A*c*(c*d - a*f) - b*B*
(c*d + a*f))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 +
 4*a*c]*(B*c*d + A*b*f - a*B*f)*(-Log[d + f*x^2] + Log[a + x*(b +
 c*x)])))/(2*Sqrt[-b^2 + 4*a*c]*Sqrt[d]*(c^2*d^2 - 2*a*c*d*f + f*
(b^2*d + a^2*f)))
```

Maple [B] time = 0.012, size = 745, normalized size = 2.7

$$\frac{\ln\left(cx^2+bx+a\right)Abf}{2a^2f^2-4acdf+2b^2df+2c^2d^2}-\frac{\ln\left(cx^2+bx+a\right)Baf}{2a^2f^2-4acdf+2b^2df+2c^2d^2}$$
$$+\frac{c\ln\left(cx^2+bx+a\right)Bd}{2a^2f^2-4acdf+2b^2df+2c^2d^2}$$
$$-2\frac{Aacf}{\left(a^2f^2-2acdf+b^2df+c^2d^2\right)\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)$$
$$+\frac{Ab^2f}{a^2f^2-2acdf+b^2df+c^2d^2}\arctan\left((2cx+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$
$$+2\frac{Ac^2d}{\left(a^2f^2-2acdf+b^2df+c^2d^2\right)\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)$$
$$-\frac{Babf}{a^2f^2-2acdf+b^2df+c^2d^2}\arctan\left((2cx+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$
$$-\frac{Bbcd}{a^2f^2-2acdf+b^2df+c^2d^2}\arctan\left((2cx+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$
$$-\frac{f\ln\left(fx^2+d\right)Ab}{2a^2f^2-4acdf+2b^2df+2c^2d^2}$$
$$+\frac{f\ln\left(fx^2+d\right)aB}{2a^2f^2-4acdf+2b^2df+2c^2d^2}-\frac{\ln\left(fx^2+d\right)Bcd}{2a^2f^2-4acdf+2b^2df+2c^2d^2}$$
$$+\frac{Aaf^2}{a^2f^2-2acdf+b^2df+c^2d^2}\arctan\left(fx\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}$$
$$-\frac{Acdf}{a^2f^2-2acdf+b^2df+c^2d^2}\arctan\left(fx\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}$$
$$+\frac{Bbdf}{a^2f^2-2acdf+b^2df+c^2d^2}\arctan\left(fx\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x)
```

```
[Out] 1/2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*ln(c*x^2+b*x+a)*A*b*f-1/2
/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*ln(c*x^2+b*x+a)*B*a*f+1/2/(a
^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*c*ln(c*x^2+b*x+a)*B*d-2/(a^2*f^
2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(
4*a*c-b^2)^(1/2))*A*a*c*f+1/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(
4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*b^2*f+2/(a
^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x
+b)/(4*a*c-b^2)^(1/2))*A*c^2*d-1/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d
^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*B*a*b*f
-1/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2))*B*b*c*d-1/2*f/(a^2*f^2-2*a*c*d*f+b^2*
d*f+c^2*d^2)*ln(f*x^2+d)*A*b+1/2*f/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2
*d^2)*ln(f*x^2+d)*a*B-1/2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*ln(
f*x^2+d)*B*c*d+f^2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(d*f)^(1/2
)*arctan(x*f/(d*f)^(1/2))*a*A-f/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^
2)/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*A*c*d+f/(a^2*f^2-2*a*c*d*f
```

```
+b^2*d*f+c^2*d^2)/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*B*b*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + b*x + a)*(f*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + b*x + a)*(f*x^2 + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.266682, size = 359, normalized size = 1.31

$$\frac{(Bcd - Baf + Abf)\ln\left(cx^2 + bx + a\right)}{2\,(c^2d^2 + b^2df - 2\,acdf + a^2f^2)} - \frac{(Bcd - Baf + Abf)\ln\left(fx^2 + d\right)}{2\,(c^2d^2 + b^2df - 2\,acdf + a^2f^2)} + \frac{\left(Bbdf - Acdf + Aaf^2\right)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{(c^2d^2 + b^2df - 2\,acdf + a^2f^2)\sqrt{df}} - \frac{\left(Bbcd - 2\,Ac^2d + Babf - Ab^2f + 2\,Aacf\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(c^2d^2 + b^2df - 2\,acdf + a^2f^2)\sqrt{-b^2 + 4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + b*x + a)*(f*x^2 + d)),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*d - B*a*f + A*b*f)*ln(c*x^2 + b*x + a)/(c^2*d^2 + b^2*d*
f - 2*a*c*d*f + a^2*f^2) - 1/2*(B*c*d - B*a*f + A*b*f)*ln(f*x^2 +
 d)/(c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2) + (B*b*d*f - A*c*d*
f + A*a*f^2)*arctan(f*x/sqrt(d*f))/((c^2*d^2 + b^2*d*f - 2*a*c*d*
f + a^2*f^2)*sqrt(d*f)) - (B*b*c*d - 2*A*c^2*d + B*a*b*f - A*b^2*
f + 2*A*a*c*f)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^2*d^2 +
 b^2*d*f - 2*a*c*d*f + a^2*f^2)*sqrt(-b^2 + 4*a*c))
```

3.5 $\int \frac{A+Bx}{(a+bx+cx^2)^2(d+fx^2)}\,dx$

Optimal. Leaf size=596

$$-\frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(-A(cd-af)^2+2bBd(cd-af)+Ab^2df\right)}{\sqrt{d}\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}-\frac{f\log\left(a+bx+cx^2\right)\left(B\left(-f\left(b^2d-a^2f\right)-2acdf+c^2d^2\right)+2Abf(cd-af)\right)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}+\frac{f\log\left(d+fx^2\right)\left(B\left(-f\left(b^2d-a^2f\right)-2acdf+c^2d^2\right)+2Abf(cd-af)\right)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-4Ab^2cf\left(3a^2f^2-3acdf+2c^2d^2\right)+b^3Bf\left(-a^2f^2-4acdf+5c^2d^2\right)+2bBc\left(3a^3f^3+3a^2cdf^2-7ac\right.\right.}{\left(b^2-4ac\right)^{3/2}\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}+\frac{-(Ab-aB)\left(-2acf+b^2f+2c^2d\right)-cx\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)+Abc(af+cd)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left((cd-af)^2+b^2df\right)}$$

```
[Out] (A*b*c*(c*d + a*f) - (A*b - a*B)*(2*c^2*d + b^2*f - 2*a*c*f) - c*
(A*b^2*f + 2*A*c*(c*d - a*f) - b*B*(c*d + a*f))*x)/((b^2 - 4*a*c)
*(b^2*d*f + (c*d - a*f)^2)*(a + b*x + c*x^2)) - (f^(3/2)*(A*b^2*d
*f + 2*b*B*d*(c*d - a*f) - A*(c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)/Sq
rt[d]])/(Sqrt[d]*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2) - (
(b^5*B*d*f^2 - 2*A*b^4*f^2*(c*d - a*f) - 4*A*c^2*(c*d - 3*a*f)*(c
*d - a*f)^2 + b^3*B*f*(5*c^2*d^2 - 4*a*c*d*f - a^2*f^2) - 4*A*b^2
*c*f*(2*c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2) + 2*b*B*c*(c^3*d^3 - 7*a
*c^2*d^2*f + 3*a^2*c*d*f^2 + 3*a^3*f^3))*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c^2*d^2 - 2*a*c*d*f + f*(b^
2*d + a^2*f))^2) - (f*(2*A*b*f*(c*d - a*f) + B*(c^2*d^2 - 2*a*c*d
*f - f*(b^2*d - a^2*f)))*Log[a + b*x + c*x^2])/(2*(c^2*d^2 - 2*a*
c*d*f + f*(b^2*d + a^2*f))^2) + (f*(2*A*b*f*(c*d - a*f) + B*(c^2*
d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[d + f*x^2])/(2*(c^2*d^2
 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2)
```

Rubi [A] time = 3.94124, antiderivative size = 596, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$

$$-\frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(-A(cd-af)^2+2bBd(cd-af)+Ab^2df\right)}{\sqrt{d}\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}-\frac{f\log\left(a+bx+cx^2\right)\left(B\left(-f\left(b^2d-a^2f\right)-2acdf+c^2d^2\right)+2Abf(cd-af)\right)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}+\frac{f\log\left(d+fx^2\right)\left(B\left(-f\left(b^2d-a^2f\right)-2acdf+c^2d^2\right)+2Abf(cd-af)\right)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-4Ab^2cf\left(3a^2f^2-3acdf+2c^2d^2\right)+b^3Bf\left(-a^2f^2-4acdf+5c^2d^2\right)+2bBc\left(3a^3f^3+3a^2cdf^2-7ac\right.\right.}{\left(b^2-4ac\right)^{3/2}\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}+\frac{-(Ab-aB)\left(-2acf+b^2f+2c^2d\right)-cx\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)+Abc(af+cd)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left((cd-af)^2+b^2df\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/((a + b*x + c*x^2)^2*(d + f*x^2)),x]
```

```
[Out] (A*b*c*(c*d + a*f) - (A*b - a*B)*(2*c^2*d + b^2*f - 2*a*c*f) - c*
(A*b^2*f + 2*A*c*(c*d - a*f) - b*B*(c*d + a*f))*x)/((b^2 - 4*a*c)
*(b^2*d*f + (c*d - a*f)^2)*(a + b*x + c*x^2)) - (f^(3/2)*(A*b^2*d
*f + 2*b*B*d*(c*d - a*f) - A*(c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)/Sq
rt[d]])/(Sqrt[d]*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2) - (
(b^5*B*d*f^2 - 2*A*b^4*f^2*(c*d - a*f) - 4*A*c^2*(c*d - 3*a*f)*(c
```

```
*d - a*f)^2 + b^3*B*f*(5*c^2*d^2 - 4*a*c*d*f - a^2*f^2) - 4*A*b^2
*c*f*(2*c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2) + 2*b*B*c*(c^3*d^3 - 7*a
*c^2*d^2*f + 3*a^2*c*d*f^2 + 3*a^3*f^3))*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c^2*d^2 - 2*a*c*d*f + f*(b^
2*d + a^2*f))^2) - (f*(2*A*b*f*(c*d - a*f) + B*(c^2*d^2 - 2*a*c*d
*f - f*(b^2*d - a^2*f)))*Log[a + b*x + c*x^2])/(2*(c^2*d^2 - 2*a*
c*d*f + f*(b^2*d + a^2*f))^2) + (f*(2*A*b*f*(c*d - a*f) + B*(c^2*
d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[d + f*x^2])/(2*(c^2*d^2
 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/(c*x**2+b*x+a)**2/(f*x**2+d),x)

[Out] Timed out
```

Mathematica [A] time = 4.05286, size = 523, normalized size = 0.88

$$\frac{f\log\left(d+fx^2\right)\left(B\left(f\left(a^2f-b^2d\right)-2acdf+c^2d^2\right)+2Abf(cd-af)\right)+f\log(a+x(b+cx))\left(B\left(f\left(b^2d-a^2f\right)+2acdf-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/((a + b*x + c*x^2)^2*(d + f*x^2)),x]

[Out] ((-2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))*(A*(b^3*f + b*c*(c
*d - 3*a*f) + b^2*c*f*x + 2*c^2*(c*d - a*f)*x) + B*(2*a^2*c*f - b
*c^2*d*x - a*(2*c^2*d + b^2*f + b*c*f*x))))/((b^2 - 4*a*c)*(a + x
*(b + c*x))) + (2*f^(3/2)*(-(A*b^2*d*f) + A*(c*d - a*f)^2 + 2*b*B
*d*(-(c*d) + a*f))*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/Sqrt[d] - (2*(b^5
*B*d*f^2 - 4*A*c^2*(c*d - 3*a*f)*(c*d - a*f)^2 + 2*A*b^4*f^2*(-(c
*d) + a*f) - b^3*B*f*(-5*c^2*d^2 + 4*a*c*d*f + a^2*f^2) - 4*A*b^2
*c*f*(2*c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2) + 2*b*B*c*(c^3*d^3 - 7*a
*c^2*d^2*f + 3*a^2*c*d*f^2 + 3*a^3*f^3))*ArcTan[(b + 2*c*x)/Sqrt[
-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + f*(2*A*b*f*(c*d - a*f) + B
*(c^2*d^2 - 2*a*c*d*f + f*(-(b^2*d) + a^2*f)))*Log[d + f*x^2] + f
*(2*A*b*f*(-(c*d) + a*f) + B*(-(c^2*d^2) + 2*a*c*d*f + f*(b^2*d -
 a^2*f)))*Log[a + x*(b + c*x)])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*
d + a^2*f))^2)
```

Maple [B] time = 0.04, size = 10537, normalized size = 17.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + b*x + a)^2*(f*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + b*x + a)^2*(f*x^2 + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**2/(f*x**2+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.271936, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + b*x + a)^2*(f*x^2 + d)),x, algorithm="giac")
```

```
[Out] Done
```

3.6 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

Optimal. Leaf size=331

$$-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}f^{3/2}}+\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}f^{3/2}}-\frac{(2Ac+bB)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f}-\frac{B\sqrt{a+bx+cx^2}}{f}$$

```
[Out] -((B*Sqrt[a + b*x + c*x^2])/f) - ((b*B + 2*A*c)*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f) - ((B*Sqrt[d
] - A*Sqrt[f])*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqr
t[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b
*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*f^(3/
2)) + ((B*Sqrt[d] + A*Sqrt[f])*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f
]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)
/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/
(2*Sqrt[d]*f^(3/2))
```

Rubi [A] time = 1.32177, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}f^{3/2}}+\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}f^{3/2}}-\frac{(2Ac+bB)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f}-\frac{B\sqrt{a+bx+cx^2}}{f}$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -((B*Sqrt[a + b*x + c*x^2])/f) - ((b*B + 2*A*c)*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f) - ((B*Sqrt[d
] - A*Sqrt[f])*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqr
t[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b
*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*f^(3/
2)) + ((B*Sqrt[d] + A*Sqrt[f])*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f
]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)
/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/
(2*Sqrt[d]*f^(3/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.99992, size = 450, normalized size = 1.36

$$-\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\log\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}{\sqrt{d}}+\frac{\left(A\sqrt{f}-B\sqrt{d}\right)\log\left(\sqrt{d}\sqrt{f}+fx\right)\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}{\sqrt{d}}+\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\log\left(\sqrt{d}\left(2\sqrt{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] (-2*B*Sqrt[f]*Sqrt[a + x*(b + c*x)] - ((B*Sqrt[d] + A*Sqrt[f])*Sq
rt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Log[Sqrt[d]*Sqrt[f] - f*x])/Sqr
t[d] + ((-(B*Sqrt[d]) + A*Sqrt[f])*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] +
 a*f]*Log[Sqrt[d]*Sqrt[f] + f*x])/Sqrt[d] - ((b*B + 2*A*c)*Sqrt[f
]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/Sqrt[c] + ((B
*Sqrt[d] - A*Sqrt[f])*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Log[Sqr
t[d]*(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x +
2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sq
rt[d] + ((B*Sqrt[d] + A*Sqrt[f])*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a
*f]*Log[Sqrt[d]*(b*(Sqrt[d] + Sqrt[f]*x) + 2*(a*Sqrt[f] + c*Sqrt[
d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])
)])/Sqrt[d])/(2*f^(3/2))
```

Maple [B] time = 0.075, size = 3358, normalized size = 10.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)
*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*A-1/2/f*((
x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)
+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*B-1/2/f*ln((1/2/f*(-2*c*(d*f
)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+
1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+
f*a+c*d))^(1/2))*c^(1/2)*A+1/2*(d*f)^(1/2)/f^2*ln((1/2/f*(-2*c*(d
*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*
c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2
)+f*a+c*d))^(1/2))*c^(1/2)*B+1/4/(d*f)^(1/2)*ln((1/2/f*(-2*c*(d*f
)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+
1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+
f*a+c*d))^(1/2))/c^(1/2)*b*A-1/4/f*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*
f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*
(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^
(1/2))/c^(1/2)*b*B+1/2/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln(
(2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/
f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)
^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*A-1/2*(d*f)^(1/2)/f^
2/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*
a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d
*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^
(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))
```

```
/(x+(d*f)^(1/2)/f))*b*B-1/2/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+f*a+
c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2
)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*(
(x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f
)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a*A+1/2
/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f
*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(
d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)
^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)
)/(x+(d*f)^(1/2)/f))*a*B-1/2/(d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+f
*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(
1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2
)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2
)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c*d*
A+1/2/f^2/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^
(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/
f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*
c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d)
)^(1/2))/(x+(d*f)^(1/2)/f))*c*d*B-1/2/(d*f)^(1/2)*((x-(d*f)^(1/2)
/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+
f*a+c*d)/f)^(1/2)*A-1/2/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)
+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*B-1/2/
f*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((
x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b
*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))*c^(1/2)*A-1/2*(d*f)^(1/2)/f^2*ln(
(1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*
f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f
)^(1/2)+f*a+c*d)/f)^(1/2))*c^(1/2)*B-1/4/(d*f)^(1/2)*ln((1/2*(2*c
*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/
f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f
*a+c*d)/f)^(1/2))/c^(1/2)*b*A-1/4/f*ln((1/2*(2*c*(d*f)^(1/2)+b*f)
/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)
^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))
/c^(1/2)*b*B+1/2/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*
f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*(
(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)
)/(x-(d*f)^(1/2)/f))*b*A+1/2*(d*f)^(1/2)/f^2/((b*(d*f)^(1/2)+f*a+
c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*
f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d
*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*
f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*B+1/2/(d*f)^(1/2
)/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)
/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*
a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x
-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/
f))*a*A+1/2/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1
/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d
*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-
(d*f)^(1/2)/f))*a*B+1/2/(d*f)^(1/2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)
^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x
-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/
2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2
)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c*d*A+1/2/f^2/((b*(d*f)^(
1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)
^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/
2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/
f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c*d*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(c*x^2 + b*x + a)*(B*x + A)/(f*x^2 - d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(c*x^2 + b*x + a)*(B*x + A)/(f*x^2 - d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{A\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{Bx\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(A*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(B
*x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(c*x^2 + b*x + a)*(B*x + A)/(f*x^2 - d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.7 $\int \frac{A+Bx}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal. Leaf size=249

$$\frac{\left(\frac{A\sqrt{f}}{\sqrt{d}}+B\right)\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

```
[Out] -((B - (A*Sqrt[f])/Sqrt[d])*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2
*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]
*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f]
 + a*f]) + ((B + (A*Sqrt[f])/Sqrt[d])*ArcTanh[(b*Sqrt[d] + 2*a*Sq
rt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[f]*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f])
```

Rubi [A] time = 0.491878, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{\left(\frac{A\sqrt{f}}{\sqrt{d}}+B\right)\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -((B - (A*Sqrt[f])/Sqrt[d])*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2
*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]
*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f]
 + a*f]) + ((B + (A*Sqrt[f])/Sqrt[d])*ArcTanh[(b*Sqrt[d] + 2*a*Sq
rt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[f]*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f])
```

Rubi in Sympy [A] time = 64.3761, size = 238, normalized size = 0.96

$$\frac{\left(A\sqrt{f}-B\sqrt{d}\right)\operatorname{atanh}\left(\frac{-2a\sqrt{f}+b\sqrt{d}+x\left(-b\sqrt{f}+2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{f}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}-\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\operatorname{atanh}\left(\frac{-2a\sqrt{f}-b\sqrt{d}+x\left(-b\sqrt{f}-2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] (A*sqrt(f) - B*sqrt(d))*atanh((-2*a*sqrt(f) + b*sqrt(d) + x*(-b*s
qrt(f) + 2*c*sqrt(d)))/(2*sqrt(a + b*x + c*x**2)*sqrt(a*f - b*sqr
t(d)*sqrt(f) + c*d)))/(2*sqrt(d)*sqrt(f)*sqrt(a*f - b*sqrt(d)*sqr
t(f) + c*d)) - (A*sqrt(f) + B*sqrt(d))*atanh((-2*a*sqrt(f) - b*sq
rt(d) + x*(-b*sqrt(f) - 2*c*sqrt(d)))/(2*sqrt(a + b*x + c*x**2)*s
qrt(a*f + b*sqrt(d)*sqrt(f) + c*d)))/(2*sqrt(d)*sqrt(f)*sqrt(a*f
+ b*sqrt(d)*sqrt(f) + c*d))
```

Mathematica [A] time = 2.25491, size = 370, normalized size = 1.49

$$\frac{-\frac{\left(Af+B\sqrt{d}\sqrt{f}\right)\log\left(\sqrt{d}\sqrt{f}-fx\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\left(B\sqrt{d}\sqrt{f}-Af\right)\log\left(\sqrt{d}\sqrt{f}+fx\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\left(B\sqrt{d}\sqrt{f}-Af\right)\log\left(\sqrt{d}\left(2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}+2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\cdots\right.\right.}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\cdots}{2\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] (-(((B*Sqrt[d]*Sqrt[f] + A*f)*Log[Sqrt[d]*Sqrt[f] - f*x])/Sqrt[c*
d + b*Sqrt[d]*Sqrt[f] + a*f]) - ((B*Sqrt[d]*Sqrt[f] - A*f)*Log[Sq
rt[d]*Sqrt[f] + f*x])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] + ((B*S
qrt[d]*Sqrt[f] - A*f)*Log[Sqrt[d]*(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2
*c*Sqrt[d]*x + b*Sqrt[f]*x + 2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f
]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] +
((B*Sqrt[d]*Sqrt[f] + A*f)*Log[Sqrt[d]*(b*(Sqrt[d] + Sqrt[f]*x) +
 2*(a*Sqrt[f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]
*Sqrt[a + x*(b + c*x)]))])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])/(
2*Sqrt[d]*f)
```

Maple [B] time = 0.029, size = 714, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b
*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f
)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1
/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f
*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*A+1/2/f/(1/f*(-b*(d*f)^(1/2)+f
*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(
1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2
)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2
)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*B+1/
2/(d*f)^(1/2)/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1
/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d
*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-
(d*f)^(1/2)/f))*A+1/2/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(
b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f
)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c
*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^
(1/2))/(x-(d*f)^(1/2)/f))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(B*x + A)/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 53.1511, size = 8253, normalized size = 33.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(B*x + A)/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*c)*d*f +
 (c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^4*b^2 -
 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2
- 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b
 + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d
^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a
^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2))
*log(-((B^4*b^2 - 2*A*B^3*b*c)*d^2 - 2*(A*B^3*a*b - A^3*B*b*c)*d*
f + (2*A^3*B*a*b - A^4*b^2)*f^2 + 2*((2*A^3*B*a - A^4*b)*c*f^2 +
(B^4*b*c - 2*A*B^3*c^2)*d^2 - 2*(A*B^3*a*c - A^3*B*c^2)*d*f)*x +
2*((B^3*b^2 - 3*A*B^2*b*c + 2*A^2*B*c^2)*d^2*f - (3*A*B^2*a*b - A
^2*B*b^2 - (4*A^2*B*a - A^3*b)*c)*d*f^2 + (2*A^2*B*a^2 - A^3*a*b)
*f^3 - (B*c^3*d^4*f - (B*b^2*c - (3*B*a - A*b)*c^2)*d^3*f^2 - (B*
a*b^2 - A*b^3 - (3*B*a^2 - 2*A*a*b)*c)*d^2*f^3 + (B*a^3 - A*a^2*b
)*d*f^4)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2
*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*
A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5
- 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d
^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))*sqrt(c*x^2 + b*x + a)*s
qrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*c)*d*f + (c^2
*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^4*b^2 - 4*A*
B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(
2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^
4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^
2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)
*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)) - (2
*B^2*a*c^2*d^3*f - 2*A^2*a^3*f^4 - 2*(B^2*a*b^2 - 2*B^2*a^2*c + A
^2*a*c^2)*d^2*f^2 + 2*(B^2*a^3 + A^2*a*b^2 - 2*A^2*a^2*c)*d*f^3 +
 (B^2*b*c^2*d^3*f - A^2*a^2*b*f^4 - (B^2*b^3 - 2*B^2*a*b*c + A^2*
b*c^2)*d^2*f^2 + (B^2*a^2*b + A^2*b^3 - 2*A^2*a*b*c)*d*f^3)*x)*sq
rt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b
- A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2
 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^
2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*
(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/x) - 1/4*sqrt((B^2*c*d^2 + A^2*a*f
^2 + (B^2*a - 2*A*B*b + A^2*c)*d*f + (c^2*d^3*f + a^2*d*f^3 - (b^
2 - 2*a*c)*d^2*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)
*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c
)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f +
 a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6
*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f +
 a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2))*log(-((B^4*b^2 - 2*A*B^3*b*c
)*d^2 - 2*(A*B^3*a*b - A^3*B*b*c)*d*f + (2*A^3*B*a*b - A^4*b^2)*f
^2 + 2*((2*A^3*B*a - A^4*b)*c*f^2 + (B^4*b*c - 2*A*B^3*c^2)*d^2 -
 2*(A*B^3*a*c - A^3*B*c^2)*d*f)*x - 2*((B^3*b^2 - 3*A*B^2*b*c + 2
*A^2*B*c^2)*d^2*f - (3*A*B^2*a*b - A^2*B*b^2 - (4*A^2*B*a - A^3*b
)*c)*d*f^2 + (2*A^2*B*a^2 - A^3*a*b)*f^3 - (B*c^3*d^4*f - (B*b^2*
c - (3*B*a - A*b)*c^2)*d^3*f^2 - (B*a*b^2 - A*b^3 - (3*B*a^2 - 2*
A*a*b)*c)*d^2*f^3 + (B*a^3 - A*a^2*b)*d*f^4)*sqrt(((B^4*b^2 - 4*A
*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*
(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A
^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f
^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c
)*d^2*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt((B^2*c*d^2 + A^2*a*f^2 +
(B^2*a - 2*A*B*b + A^2*c)*d*f + (c^2*d^3*f + a^2*d*f^3 - (b^2 - 2
*a*c)*d^2*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2
- 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f
 + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*
d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*
c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*
```

```
d*f^3 - (b^2 - 2*a*c)*d^2*f^2)) - (2*B^2*a*c^2*d^3*f - 2*A^2*a^3*
f^4 - 2*(B^2*a*b^2 - 2*B^2*a^2*c + A^2*a*c^2)*d^2*f^2 + 2*(B^2*a^
3 + A^2*a*b^2 - 2*A^2*a^2*c)*d*f^3 + (B^2*b*c^2*d^3*f - A^2*a^2*b
*f^4 - (B^2*b^3 - 2*B^2*a*b*c + A^2*b*c^2)*d^2*f^2 + (B^2*a^2*b +
 A^2*b^3 - 2*A^2*a*b*c)*d*f^3)*x)*sqrt(((B^4*b^2 - 4*A*B^3*b*c +
4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*
a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2
)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 -
 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4))
)/x) + 1/4*sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*c
)*d*f - (c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^
4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B
^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^
3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a
*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^
2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^
2*f^2))*log(-((B^4*b^2 - 2*A*B^3*b*c)*d^2 - 2*(A*B^3*a*b - A^3*B*
b*c)*d*f + (2*A^3*B*a*b - A^4*b^2)*f^2 + 2*((2*A^3*B*a - A^4*b)*c
*f^2 + (B^4*b*c - 2*A*B^3*c^2)*d^2 - 2*(A*B^3*a*c - A^3*B*c^2)*d*
f)*x + 2*((B^3*b^2 - 3*A*B^2*b*c + 2*A^2*B*c^2)*d^2*f - (3*A*B^2*
a*b - A^2*B*b^2 - (4*A^2*B*a - A^3*b)*c)*d*f^2 + (2*A^2*B*a^2 - A
^3*a*b)*f^3 + (B*c^3*d^4*f - (B*b^2*c - (3*B*a - A*b)*c^2)*d^3*f^
2 - (B*a*b^2 - A*b^3 - (3*B*a^2 - 2*A*a*b)*c)*d^2*f^3 + (B*a^3 -
A*a^2*b)*d*f^4)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2
 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*
f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4
*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2
*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))*sqrt(c*x^2 + b*x
 + a)*sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*c)*d*f
 - (c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^4*b^2
 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^
2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a
*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)
*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2
*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2
)) + (2*B^2*a*c^2*d^3*f - 2*A^2*a^3*f^4 - 2*(B^2*a*b^2 - 2*B^2*a^
2*c + A^2*a*c^2)*d^2*f^2 + 2*(B^2*a^3 + A^2*a*b^2 - 2*A^2*a^2*c)*
d*f^3 + (B^2*b*c^2*d^3*f - A^2*a^2*b*f^4 - (B^2*b^3 - 2*B^2*a*b*c
 + A^2*b*c^2)*d^2*f^2 + (B^2*a^2*b + A^2*b^3 - 2*A^2*a*b*c)*d*f^3
)*x)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B
^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*
B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*
(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f
^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/x) - 1/4*sqrt((B^2*c*d^2 +
A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*c)*d*f - (c^2*d^3*f + a^2*d*f^
3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B
^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3
*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*
d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*
d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2))*log(-((B^4*b^2 - 2*A*
B^3*b*c)*d^2 - 2*(A*B^3*a*b - A^3*B*b*c)*d*f + (2*A^3*B*a*b - A^4
*b^2)*f^2 + 2*((2*A^3*B*a - A^4*b)*c*f^2 + (B^4*b*c - 2*A*B^3*c^2
)*d^2 - 2*(A*B^3*a*c - A^3*B*c^2)*d*f)*x - 2*((B^3*b^2 - 3*A*B^2*
b*c + 2*A^2*B*c^2)*d^2*f - (3*A*B^2*a*b - A^2*B*b^2 - (4*A^2*B*a
- A^3*b)*c)*d*f^2 + (2*A^2*B*a^2 - A^3*a*b)*f^3 + (B*c^3*d^4*f -
(B*b^2*c - (3*B*a - A*b)*c^2)*d^3*f^2 - (B*a*b^2 - A*b^3 - (3*B*a
^2 - 2*A*a*b)*c)*d^2*f^3 + (B*a^3 - A*a^2*b)*d*f^4)*sqrt(((B^4*b^
2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b
^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*
a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3
)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 -
2*a^3*c)*d^2*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt((B^2*c*d^2 + A^2*a
*f^2 + (B^2*a - 2*A*B*b + A^2*c)*d*f - (c^2*d^3*f + a^2*d*f^3 - (
b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^
2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)
*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f
 + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c +
 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f
 + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)) + (2*B^2*a*c^2*d^3*f - 2*A
^2*a^3*f^4 - 2*(B^2*a*b^2 - 2*B^2*a^2*c + A^2*a*c^2)*d^2*f^2 + 2*
(B^2*a^3 + A^2*a*b^2 - 2*A^2*a^2*c)*d*f^3 + (B^2*b*c^2*d^3*f - A^
```

```
2*a^2*b*f^4 - (B^2*b^3 - 2*B^2*a*b*c + A^2*b*c^2)*d^2*f^2 + (B^2*
a^2*b + A^2*b^3 - 2*A^2*a*b*c)*d*f^3)*x)*sqrt(((B^4*b^2 - 4*A*B^3
*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A
^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b
^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 +
 (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^
2*f^4)))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{A}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx-\int \frac{Bx}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(A/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*
x**2)), x) - Integral(B*x/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqr
t(a + b*x + c*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(B*x + A)/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.8 $\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

Optimal. Leaf size=381

$$-\frac{2\left(A\left(b^3f-bc(3af+cd)\right)+cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)+aB\left(2acf+b^2(-f)+2c^2d\right)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}-\frac{\sqrt{f}\left(B\sqrt{d}-A\sqrt{f}\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\sqrt{f}\left(A\sqrt{f}+B\sqrt{d}\right)\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

```
[Out] (-2*(a*B*(2*c^2*d - b^2*f + 2*a*c*f) + A*(b^3*f - b*c*(c*d + 3*a*
f)) + c*(A*b^2*f + b*B*(c*d - a*f) - 2*A*c*(c*d + a*f))*x))/((b^2
 - 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ((B*
Sqrt[d] - A*Sqrt[f])*Sqrt[f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (
2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f
]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] +
a*f)^(3/2)) + ((B*Sqrt[d] + A*Sqrt[f])*Sqrt[f]*ArcTanh[(b*Sqrt[d]
 + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d + b*
Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rubi [A] time = 1.92169, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$-\frac{2\left(cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)-Abc(3af+cd)+aB\left(2acf+b^2(-f)+2c^2d\right)+Ab^3f\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}-\frac{\sqrt{f}\left(B\sqrt{d}-A\sqrt{f}\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\sqrt{f}\left(A\sqrt{f}+B\sqrt{d}\right)\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(A*b^3*f - A*b*c*(c*d + 3*a*f) + a*B*(2*c^2*d - b^2*f + 2*a*c
*f) + c*(A*b^2*f + b*B*(c*d - a*f) - 2*A*c*(c*d + a*f))*x))/((b^2
 - 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ((B*
Sqrt[d] - A*Sqrt[f])*Sqrt[f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (
2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f
]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] +
a*f)^(3/2)) + ((B*Sqrt[d] + A*Sqrt[f])*Sqrt[f]*ArcTanh[(b*Sqrt[d]
 + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d + b*
Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rubi in Sympy [A] time = 167.217, size = 439, normalized size = 1.15

$$-\frac{2\left(-Abc\left(af-cd\right)+cx\left(A\left(-2acf+b^2f-2c^2d\right)-Bb\left(af-cd\right)\right)+\left(Ab-Ba\right)\left(-2acf+b^2f-2c^2d\right)\right)}{\left(-4ac+b^2\right)\left(b^2df-\left(af+cd\right)^2\right)\sqrt{a+bx+cx^2}}-\frac{\sqrt{f}\left(\sqrt{d}\left(Abf-Baf-Bcd\right)-\sqrt{f}\left(Aaf+Acd-Bbd\right)\right)\operatorname{atanh}\left(\frac{-2a\sqrt{f}-b\sqrt{d}+x\left(-b\sqrt{f}-2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(b^2df-\left(af+cd\right)^2\right)\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\sqrt{f}\left(\sqrt{d}\left(Abf-Baf-Bcd\right)+\sqrt{f}\left(Aaf+Acd-Bbd\right)\right)\operatorname{atanh}\left(\frac{-2a\sqrt{f}+b\sqrt{d}+x\left(-b\sqrt{f}+2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(b^2df-\left(af+cd\right)^2\right)\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -2*(-A*b*c*(a*f - c*d) + c*x*(A*(-2*a*c*f + b**2*f - 2*c**2*d) -
B*b*(a*f - c*d)) + (A*b - B*a)*(-2*a*c*f + b**2*f - 2*c**2*d))/((
-4*a*c + b**2)*(b**2*d*f - (a*f + c*d)**2)*sqrt(a + b*x + c*x**2)
) - sqrt(f)*(sqrt(d)*(A*b*f - B*a*f - B*c*d) - sqrt(f)*(A*a*f + A
*c*d - B*b*d))*atanh((-2*a*sqrt(f) - b*sqrt(d) + x*(-b*sqrt(f) -
2*c*sqrt(d)))/(2*sqrt(a + b*x + c*x**2)*sqrt(a*f + b*sqrt(d)*sqrt
(f) + c*d)))/(2*sqrt(d)*(b**2*d*f - (a*f + c*d)**2)*sqrt(a*f + b*
sqrt(d)*sqrt(f) + c*d)) - sqrt(f)*(sqrt(d)*(A*b*f - B*a*f - B*c*d
) + sqrt(f)*(A*a*f + A*c*d - B*b*d))*atanh((-2*a*sqrt(f) + b*sqrt
(d) + x*(-b*sqrt(f) + 2*c*sqrt(d)))/(2*sqrt(a + b*x + c*x**2)*sqr
t(a*f - b*sqrt(d)*sqrt(f) + c*d)))/(2*sqrt(d)*(b**2*d*f - (a*f +
c*d)**2)*sqrt(a*f - b*sqrt(d)*sqrt(f) + c*d))
```

Mathematica [A] time = 2.78083, size = 517, normalized size = 1.36

$$\frac{1}{2}\left(-\frac{4\left(B\left(2a^2cf+a\left(b^2(-f)-bcfx+2c^2d\right)+bc^2dx\right)+A\left(-bc(3af+cd)-2c^2x(af+cd)+b^3f+b^2cfx\right)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(f\left(b^2d-a^2f\right)-2acdf-c^2d^2\right)}-\frac{\left(Af+B\sqrt{d}\sqrt{f}\right)\log\left(\sqrt{d}\sqrt{f}-fx\right)}{\sqrt{d}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}-\frac{\left(B\sqrt{d}\sqrt{f}-Af\right)\log\left(\sqrt{d}\sqrt{f}+fx\right)}{\sqrt{d}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\left(B\sqrt{d}\sqrt{f}-Af\right)\log\left(\sqrt{d}\left(2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}+2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x\right)\right)}{\sqrt{d}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\left(Af+B\sqrt{d}\sqrt{f}\right)\log\left(\sqrt{d}\left(2\left(\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}+a\sqrt{f}+c\sqrt{d}x\right)+b\left(\sqrt{d}+\sqrt{f}x\right)\right)\right)}{\sqrt{d}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] ((-4*(A*(b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f
)*x) + B*(2*a^2*c*f + b*c^2*d*x + a*(2*c^2*d - b^2*f - b*c*f*x)))
)/((b^2 - 4*a*c)*(-(c^2*d^2) - 2*a*c*d*f + f*(b^2*d - a^2*f))*Sqr
t[a + x*(b + c*x)]) - ((B*Sqrt[d]*Sqrt[f] + A*f)*Log[Sqrt[d]*Sqrt
[f] - f*x])/(Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) - ((B
*Sqrt[d]*Sqrt[f] - A*f)*Log[Sqrt[d]*Sqrt[f] + f*x])/(Sqrt[d]*(c*d
 - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + ((B*Sqrt[d]*Sqrt[f] - A*f)*L
```

```
og[Sqrt[d]*(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f
]*x + 2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]
)])/(Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + ((B*Sqrt[d]
*Sqrt[f] + A*f)*Log[Sqrt[d]*(b*(Sqrt[d] + Sqrt[f]*x) + 2*(a*Sqrt[
f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x
*(b + c*x)]))])/(Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)))/
2
```

Maple [B] time = 0.034, size = 2758, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/(d*f)^(1/2)*f/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*f)^(1/2)/f)^2*c
+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)
+f*a+c*d))^(1/2)*A-1/2/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*f)^(1/2)/f
)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^
(1/2)+f*a+c*d))^(1/2)*B+2/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((
x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)
+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*x*c^2*A-2*(d*f)^(1/2)/f/(-b*
(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c
*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))
^(1/2)*x*c^2*B-1/(d*f)^(1/2)*f/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^
2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/
2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*x*b*c*A+1/(-b*(d*f)^(1/
2)+f*a+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1
/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*x*
b*c*B+1/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2
*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/
2)+f*a+c*d))^(1/2)*b*c*A-(d*f)^(1/2)/f/(-b*(d*f)^(1/2)+f*a+c*d)/(
4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(
d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*b*c*B-1/2/(d*f)
^(1/2)*f/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^
2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1
/2)+f*a+c*d))^(1/2)*b^2*A+1/2/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2
)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2
)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*b^2*B-1/2/(d*f)^(1/2)*f/
(-b*(d*f)^(1/2)+f*a+c*d)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln(
(2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/
f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)
^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*A+1/2/(-b*(d*f)^(1/2)+
f*a+c*d)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(
1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f
*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c
*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))
^(1/2))/(x+(d*f)^(1/2)/f))*B-1/2/(d*f)^(1/2)*f/(b*(d*f)^(1/2)+f*a
+c*d)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/
2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*A-1/2/(b*(d*f)^(1/2)+f*a+c
*d)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)
/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*B+2/(b*(d*f)^(1/2)+f*a+c*d)/
(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*
f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*x*c^2*A+2*(d*f)^(1/2
)/f/(b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2
*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f
)^(1/2)*x*c^2*B+1/(d*f)^(1/2)*f/(b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^
2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/
f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*x*b*c*A+1/(b*(d*f)^(1/2)+f*a+
c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(
x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*x*b*c*B+1/(b*(d
*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^
(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*b
*c*A+(d*f)^(1/2)/f/(b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-(d*f)^
(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(
```

```
1/2)+f*a+c*d)/f)^(1/2)*b*c*B+1/2/(d*f)^(1/2)*f/(b*(d*f)^(1/2)+f*a
+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*
(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*b^2*A+1/2/(b*(
d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)
^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*
b^2*B+1/2/(d*f)^(1/2)*f/(b*(d*f)^(1/2)+f*a+c*d)/((b*(d*f)^(1/2)+f
*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)
+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x
-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*
(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*A+1/2/(b*(d*f)^
(1/2)+f*a+c*d)/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(
1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(
d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1
/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x
-(d*f)^(1/2)/f))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(B*x + A)/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(B*x + A)/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(B*x + A)/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)),x, algorithm="giac")

[Out] Exception raised: RuntimeError
```

3.9 $\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}(d-fx^2)}\,dx$

Optimal. Leaf size=797

$$-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\tanh^{-1}\left(\frac{-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{2\sqrt{-\sqrt{d}\sqrt{f}b+cd+af}\sqrt{cx^2+bx+a}}\right)f^{3/2}}{2\sqrt{d}\left(-\sqrt{d}\sqrt{f}b+cd+af\right)^{5/2}}+\frac{\left(\sqrt{f}A+B\sqrt{d}\right)\tanh^{-1}\left(\frac{2\sqrt{f}a+\left(\sqrt{f}b+2c\sqrt{d}\right)x+b\sqrt{d}}{2\sqrt{\sqrt{d}\sqrt{f}b+cd+af}\sqrt{cx^2+bx+a}}\right)f^{3/2}}{2\sqrt{d}\left(\sqrt{d}\sqrt{f}b+cd+af\right)^{5/2}}$$

$$-\frac{2\left(3Bdf^2b^6-Af^2(7cd+6af)b^5-Bf\left(7c^2d^2+14acfd-3a^2f^2\right)b^4+Acf\left(15c^2d^2+46acfd+43a^2f^2\right)b^3+2Bc\left(2c^3d^3\right.\right.}{}$$

$$-\frac{2\left(aB\left(-fb^2+2c^2d+2acf\right)+A\left(b^3f-bc(cd+3af)\right)+c\left(Afb^2+B(cd-af)b-2Ac(cd+af)\right)x\right)}{3\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\left(cx^2+bx+a\right)^{3/2}}$$

```
[Out] (-2*(a*B*(2*c^2*d - b^2*f + 2*a*c*f) + A*(b^3*f - b*c*(c*d + 3*a*
f)) + c*(A*b^2*f + b*B*(c*d - a*f) - 2*A*c*(c*d + a*f))*x))/(3*(b
^2 - 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*(a + b*x + c*x^2)^(3/2)) -
(2*(3*b^6*B*d*f^2 + 24*a^2*B*c^2*f*(c*d + a*f)^2 - A*b^5*f^2*(7*c
*d + 6*a*f) - b^4*B*f*(7*c^2*d^2 + 14*a*c*d*f - 3*a^2*f^2) + A*b^
3*c*f*(15*c^2*d^2 + 46*a*c*d*f + 43*a^2*f^2) + 2*b^2*B*c*(2*c^3*d
^3 + 5*a*c^2*d^2*f + 4*a^2*c*d*f^2 - 11*a^3*f^3) - 4*A*b*c^2*(2*c
^3*d^3 + 9*a*c^2*d^2*f + 24*a^2*c*d*f^2 + 17*a^3*f^3) + c*(3*b^5*
B*d*f^2 - 2*A*b^4*f^2*(4*c*d + 3*a*f) - 8*A*c^2*(c*d + a*f)^2*(2*
c*d + 5*a*f) - b^3*B*f*(17*c^2*d^2 + 10*a*c*d*f - 3*a^2*f^2) + 2*
A*b^2*c*f*(15*c^2*d^2 + 22*a*c*d*f + 19*a^2*f^2) + 4*b*B*c*(2*c^3
*d^3 + 11*a*c^2*d^2*f + 4*a^2*c*d*f^2 - 5*a^3*f^3))*x))/(3*(b^2 -
 4*a*c)^2*(c^2*d^2 + 2*a*c*d*f - f*(b^2*d - a^2*f))^2*Sqrt[a + b*
x + c*x^2]) - ((B*Sqrt[d] - A*Sqrt[f])*f^(3/2)*ArcTanh[(b*Sqrt[d]
 - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d - b*
Sqrt[d]*Sqrt[f] + a*f)^(5/2)) + ((B*Sqrt[d] + A*Sqrt[f])*f^(3/2)*
ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(
2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2
*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(5/2))
```

Rubi [A] time = 4.31787, antiderivative size = 796, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\tanh^{-1}\left(\frac{-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{2\sqrt{-\sqrt{d}\sqrt{f}b+cd+af}\sqrt{cx^2+bx+a}}\right)f^{3/2}}{2\sqrt{d}\left(-\sqrt{d}\sqrt{f}b+cd+af\right)^{5/2}}+\frac{\left(\sqrt{f}A+B\sqrt{d}\right)\tanh^{-1}\left(\frac{2\sqrt{f}a+\left(\sqrt{f}b+2c\sqrt{d}\right)x+b\sqrt{d}}{2\sqrt{\sqrt{d}\sqrt{f}b+cd+af}\sqrt{cx^2+bx+a}}\right)f^{3/2}}{2\sqrt{d}\left(\sqrt{d}\sqrt{f}b+cd+af\right)^{5/2}}$$

$$-\frac{2\left(3Bdf^2b^6-Af^2(7cd+6af)b^5-Bf\left(7c^2d^2+14acfd-3a^2f^2\right)b^4+Acf\left(15c^2d^2+46acfd+43a^2f^2\right)b^3+2Bc\left(2c^3d^3\right.\right.}{}$$

$$-\frac{2\left(Afb^3-Ac(cd+3af)b+aB\left(-fb^2+2c^2d+2acf\right)+c\left(Afb^2+B(cd-af)b-2Ac(cd+af)\right)x\right)}{3\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\left(cx^2+bx+a\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/((a + b*x + c*x^2)^(5/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(A*b^3*f - A*b*c*(c*d + 3*a*f) + a*B*(2*c^2*d - b^2*f + 2*a*c
*f) + c*(A*b^2*f + b*B*(c*d - a*f) - 2*A*c*(c*d + a*f))*x))/(3*(b
^2 - 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*(a + b*x + c*x^2)^(3/2)) -
(2*(3*b^6*B*d*f^2 + 24*a^2*B*c^2*f*(c*d + a*f)^2 - A*b^5*f^2*(7*c
*d + 6*a*f) - b^4*B*f*(7*c^2*d^2 + 14*a*c*d*f - 3*a^2*f^2) + A*b^
3*c*f*(15*c^2*d^2 + 46*a*c*d*f + 43*a^2*f^2) + 2*b^2*B*c*(2*c^3*d
^3 + 5*a*c^2*d^2*f + 4*a^2*c*d*f^2 - 11*a^3*f^3) - 4*A*b*c^2*(2*c
^3*d^3 + 9*a*c^2*d^2*f + 24*a^2*c*d*f^2 + 17*a^3*f^3) + c*(3*b^5*
B*d*f^2 - 2*A*b^4*f^2*(4*c*d + 3*a*f) - 8*A*c^2*(c*d + a*f)^2*(2*
c*d + 5*a*f) - b^3*B*f*(17*c^2*d^2 + 10*a*c*d*f - 3*a^2*f^2) + 2*
A*b^2*c*f*(15*c^2*d^2 + 22*a*c*d*f + 19*a^2*f^2) + 4*b*B*c*(2*c^3
*d^3 + 11*a*c^2*d^2*f + 4*a^2*c*d*f^2 - 5*a^3*f^3))*x))/(3*(b^2 -
 4*a*c)^2*(c^2*d^2 + 2*a*c*d*f - f*(b^2*d - a^2*f))^2*Sqrt[a + b*
x + c*x^2]) - ((B*Sqrt[d] - A*Sqrt[f])*f^(3/2)*ArcTanh[(b*Sqrt[d]
 - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d - b*
Sqrt[d]*Sqrt[f] + a*f)^(5/2)) + ((B*Sqrt[d] + A*Sqrt[f])*f^(3/2)*
ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(
2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2
*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(5/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/(c*x**2+b*x+a)**(5/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Mathematica [B] time = 7.37118, size = 1847, normalized size = 2.32

result too large to display

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/((a + b*x + c*x^2)^(5/2)*(d - f*x^2)),x]
```

```
[Out] ((a + b*x + c*x^2)^3*((-2*(-(A*b*c^2*d) + 2*a*B*c^2*d + A*b^3*f -
 a*b^2*B*f - 3*a*A*b*c*f + 2*a^2*B*c*f + b*B*c^2*d*x - 2*A*c^3*d*
x + A*b^2*c*f*x - a*b*B*c*f*x - 2*a*A*c^2*f*x))/(3*(b^2 - 4*a*c)*
(-(c^2*d^2) + b^2*d*f - 2*a*c*d*f - a^2*f^2)*(a + b*x + c*x^2)^2)
 - (2*(4*b^2*B*c^4*d^3 - 8*A*b*c^5*d^3 - 7*b^4*B*c^2*d^2*f + 15*A
*b^3*c^3*d^2*f + 10*a*b^2*B*c^3*d^2*f - 36*a*A*b*c^4*d^2*f + 24*a
^2*B*c^4*d^2*f + 3*b^6*B*d*f^2 - 7*A*b^5*c*d*f^2 - 14*a*b^4*B*c*d
*f^2 + 46*a*A*b^3*c^2*d*f^2 + 8*a^2*b^2*B*c^2*d*f^2 - 96*a^2*A*b*
c^3*d*f^2 + 48*a^3*B*c^3*d*f^2 - 6*a*A*b^5*f^3 + 3*a^2*b^4*B*f^3
+ 43*a^2*A*b^3*c*f^3 - 22*a^3*b^2*B*c*f^3 - 68*a^3*A*b*c^2*f^3 +
24*a^4*B*c^2*f^3 + 8*b*B*c^5*d^3*x - 16*A*c^6*d^3*x - 17*b^3*B*c^
3*d^2*f*x + 30*A*b^2*c^4*d^2*f*x + 44*a*b*B*c^4*d^2*f*x - 72*a*A*
c^5*d^2*f*x + 3*b^5*B*c*d*f^2*x - 8*A*b^4*c^2*d*f^2*x - 10*a*b^3*
B*c^2*d*f^2*x + 44*a*A*b^2*c^3*d*f^2*x + 16*a^2*b*B*c^3*d*f^2*x -
 96*a^2*A*c^4*d*f^2*x - 6*a*A*b^4*c*f^3*x + 3*a^2*b^3*B*c*f^3*x +
 38*a^2*A*b^2*c^2*f^3*x - 20*a^3*b*B*c^2*f^3*x - 40*a^3*A*c^3*f^3
*x))/(3*(b^2 - 4*a*c)^2*(-(c^2*d^2) + b^2*d*f - 2*a*c*d*f - a^2*f
^2)^2*(a + b*x + c*x^2))))/(a + x*(b + c*x))^(5/2) - (f*(B*c^2*d^
(5/2)*Sqrt[f] - 2*b*B*c*d^2*f + A*c^2*d^2*f + b^2*B*d^(3/2)*f^(3/
2) - 2*A*b*c*d^(3/2)*f^(3/2) + 2*a*B*c*d^(3/2)*f^(3/2) + A*b^2*d*
f^2 - 2*a*b*B*d*f^2 + 2*a*A*c*d*f^2 - 2*a*A*b*Sqrt[d]*f^(5/2) + a
^2*B*Sqrt[d]*f^(5/2) + a^2*A*f^3)*(a + b*x + c*x^2)^(5/2)*Log[Sqr
t[d]*Sqrt[f] - f*x])/(2*Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*
f]*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2)^2*(a + x*(b + c*x))^
```

```
(5/2)) + (f*(-(B*c^2*d^(5/2)*Sqrt[f]) - 2*b*B*c*d^2*f + A*c^2*d^2
*f - b^2*B*d^(3/2)*f^(3/2) + 2*A*b*c*d^(3/2)*f^(3/2) - 2*a*B*c*d^
(3/2)*f^(3/2) + A*b^2*d*f^2 - 2*a*b*B*d*f^2 + 2*a*A*c*d*f^2 + 2*a
*A*b*Sqrt[d]*f^(5/2) - a^2*B*Sqrt[d]*f^(5/2) + a^2*A*f^3)*(a + b*
x + c*x^2)^(5/2)*Log[Sqrt[d]*Sqrt[f] + f*x])/(2*Sqrt[d]*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f
^2)^2*(a + x*(b + c*x))^(5/2)) - (f*(-(B*c^2*d^(5/2)*Sqrt[f]) - 2
*b*B*c*d^2*f + A*c^2*d^2*f - b^2*B*d^(3/2)*f^(3/2) + 2*A*b*c*d^(3
/2)*f^(3/2) - 2*a*B*c*d^(3/2)*f^(3/2) + A*b^2*d*f^2 - 2*a*b*B*d*f
^2 + 2*a*A*c*d*f^2 + 2*a*A*b*Sqrt[d]*f^(5/2) - a^2*B*Sqrt[d]*f^(5
/2) + a^2*A*f^3)*(a + b*x + c*x^2)^(5/2)*Log[-(b*d) + 2*a*Sqrt[d]
*Sqrt[f] - 2*c*d*x + b*Sqrt[d]*Sqrt[f]*x + 2*Sqrt[d]*Sqrt[c*d - b
*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2]])/(2*Sqrt[d]*Sqrt[c
*d - b*Sqrt[d]*Sqrt[f] + a*f]*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^
2*f^2)^2*(a + x*(b + c*x))^(5/2)) + (f*(B*c^2*d^(5/2)*Sqrt[f] - 2
*b*B*c*d^2*f + A*c^2*d^2*f + b^2*B*d^(3/2)*f^(3/2) - 2*A*b*c*d^(3
/2)*f^(3/2) + 2*a*B*c*d^(3/2)*f^(3/2) + A*b^2*d*f^2 - 2*a*b*B*d*f
^2 + 2*a*A*c*d*f^2 - 2*a*A*b*Sqrt[d]*f^(5/2) + a^2*B*Sqrt[d]*f^(5
/2) + a^2*A*f^3)*(a + b*x + c*x^2)^(5/2)*Log[b*d + 2*a*Sqrt[d]*Sq
rt[f] + 2*c*d*x + b*Sqrt[d]*Sqrt[f]*x + 2*Sqrt[d]*Sqrt[c*d + b*Sq
rt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2]])/(2*Sqrt[d]*Sqrt[c*d
+ b*Sqrt[d]*Sqrt[f] + a*f]*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f
^2)^2*(a + x*(b + c*x))^(5/2))
```

Maple [B] time = 0.04, size = 6422, normalized size = 8.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(B*x + A)/((c*x^2 + b*x + a)^(5/2)*(f*x^2 - d)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(B*x + A)/((c*x^2 + b*x + a)^(5/2)*(f*x^2 - d)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/(c*x**2+b*x+a)**(5/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(B*x + A)/((c*x^2 + b*x + a)^(5/2)*(f*x^2 - d)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.10 $\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx$

Optimal. Leaf size=47

$$\frac{3}{2}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)-\frac{1}{2}\tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)$$

```
[Out] -ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])]/2 + (3*ArcTanh[(1 - 3*x)/
(2*Sqrt[-1 + x + x^2])])/2
```

Rubi [A] time = 0.0929122, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174

$$\frac{3}{2}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)-\frac{1}{2}\tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x)/((-1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] -ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])]/2 + (3*ArcTanh[(1 - 3*x)/
(2*Sqrt[-1 + x + x^2])])/2
```

Rubi in Sympy [A] time = 15.2647, size = 44, normalized size = 0.94

$$-\frac{\operatorname{atan}\left(-\frac{-x-3}{2\sqrt{x^2+x-1}}\right)}{2}-\frac{3\operatorname{atanh}\left(\frac{3x-1}{2\sqrt{x^2+x-1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+2*x)/(x**2-1)/(x**2+x-1)**(1/2),x)
```

```
[Out] -atan(-(-x - 3)/(2*sqrt(x**2 + x - 1)))/2 - 3*atanh((3*x - 1)/(2*
sqrt(x**2 + x - 1)))/2
```

Mathematica [A] time = 0.0264789, size = 55, normalized size = 1.17

$$-\frac{3}{2}\log\left(-2\sqrt{x^2+x-1}-3x+1\right)-\frac{1}{2}\tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)+\frac{3}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x)/((-1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] -ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])]/2 + (3*Log[1 - x])/2 - (3
*Log[1 - 3*x - 2*Sqrt[-1 + x + x^2]])/2
```

Maple [A] time = 0.015, size = 46, normalized size = 1.

$$\frac{1}{2}\arctan\left(\frac{-3-x}{2}\frac{1}{\sqrt{(1+x)^2-2-x}}\right)-\frac{3}{2}\mathit{Artanh}\left(\frac{-1+3x}{2}\frac{1}{\sqrt{(-1+x)^2+3x-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x)
```

```
[Out] 1/2*arctan(1/2*(-3-x)/((1+x)^2-2-x)^(1/2))-3/2*arctanh(1/2*(-1+3*
x)/((-1+x)^2+3*x-2)^(1/2))
```

Maxima [A] time = 0.756138, size = 88, normalized size = 1.87

$$-\frac{1}{2}\arcsin\left(\frac{2\sqrt{5}x}{5\,|2x+2|}+\frac{6\sqrt{5}}{5\,|2x+2|}\right)-\frac{3}{2}\log\left(\frac{2\sqrt{x^2+x-1}}{|2x-2|}+\frac{2}{|2x-2|}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 1)/(sqrt(x^2 + x - 1)*(x^2 - 1)),x, algorithm="maxima")
```

```
[Out] -1/2*arcsin(2/5*sqrt(5)*x/abs(2*x + 2) + 6/5*sqrt(5)/abs(2*x + 2)
) - 3/2*log(2*sqrt(x^2 + x - 1)/abs(2*x - 2) + 2/abs(2*x - 2) + 3
/2)
```

Fricas [A] time = 0.285837, size = 62, normalized size = 1.32

$$\arctan\left(-x+\sqrt{x^2+x-1}-1\right)-\frac{3}{2}\log\left(-x+\sqrt{x^2+x-1}+2\right)+\frac{3}{2}\log\left(-x+\sqrt{x^2+x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 1)/(sqrt(x^2 + x - 1)*(x^2 - 1)),x, algorithm="fricas")
```

```
[Out] arctan(-x + sqrt(x^2 + x - 1) - 1) - 3/2*log(-x + sqrt(x^2 + x -
1) + 2) + 3/2*log(-x + sqrt(x^2 + x - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{2x+1}{(x-1)(x+1)\sqrt{x^2+x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+2*x)/(x**2-1)/(x**2+x-1)**(1/2),x)
```

```
[Out] Integral((2*x + 1)/((x - 1)*(x + 1)*sqrt(x**2 + x - 1)), x)
```

GIAC/XCAS [A] time = 0.269572, size = 65, normalized size = 1.38

$$\arctan\left(-x+\sqrt{x^2+x-1}-1\right)-\frac{3}{2}\ln\left(\left|-x+\sqrt{x^2+x-1}+2\right|\right)+\frac{3}{2}\ln\left(\left|-x+\sqrt{x^2+x-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 1)/(sqrt(x^2 + x - 1)*(x^2 - 1)),x, algorithm="giac")
```

```
[Out] arctan(-x + sqrt(x^2 + x - 1) - 1) - 3/2*ln(abs(-x + sqrt(x^2 + x
 - 1) + 2)) + 3/2*ln(abs(-x + sqrt(x^2 + x - 1)))
```

3.11 $\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx$

Optimal. Leaf size=117

$$\sqrt{\frac{1}{2}\left(\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{5}x-2\sqrt{5}+5}{\sqrt{10\left(\sqrt{5}-2\right)}\sqrt{x^2+x-1}}\right)-\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{-\sqrt{5}x+2\sqrt{5}+5}{\sqrt{10\left(2+\sqrt{5}\right)}\sqrt{x^2+x-1}}\right)$$

```
[Out] -(Sqrt[(2 + Sqrt[5])/2]*ArcTan[(5 + 2*Sqrt[5] - Sqrt[5]*x)/(Sqrt[
10*(2 + Sqrt[5])]*Sqrt[-1 + x + x^2])]) + Sqrt[(-2 + Sqrt[5])/2]*
ArcTanh[(5 - 2*Sqrt[5] + Sqrt[5]*x)/(Sqrt[10*(-2 + Sqrt[5])]*Sqrt
[-1 + x + x^2])]
```

Rubi [A] time = 0.380311, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\sqrt{\frac{1}{2}\left(\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{5}x-2\sqrt{5}+5}{\sqrt{10\left(\sqrt{5}-2\right)}\sqrt{x^2+x-1}}\right)-\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{-\sqrt{5}x+2\sqrt{5}+5}{\sqrt{10\left(2+\sqrt{5}\right)}\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x)/((1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] -(Sqrt[(2 + Sqrt[5])/2]*ArcTan[(5 + 2*Sqrt[5] - Sqrt[5]*x)/(Sqrt[
10*(2 + Sqrt[5])]*Sqrt[-1 + x + x^2])]) + Sqrt[(-2 + Sqrt[5])/2]*
ArcTanh[(5 - 2*Sqrt[5] + Sqrt[5]*x)/(Sqrt[10*(-2 + Sqrt[5])]*Sqrt
[-1 + x + x^2])]
```

Rubi in Sympy [A] time = 29.9711, size = 138, normalized size = 1.18

$$\frac{\sqrt{10}\left(2\sqrt{5}+5\right)\operatorname{atan}\left(\frac{\sqrt{10}\left(\sqrt{5}x-5-2\sqrt{5}\right)}{10\sqrt{2+\sqrt{5}}\sqrt{x^2+x-1}}\right)}{10\sqrt{2+\sqrt{5}}}-\frac{\sqrt{10}\left(-2\sqrt{5}+5\right)\operatorname{atanh}\left(\frac{\sqrt{10}\left(-\sqrt{5}x-5+2\sqrt{5}\right)}{10\sqrt{-2+\sqrt{5}}\sqrt{x^2+x-1}}\right)}{10\sqrt{-2+\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+2*x)/(x**2+1)/(x**2+x-1)**(1/2),x)
```

```
[Out] sqrt(10)*(2*sqrt(5) + 5)*atan(sqrt(10)*(sqrt(5)*x - 5 - 2*sqrt(5)
)/(10*sqrt(2 + sqrt(5))*sqrt(x**2 + x - 1)))/(10*sqrt(2 + sqrt(5)
)) - sqrt(10)*(-2*sqrt(5) + 5)*atanh(sqrt(10)*(-sqrt(5)*x - 5 + 2
*sqrt(5))/(10*sqrt(-2 + sqrt(5))*sqrt(x**2 + x - 1)))/(10*sqrt(-2
 + sqrt(5)))
```

Mathematica [C] time = 0.646251, size = 394, normalized size = 3.37

$$\frac{1}{4}\left(i\left(\left(\sqrt{2-i}+\sqrt{2+i}\right)\log\left(x^2+1\right)\right.\right.$$
$$-\sqrt{2-i}\log\left((-13+4i)x^2+8\sqrt{2-i}\sqrt{x^2+x-1}x+4\sqrt{2-i}\sqrt{x^2+x-1}-(8-4i)x+(3-4i)\right)$$
$$\left.-\sqrt{2+i}\log\left((13+4i)x^2+8\sqrt{2+i}\sqrt{x^2+x-1}x+4\sqrt{2+i}\sqrt{x^2+x-1}+(8+4i)x-(3+4i)\right)\right)$$
$$+2\sqrt{2-i}\tan^{-1}\left(\frac{8ix^3+\left(2-(2-4i)\sqrt{2-i}\sqrt{x^2+x-1}\right)x^2-2i\left(1+5\sqrt{2-i}\sqrt{x^2+x-1}\right)x+\frac{20\sqrt{x^2+x-1}}{\sqrt{2-i}}-8}{(5+6i)x^3-(6-5i)x^2-(15+14i)x+(14+5i)}\right)$$
$$\left.+2\sqrt{2+i}\tan^{-1}\left(\frac{2\left(-4x^3+\left(\frac{5\sqrt{x^2+x-1}}{\sqrt{2+i}}-i\right)x^2-5\sqrt{2+i}\sqrt{x^2+x-1}x+(2+4i)\sqrt{2+i}\sqrt{x^2+x-1}+x+4i\right)}{(6+5i)x^3+(5-6i)x^2-(14+15i)x+(5+14i)}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x)/((1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] (2*Sqrt[2 - I]*ArcTan[(-8 + (8*I)*x^3 + (20*Sqrt[-1 + x + x^2])/S
qrt[2 - I] + x^2*(2 - (2 - 4*I)*Sqrt[2 - I]*Sqrt[-1 + x + x^2]) -
 (2*I)*x*(1 + 5*Sqrt[2 - I]*Sqrt[-1 + x + x^2]))/((14 + 5*I) - (1
5 + 14*I)*x - (6 - 5*I)*x^2 + (5 + 6*I)*x^3)] + 2*Sqrt[2 + I]*Arc
Tan[(2*(4*I + x - 4*x^3 + (2 + 4*I)*Sqrt[2 + I]*Sqrt[-1 + x + x^2
] - 5*Sqrt[2 + I]*x*Sqrt[-1 + x + x^2] + x^2*(-I + (5*Sqrt[-1 + x
 + x^2])/Sqrt[2 + I])))/((5 + 14*I) - (14 + 15*I)*x + (5 - 6*I)*x
^2 + (6 + 5*I)*x^3)] + I*((Sqrt[2 - I] + Sqrt[2 + I])*Log[1 + x^2
] - Sqrt[2 - I]*Log[(3 - 4*I) - (8 - 4*I)*x - (13 - 4*I)*x^2 + 4*
Sqrt[2 - I]*Sqrt[-1 + x + x^2] + 8*Sqrt[2 - I]*x*Sqrt[-1 + x + x^
2]] - Sqrt[2 + I]*Log[(-3 - 4*I) + (8 + 4*I)*x + (13 + 4*I)*x^2 +
 4*Sqrt[2 + I]*Sqrt[-1 + x + x^2] + 8*Sqrt[2 + I]*x*Sqrt[-1 + x +
 x^2]]))/4
```

Maple [B] time = 0.171, size = 637, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x)
```

```
[Out] (10*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2-5*5^(1/2)*(-5^(1/2)-2+x)^2/
(-5^(1/2)+2-x)^2+10+5*5^(1/2))^(1/2)*5^(1/2)*(arctan(1/5*(-5^(1/2
)-2+x)/(-5^(1/2)+2-x)*(-2+5^(1/2))*(5^(1/2)*(-5^(1/2)-2+x)^2/(-5^
(1/2)+2-x)^2+2*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2-5^(1/2)+2)*(20+1
0*5^(1/2))^(1/2)*((-2+5^(1/2))*(-(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^
2+4*5^(1/2)+9))^(1/2)*5^(1/2)/((-5^(1/2)-2+x)^4/(-5^(1/2)+2-x)^4-
18*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2+1))*5^(1/2)+arctanh((10*(-5^
(1/2)-2+x)^2/(-5^(1/2)+2-x)^2-5*5^(1/2)*(-5^(1/2)-2+x)^2/(-5^(1/2
)+2-x)^2+10+5*5^(1/2))^(1/2)/(20+10*5^(1/2))^(1/2))+2*arctan(1/5*
(-5^(1/2)-2+x)/(-5^(1/2)+2-x)*(-2+5^(1/2))*(5^(1/2)*(-5^(1/2)-2+x
)^2/(-5^(1/2)+2-x)^2+2*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2-5^(1/2)+
2)*(20+10*5^(1/2))^(1/2)*((-2+5^(1/2))*(-(-5^(1/2)-2+x)^2/(-5^(1/
2)+2-x)^2+4*5^(1/2)+9))^(1/2)*5^(1/2)/((-5^(1/2)-2+x)^4/(-5^(1/2)
+2-x)^4-18*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2+1)))/(-5*(5^(1/2)*(-
5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2-2*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)
^2-5^(1/2)-2)/(1+(-5^(1/2)-2+x)/(-5^(1/2)+2-x))^2)^(1/2)/(1+(-5^(
1/2)-2+x)/(-5^(1/2)+2-x))/(20+10*5^(1/2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+1}{\sqrt{x^2+x-1}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 1)/(sqrt(x^2 + x - 1)*(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)/(sqrt(x^2 + x - 1)*(x^2 + 1)), x)
```

Fricas [A] time = 0.304426, size = 1227, normalized size = 10.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 1)/(sqrt(x^2 + x - 1)*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*(5^(1/4)*(sqrt(5) - 2)*log(-1/5*(5^(1/4)*sqrt(2)*(sqr
t(5)*(682*x + 161) - 1525*x - 360)*sqrt((2*sqrt(5) - 5)/(4*sqrt(5
) - 9)) + 1610*x^2 - 360*sqrt(5)*(2*x^2 + x) - (5^(1/4)*sqrt(2)*(
682*sqrt(5) - 1525)*sqrt((2*sqrt(5) - 5)/(4*sqrt(5) - 9)) - 720*s
qrt(5)*x + 1610*x)*sqrt(x^2 + x - 1) - 5*sqrt(5)*(72*sqrt(5) - 16
1) + 805*x)/(72*sqrt(5) - 161)) - 5^(1/4)*(sqrt(5) - 2)*log(1/5*(
5^(1/4)*sqrt(2)*(sqrt(5)*(682*x + 161) - 1525*x - 360)*sqrt((2*sq
rt(5) - 5)/(4*sqrt(5) - 9)) - 1610*x^2 + 360*sqrt(5)*(2*x^2 + x)
- (5^(1/4)*sqrt(2)*(682*sqrt(5) - 1525)*sqrt((2*sqrt(5) - 5)/(4*s
qrt(5) - 9)) + 720*sqrt(5)*x - 1610*x)*sqrt(x^2 + x - 1) + 5*sqrt
(5)*(72*sqrt(5) - 161) - 805*x)/(72*sqrt(5) - 161)) - 4*5^(1/4)*a
rctan((sqrt(2)*(sqrt(5) - 2)*sqrt((2*sqrt(5) - 5)/(4*sqrt(5) - 9)
) + 5^(1/4))/(sqrt(2)*sqrt(1/5)*(sqrt(5) - 2)*sqrt(-(5^(1/4)*sqrt
(2)*(sqrt(5)*(682*x + 161) - 1525*x - 360)*sqrt((2*sqrt(5) - 5)/(
4*sqrt(5) - 9)) + 1610*x^2 - 360*sqrt(5)*(2*x^2 + x) - (5^(1/4)*s
qrt(2)*(682*sqrt(5) - 1525)*sqrt((2*sqrt(5) - 5)/(4*sqrt(5) - 9))
 - 720*sqrt(5)*x + 1610*x)*sqrt(x^2 + x - 1) - 5*sqrt(5)*(72*sqrt
(5) - 161) + 805*x)/(72*sqrt(5) - 161))*sqrt((2*sqrt(5) - 5)/(4*s
qrt(5) - 9)) + sqrt(2)*sqrt(x^2 + x - 1)*(sqrt(5) - 2)*sqrt((2*sq
rt(5) - 5)/(4*sqrt(5) - 9)) - sqrt(2)*(sqrt(5)*x - 2*x)*sqrt((2*s
qrt(5) - 5)/(4*sqrt(5) - 9)) + 5^(1/4)*(sqrt(5) - 2))) + 4*5^(1/4
)*arctan((sqrt(2)*(sqrt(5) - 2)*sqrt((2*sqrt(5) - 5)/(4*sqrt(5) -
 9)) - 5^(1/4))/(sqrt(2)*sqrt(1/5)*(sqrt(5) - 2)*sqrt((5^(1/4)*sq
rt(2)*(sqrt(5)*(682*x + 161) - 1525*x - 360)*sqrt((2*sqrt(5) - 5)
/(4*sqrt(5) - 9)) - 1610*x^2 + 360*sqrt(5)*(2*x^2 + x) - (5^(1/4)
*sqrt(2)*(682*sqrt(5) - 1525)*sqrt((2*sqrt(5) - 5)/(4*sqrt(5) - 9
)) + 720*sqrt(5)*x - 1610*x)*sqrt(x^2 + x - 1) + 5*sqrt(5)*(72*sq
rt(5) - 161) - 805*x)/(72*sqrt(5) - 161))*sqrt((2*sqrt(5) - 5)/(4
*sqrt(5) - 9)) + sqrt(2)*sqrt(x^2 + x - 1)*(sqrt(5) - 2)*sqrt((2*
sqrt(5) - 5)/(4*sqrt(5) - 9)) - sqrt(2)*(sqrt(5)*x - 2*x)*sqrt((2
*sqrt(5) - 5)/(4*sqrt(5) - 9)) - 5^(1/4)*(sqrt(5) - 2))))/((sqrt(
5) - 2)*sqrt((2*sqrt(5) - 5)/(4*sqrt(5) - 9)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+1}{(x^2+1)\sqrt{x^2+x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+2*x)/(x**2+1)/(x**2+x-1)**(1/2),x)
```

```
[Out] Integral((2*x + 1)/((x**2 + 1)*sqrt(x**2 + x - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+1}{\sqrt{x^2+x-1}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 1)/(sqrt(x^2 + x - 1)*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate((2*x + 1)/(sqrt(x^2 + x - 1)*(x^2 + 1)), x)
```

3.12 $\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=484

$$-\frac{\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2-2ac+b^2+c^2}\right)+a^2+b^2}\tan^{-1}\left(\frac{b\sqrt{a^2-2ac+b^2+c^2}-x\left((a-c)\left(\sqrt{\cdots}\right.\right.}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+\cdots}\right.}}\right.}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}}$$

$$-\frac{\sqrt{-a\left(\sqrt{a^2-2ac+b^2+c^2}+2c\right)+c\left(\sqrt{a^2-2ac+b^2+c^2}+c\right)+a^2+b^2}\tanh^{-1}\left(\frac{x\left((a-c)\left(-\sqrt{a^2-2ac+b^2+c^2}+a\cdots\right.\right.}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}\sqrt{-a\left(\sqrt{a^2-2ac+b^2+c^2}\cdots\right.}}\right.}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}}$$

```
[Out] -((Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*
c - Sqrt[a^2 + b^2 - 2*a*c + c^2])]*ArcTan[(b*Sqrt[a^2 + b^2 - 2*
a*c + c^2] - (b^2 + (a - c)*(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2
]))*x)/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 +
c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a^2 + b^2 -
 2*a*c + c^2])]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*(a^2 + b^2 - 2*
a*c + c^2)^(1/4))) - (Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*
a*c + c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]*ArcTanh[(b
*Sqrt[a^2 + b^2 - 2*a*c + c^2] + (b^2 + (a - c)*(a - c - Sqrt[a^2
 + b^2 - 2*a*c + c^2]))*x)/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/
4)*Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*
c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*x + c*x^2])])/(Sqr
t[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4))
```

Rubi [A] time = 25.5193, antiderivative size = 484, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2-2ac+b^2+c^2}\right)+a^2+b^2}\tan^{-1}\left(\frac{b\sqrt{a^2-2ac+b^2+c^2}-x\left((a-c)\left(\sqrt{\cdots}\right.\right.}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+\cdots}\right.}}\right.}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}}$$

$$-\frac{\sqrt{-a\left(\sqrt{a^2-2ac+b^2+c^2}+2c\right)+c\left(\sqrt{a^2-2ac+b^2+c^2}+c\right)+a^2+b^2}\tanh^{-1}\left(\frac{x\left((a-c)\left(-\sqrt{a^2-2ac+b^2+c^2}+a\cdots\right.\right.}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}\sqrt{-a\left(\sqrt{a^2-2ac+b^2+c^2}\cdots\right.}}\right.}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a - c + b*x)/((1 + x^2)*Sqrt[a + b*x + c*x^2]),x]
```


```
[Out] -((Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*
c - Sqrt[a^2 + b^2 - 2*a*c + c^2])]*ArcTan[(b*Sqrt[a^2 + b^2 - 2*
a*c + c^2] - (b^2 + (a - c)*(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2
]))*x)/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 +
c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a^2 + b^2 -
 2*a*c + c^2])]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*(a^2 + b^2 - 2*
a*c + c^2)^(1/4))) - (Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*
a*c + c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]*ArcTanh[(b
*Sqrt[a^2 + b^2 - 2*a*c + c^2] + (b^2 + (a - c)*(a - c - Sqrt[a^2
 + b^2 - 2*a*c + c^2]))*x)/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/
4)*Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*
c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*x + c*x^2])])/(Sqr
t[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a-c)/(x**2+1)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.505352, size = 182, normalized size = 0.38

$$\frac{1}{2}i\left(\sqrt{a+ib-c}\log\left(\frac{2i\left(2\sqrt{a+ib-c}\sqrt{a+x(b+cx)}+2a+b(x+i)+2icx\right)}{(x-i)(a+ib-c)^{3/2}}\right)-\sqrt{a-ib-c}\log\left(-\frac{2i\left(2\sqrt{a-ib-c}\sqrt{a+x(b+cx)}+2a+b(x-i)-2icx\right)}{(x+i)(a-ib-c)^{3/2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a - c + b*x)/((1 + x^2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (I/2)*(-(Sqrt[a - I*b - c]*Log[((-2*I)*(2*a - (2*I)*c*x + b*(-I +
 x) + 2*Sqrt[a - I*b - c]*Sqrt[a + x*(b + c*x)]))/((a - I*b - c)^
(3/2)*(I + x))]) + Sqrt[a + I*b - c]*Log[((2*I)*(2*a + (2*I)*c*x
+ b*(I + x) + 2*Sqrt[a + I*b - c]*Sqrt[a + x*(b + c*x)]))/((a + I
*b - c)^(3/2)*(-I + x))])
```

Maple [B] time = 0.623, size = 6871419, normalized size = 14197.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx+a-c}{\sqrt{cx^2+bx+a}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x + a - c)/(sqrt(c*x^2 + b*x + a)*(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a - c)/(sqrt(c*x^2 + b*x + a)*(x^2 + 1)), x)
```

Fricas [A] time = 0.648555, size = 16226, normalized size = 33.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x + a - c)/(sqrt(c*x^2 + b*x + a)*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(2)*((a^2 + b^2 - 2*a*c + c^2)^(1/4)*(a - c - sqrt(a^2 +
 b^2 - 2*a*c + c^2))*log(-(32*a^8*b^2 + 72*a^6*b^4 + 52*a^4*b^6 +
 13*a^2*b^8 + b^10 + 8*(4*a^2*b^2 + b^4)*c^6 - 48*(4*a^3*b^2 + a*
b^4)*c^5 + 8*(60*a^4*b^2 + 23*a^2*b^4 + 2*b^6)*c^4 - 32*(20*a^5*b
^2 + 13*a^3*b^4 + 2*a*b^6)*c^3 + 3*(160*a^6*b^2 + 168*a^4*b^4 + 4
4*a^2*b^6 + 3*b^8)*c^2 + (8*a^6*b^4 + 16*a^4*b^6 + 9*a^2*b^8 + b^
10 - 192*a*b^2*c^7 + 32*b^2*c^8 + 24*(20*a^2*b^2 + 3*b^4)*c^6 - 1
6*(40*a^3*b^2 + 19*a*b^4)*c^5 + 4*(120*a^4*b^2 + 126*a^2*b^4 + 13
*b^6)*c^4 - 8*(24*a^5*b^2 + 52*a^3*b^4 + 17*a*b^6)*c^3 + (32*a^6*
b^2 + 184*a^4*b^4 + 132*a^2*b^6 + 13*b^8)*c^2 - 2*(24*a^5*b^4 + 3
2*a^3*b^6 + 9*a*b^8)*c)*x^2 + 2*sqrt(2)*(4*a^4*b^5 + 5*a^2*b^7 +
b^9 - 80*a*b^3*c^5 + 16*b^3*c^6 + 4*(40*a^2*b^3 + 7*b^5)*c^4 - 8*
(20*a^3*b^3 + 11*a*b^5)*c^3 + (80*a^4*b^3 + 96*a^2*b^5 + 13*b^7)*
c^2 - 2*(8*a^5*b^3 + 20*a^3*b^5 + 9*a*b^7)*c - 2*(4*a^5*b^4 + 6*a
^3*b^6 + 2*a*b^8 + 96*a*b^2*c^6 - 16*b^2*c^7 - 16*(15*a^2*b^2 + 2
*b^4)*c^5 + 4*(80*a^3*b^2 + 33*a*b^4)*c^4 - (240*a^4*b^2 + 208*a^
2*b^4 + 19*b^6)*c^3 + 4*(24*a^5*b^2 + 38*a^3*b^4 + 11*a*b^6)*c^2
- (16*a^6*b^2 + 48*a^4*b^4 + 31*a^2*b^6 + 3*b^8)*c)*x - (4*a^3*b^
5 + 3*a*b^7 + 64*a*b^3*c^4 - 16*b^3*c^5 - 4*(24*a^2*b^3 + 5*b^5)*
c^3 + 4*(16*a^3*b^3 + 11*a*b^5)*c^2 - (16*a^4*b^3 + 28*a^2*b^5 +
5*b^7)*c - (8*a^4*b^4 + 8*a^2*b^6 + b^8 - 160*a*b^2*c^5 + 32*b^2*
c^6 + 16*(20*a^2*b^2 + 3*b^4)*c^4 - 8*(40*a^3*b^2 + 19*a*b^4)*c^3
 + 2*(80*a^4*b^2 + 84*a^2*b^4 + 9*b^6)*c^2 - 2*(16*a^5*b^2 + 36*a
^3*b^4 + 13*a*b^6)*c)*x)*sqrt(a^2 + b^2 - 2*a*c + c^2))*sqrt(c*x^
2 + b*x + a)*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*sqrt((a^2 + b^2 - 2*
a*c + c^2 - sqrt(a^2 + b^2 - 2*a*c + c^2)*(a - c))/(2*a^2 + b^2 -
 4*a*c + 2*c^2 - 2*sqrt(a^2 + b^2 - 2*a*c + c^2)*(a - c))) - 2*(9
6*a^7*b^2 + 152*a^5*b^4 + 68*a^3*b^6 + 9*a*b^8)*c + 4*(8*a^7*b^3
+ 16*a^5*b^5 + 9*a^3*b^7 + a*b^9 - 40*a*b^3*c^6 + 8*b^3*c^7 + 8*(
9*a^2*b^3 + 2*b^5)*c^5 - 8*(5*a^3*b^3 + 6*a*b^5)*c^4 - (40*a^4*b^
3 - 32*a^2*b^5 - 9*b^7)*c^3 + (72*a^5*b^3 + 32*a^3*b^5 - 9*a*b^7)
*c^2 - (40*a^6*b^3 + 48*a^4*b^5 + 9*a^2*b^7 - b^9)*c)*x + 4*(8*a^
7*b^2 + 16*a^5*b^4 + 9*a^3*b^6 + a*b^8 - 48*a^2*b^2*c^5 + 8*a*b^2
*c^6 + 8*(15*a^3*b^2 + 2*a*b^4)*c^4 - 32*(5*a^4*b^2 + 2*a^2*b^4)*
c^3 + 3*(40*a^5*b^2 + 32*a^3*b^4 + 3*a*b^6)*c^2 - (48*a*b^2*c^6 -
 8*b^2*c^7 - 8*(15*a^2*b^2 + 2*b^4)*c^5 + 32*(5*a^3*b^2 + 2*a*b^4
)*c^4 - 3*(40*a^4*b^2 + 32*a^2*b^4 + 3*b^6)*c^3 + 2*(24*a^5*b^2 +
 32*a^3*b^4 + 9*a*b^6)*c^2 - (8*a^6*b^2 + 16*a^4*b^4 + 9*a^2*b^6
+ b^8)*c)*x^2 - 2*(24*a^6*b^2 + 32*a^4*b^4 + 9*a^2*b^6)*c + (8*a^
6*b^3 + 16*a^4*b^5 + 9*a^2*b^7 + b^9 - 48*a*b^3*c^5 + 8*b^3*c^6 +
 8*(15*a^2*b^3 + 2*b^5)*c^4 - 32*(5*a^3*b^3 + 2*a*b^5)*c^3 + 3*(4
0*a^4*b^3 + 32*a^2*b^5 + 3*b^7)*c^2 - 2*(24*a^5*b^3 + 32*a^3*b^5
+ 9*a*b^7)*c)*x - 4*(2*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 + 10*a^2*b^2
*c^4 - 2*a*b^2*c^5 - (20*a^3*b^2 + 3*a*b^4)*c^3 + (20*a^4*b^2 + 9
*a^2*b^4)*c^2 + (10*a*b^2*c^5 - 2*b^2*c^6 - (20*a^2*b^2 + 3*b^4)*
c^4 + (20*a^3*b^2 + 9*a*b^4)*c^3 - (10*a^4*b^2 + 9*a^2*b^4 + b^6)
*c^2 + (2*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c)*x^2 - (10*a^5*b^2 + 9*a
^3*b^4 + a*b^6)*c + (2*a^5*b^3 + 3*a^3*b^5 + a*b^7 + 10*a*b^3*c^4
 - 2*b^3*c^5 - (20*a^2*b^3 + 3*b^5)*c^3 + (20*a^3*b^3 + 9*a*b^5)*
c^2 - (10*a^4*b^3 + 9*a^2*b^5 + b^7)*c)*x)*sqrt(a^2 + b^2 - 2*a*c
 + c^2))*sqrt(a^2 + b^2 - 2*a*c + c^2) - 4*(8*a^7*b^2 + 14*a^5*b^
4 + 7*a^3*b^6 + a*b^8 - 2*(4*a^2*b^2 + b^4)*c^5 + 10*(4*a^3*b^2 +
 a*b^4)*c^4 - (80*a^4*b^2 + 32*a^2*b^4 + 3*b^6)*c^3 + (80*a^5*b^2
 + 56*a^3*b^4 + 9*a*b^6)*c^2 + (2*a^5*b^4 + 3*a^3*b^6 + a*b^8 + 4
0*a*b^2*c^6 - 8*b^2*c^7 - 2*(40*a^2*b^2 + 7*b^4)*c^5 + 2*(40*a^3*
b^2 + 23*a*b^4)*c^4 - (40*a^4*b^2 + 56*a^2*b^4 + 7*b^6)*c^3 + (8*
a^5*b^2 + 32*a^3*b^4 + 13*a*b^6)*c^2 - (10*a^4*b^4 + 9*a^2*b^6 +
b^8)*c)*x^2 - (40*a^6*b^2 + 46*a^4*b^4 + 13*a^2*b^6 + b^8)*c + 4*
(2*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 + 6*a*b^5*c^3 + 8*a*b^3*c^5 - 2*
b^3*c^6 - (10*a^2*b^3 + 3*b^5)*c^4 + (10*a^4*b^3 - b^7)*c^2 - 2*(
4*a^5*b^3 + 3*a^3*b^5)*c)*x)*sqrt(a^2 + b^2 - 2*a*c + c^2))/(4*(2
*a^5 + 3*a^3*b^2 + a*b^4 + 10*a*c^4 - 2*c^5 - (20*a^2 + 3*b^2)*c^
3 + (20*a^3 + 9*a*b^2)*c^2 - (10*a^4 + 9*a^2*b^2 + b^4)*c)*sqrt(a
^2 + b^2 - 2*a*c + c^2)*x^2 - (8*a^6 + 16*a^4*b^2 + 9*a^2*b^4 + b
^6 - 48*a*c^5 + 8*c^6 + 8*(15*a^2 + 2*b^2)*c^4 - 32*(5*a^3 + 2*a*
b^2)*c^3 + 3*(40*a^4 + 32*a^2*b^2 + 3*b^4)*c^2 - 2*(24*a^5 + 32*a
^3*b^2 + 9*a*b^4)*c)*x^2)) - (a^2 + b^2 - 2*a*c + c^2)^(1/4)*(a -
 c - sqrt(a^2 + b^2 - 2*a*c + c^2))*log(-(32*a^8*b^2 + 72*a^6*b^4
 + 52*a^4*b^6 + 13*a^2*b^8 + b^10 + 8*(4*a^2*b^2 + b^4)*c^6 - 48*
```

(4*a^3*b^2 + a*b^4)*c^5 + 8*(60*a^4*b^2 + 23*a^2*b^4 + 2*b^6)*c^4
 - 32*(20*a^5*b^2 + 13*a^3*b^4 + 2*a*b^6)*c^3 + 3*(160*a^6*b^2 +
168*a^4*b^4 + 44*a^2*b^6 + 3*b^8)*c^2 + (8*a^6*b^4 + 16*a^4*b^6 +
 9*a^2*b^8 + b^10 - 192*a*b^2*c^7 + 32*b^2*c^8 + 24*(20*a^2*b^2 +
 3*b^4)*c^6 - 16*(40*a^3*b^2 + 19*a*b^4)*c^5 + 4*(120*a^4*b^2 + 1
26*a^2*b^4 + 13*b^6)*c^4 - 8*(24*a^5*b^2 + 52*a^3*b^4 + 17*a*b^6)
*c^3 + (32*a^6*b^2 + 184*a^4*b^4 + 132*a^2*b^6 + 13*b^8)*c^2 - 2*
(24*a^5*b^4 + 32*a^3*b^6 + 9*a*b^8)*c)*x^2 - 2*sqrt(2)*(4*a^4*b^5
 + 5*a^2*b^7 + b^9 - 80*a*b^3*c^5 + 16*b^3*c^6 + 4*(40*a^2*b^3 +
7*b^5)*c^4 - 8*(20*a^3*b^3 + 11*a*b^5)*c^3 + (80*a^4*b^3 + 96*a^2
*b^5 + 13*b^7)*c^2 - 2*(8*a^5*b^3 + 20*a^3*b^5 + 9*a*b^7)*c - 2*(
4*a^5*b^4 + 6*a^3*b^6 + 2*a*b^8 + 96*a*b^2*c^6 - 16*b^2*c^7 - 16*
(15*a^2*b^2 + 2*b^4)*c^5 + 4*(80*a^3*b^2 + 33*a*b^4)*c^4 - (240*a
^4*b^2 + 208*a^2*b^4 + 19*b^6)*c^3 + 4*(24*a^5*b^2 + 38*a^3*b^4 +
 11*a*b^6)*c^2 - (16*a^6*b^2 + 48*a^4*b^4 + 31*a^2*b^6 + 3*b^8)*c
)*x - (4*a^3*b^5 + 3*a*b^7 + 64*a*b^3*c^4 - 16*b^3*c^5 - 4*(24*a^
2*b^3 + 5*b^5)*c^3 + 4*(16*a^3*b^3 + 11*a*b^5)*c^2 - (16*a^4*b^3
+ 28*a^2*b^5 + 5*b^7)*c - (8*a^4*b^4 + 8*a^2*b^6 + b^8 - 160*a*b^
2*c^5 + 32*b^2*c^6 + 16*(20*a^2*b^2 + 3*b^4)*c^4 - 8*(40*a^3*b^2
+ 19*a*b^4)*c^3 + 2*(80*a^4*b^2 + 84*a^2*b^4 + 9*b^6)*c^2 - 2*(16
*a^5*b^2 + 36*a^3*b^4 + 13*a*b^6)*c)*x)*sqrt(a^2 + b^2 - 2*a*c +
c^2))*sqrt(c*x^2 + b*x + a)*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*sqrt(
(a^2 + b^2 - 2*a*c + c^2 - sqrt(a^2 + b^2 - 2*a*c + c^2)*(a - c))
/(2*a^2 + b^2 - 4*a*c + 2*c^2 - 2*sqrt(a^2 + b^2 - 2*a*c + c^2)*(
a - c))) - 2*(96*a^7*b^2 + 152*a^5*b^4 + 68*a^3*b^6 + 9*a*b^8)*c
+ 4*(8*a^7*b^3 + 16*a^5*b^5 + 9*a^3*b^7 + a*b^9 - 40*a*b^3*c^6 +
8*b^3*c^7 + 8*(9*a^2*b^3 + 2*b^5)*c^5 - 8*(5*a^3*b^3 + 6*a*b^5)*c
^4 - (40*a^4*b^3 - 32*a^2*b^5 - 9*b^7)*c^3 + (72*a^5*b^3 + 32*a^3
*b^5 - 9*a*b^7)*c^2 - (40*a^6*b^3 + 48*a^4*b^5 + 9*a^2*b^7 - b^9)
*c)*x + 4*(8*a^7*b^2 + 16*a^5*b^4 + 9*a^3*b^6 + a*b^8 - 48*a^2*b^
2*c^5 + 8*a*b^2*c^6 + 8*(15*a^3*b^2 + 2*a*b^4)*c^4 - 32*(5*a^4*b^
2 + 2*a^2*b^4)*c^3 + 3*(40*a^5*b^2 + 32*a^3*b^4 + 3*a*b^6)*c^2 -
(48*a*b^2*c^6 - 8*b^2*c^7 - 8*(15*a^2*b^2 + 2*b^4)*c^5 + 32*(5*a^
3*b^2 + 2*a*b^4)*c^4 - 3*(40*a^4*b^2 + 32*a^2*b^4 + 3*b^6)*c^3 +
2*(24*a^5*b^2 + 32*a^3*b^4 + 9*a*b^6)*c^2 - (8*a^6*b^2 + 16*a^4*b
^4 + 9*a^2*b^6 + b^8)*c)*x^2 - 2*(24*a^6*b^2 + 32*a^4*b^4 + 9*a^2
*b^6)*c + (8*a^6*b^3 + 16*a^4*b^5 + 9*a^2*b^7 + b^9 - 48*a*b^3*c^
5 + 8*b^3*c^6 + 8*(15*a^2*b^3 + 2*b^5)*c^4 - 32*(5*a^3*b^3 + 2*a*
b^5)*c^3 + 3*(40*a^4*b^3 + 32*a^2*b^5 + 3*b^7)*c^2 - 2*(24*a^5*b^
3 + 32*a^3*b^5 + 9*a*b^7)*c)*x - 4*(2*a^6*b^2 + 3*a^4*b^4 + a^2*b
^6 + 10*a^2*b^2*c^4 - 2*a*b^2*c^5 - (20*a^3*b^2 + 3*a*b^4)*c^3 +
(20*a^4*b^2 + 9*a^2*b^4)*c^2 + (10*a*b^2*c^5 - 2*b^2*c^6 - (20*a^
2*b^2 + 3*b^4)*c^4 + (20*a^3*b^2 + 9*a*b^4)*c^3 - (10*a^4*b^2 + 9
*a^2*b^4 + b^6)*c^2 + (2*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c)*x^2 - (1
0*a^5*b^2 + 9*a^3*b^4 + a*b^6)*c + (2*a^5*b^3 + 3*a^3*b^5 + a*b^7
 + 10*a*b^3*c^4 - 2*b^3*c^5 - (20*a^2*b^3 + 3*b^5)*c^3 + (20*a^3*
b^3 + 9*a*b^5)*c^2 - (10*a^4*b^3 + 9*a^2*b^5 + b^7)*c)*x)*sqrt(a^
2 + b^2 - 2*a*c + c^2))*sqrt(a^2 + b^2 - 2*a*c + c^2) - 4*(8*a^7*
b^2 + 14*a^5*b^4 + 7*a^3*b^6 + a*b^8 - 2*(4*a^2*b^2 + b^4)*c^5 +
10*(4*a^3*b^2 + a*b^4)*c^4 - (80*a^4*b^2 + 32*a^2*b^4 + 3*b^6)*c^
3 + (80*a^5*b^2 + 56*a^3*b^4 + 9*a*b^6)*c^2 + (2*a^5*b^4 + 3*a^3*
b^6 + a*b^8 + 40*a*b^2*c^6 - 8*b^2*c^7 - 2*(40*a^2*b^2 + 7*b^4)*c
^5 + 2*(40*a^3*b^2 + 23*a*b^4)*c^4 - (40*a^4*b^2 + 56*a^2*b^4 + 7
*b^6)*c^3 + (8*a^5*b^2 + 32*a^3*b^4 + 13*a*b^6)*c^2 - (10*a^4*b^4
 + 9*a^2*b^6 + b^8)*c)*x^2 - (40*a^6*b^2 + 46*a^4*b^4 + 13*a^2*b^
6 + b^8)*c + 4*(2*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 + 6*a*b^5*c^3 + 8
*a*b^3*c^5 - 2*b^3*c^6 - (10*a^2*b^3 + 3*b^5)*c^4 + (10*a^4*b^3 -
 b^7)*c^2 - 2*(4*a^5*b^3 + 3*a^3*b^5)*c)*x)*sqrt(a^2 + b^2 - 2*a*
c + c^2))/(4*(2*a^5 + 3*a^3*b^2 + a*b^4 + 10*a*c^4 - 2*c^5 - (20*
a^2 + 3*b^2)*c^3 + (20*a^3 + 9*a*b^2)*c^2 - (10*a^4 + 9*a^2*b^2 +
 b^4)*c)*sqrt(a^2 + b^2 - 2*a*c + c^2)*x^2 - (8*a^6 + 16*a^4*b^2
+ 9*a^2*b^4 + b^6 - 48*a*c^5 + 8*c^6 + 8*(15*a^2 + 2*b^2)*c^4 - 3
2*(5*a^3 + 2*a*b^2)*c^3 + 3*(40*a^4 + 32*a^2*b^2 + 3*b^4)*c^2 - 2
*(24*a^5 + 32*a^3*b^2 + 9*a*b^4)*c)*x^2)) + 4*(a^2 + b^2 - 2*a*c
+ c^2)^(1/4)*sqrt(b^2)*arctan(-(2*sqrt(c*x^2 + b*x + a)*(a^2 + b^
2 - 2*a*c + c^2)^(1/4)*sqrt(b^2)*b + sqrt(2)*(sqrt(a^2 + b^2 - 2*
a*c + c^2)*sqrt(b^2)*(b*x + 2*a) - (2*a^2 - 2*a*c + (a*b - b*c)*x
)*sqrt(b^2))*sqrt((a^2 + b^2 - 2*a*c + c^2 - sqrt(a^2 + b^2 - 2*a
c + c^2)(a - c))/(2*a^2 + b^2 - 4*a*c + 2*c^2 - 2*sqrt(a^2 + b^
2 - 2*a*c + c^2)*(a - c))))/(sqrt(2)*((a - c)*x - sqrt(a^2 + b^2
- 2*a*c + c^2)*x)*sqrt(-(32*a^8*b^2 + 72*a^6*b^4 + 52*a^4*b^6 + 1
3*a^2*b^8 + b^10 + 8*(4*a^2*b^2 + b^4)*c^6 - 48*(4*a^3*b^2 + a*b^

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4)*c^5 + 8*(60*a^4*b^2 + 23*a^2*b^4 + 2*b^6)*c^4 - 32*(20*a^5*b^2
 + 13*a^3*b^4 + 2*a*b^6)*c^3 + 3*(160*a^6*b^2 + 168*a^4*b^4 + 44*
a^2*b^6 + 3*b^8)*c^2 + (8*a^6*b^4 + 16*a^4*b^6 + 9*a^2*b^8 + b^10
 - 192*a*b^2*c^7 + 32*b^2*c^8 + 24*(20*a^2*b^2 + 3*b^4)*c^6 - 16*
(40*a^3*b^2 + 19*a*b^4)*c^5 + 4*(120*a^4*b^2 + 126*a^2*b^4 + 13*b
^6)*c^4 - 8*(24*a^5*b^2 + 52*a^3*b^4 + 17*a*b^6)*c^3 + (32*a^6*b^
2 + 184*a^4*b^4 + 132*a^2*b^6 + 13*b^8)*c^2 - 2*(24*a^5*b^4 + 32*
a^3*b^6 + 9*a*b^8)*c)*x^2 + 2*sqrt(2)*(4*a^4*b^5 + 5*a^2*b^7 + b^
9 - 80*a*b^3*c^5 + 16*b^3*c^6 + 4*(40*a^2*b^3 + 7*b^5)*c^4 - 8*(2
0*a^3*b^3 + 11*a*b^5)*c^3 + (80*a^4*b^3 + 96*a^2*b^5 + 13*b^7)*c^
2 - 2*(8*a^5*b^3 + 20*a^3*b^5 + 9*a*b^7)*c - 2*(4*a^5*b^4 + 6*a^3
*b^6 + 2*a*b^8 + 96*a*b^2*c^6 - 16*b^2*c^7 - 16*(15*a^2*b^2 + 2*b
^4)*c^5 + 4*(80*a^3*b^2 + 33*a*b^4)*c^4 - (240*a^4*b^2 + 208*a^2*
b^4 + 19*b^6)*c^3 + 4*(24*a^5*b^2 + 38*a^3*b^4 + 11*a*b^6)*c^2 -
(16*a^6*b^2 + 48*a^4*b^4 + 31*a^2*b^6 + 3*b^8)*c)*x - (4*a^3*b^5
+ 3*a*b^7 + 64*a*b^3*c^4 - 16*b^3*c^5 - 4*(24*a^2*b^3 + 5*b^5)*c^
3 + 4*(16*a^3*b^3 + 11*a*b^5)*c^2 - (16*a^4*b^3 + 28*a^2*b^5 + 5*
b^7)*c - (8*a^4*b^4 + 8*a^2*b^6 + b^8 - 160*a*b^2*c^5 + 32*b^2*c^
6 + 16*(20*a^2*b^2 + 3*b^4)*c^4 - 8*(40*a^3*b^2 + 19*a*b^4)*c^3 +
 2*(80*a^4*b^2 + 84*a^2*b^4 + 9*b^6)*c^2 - 2*(16*a^5*b^2 + 36*a^3
*b^4 + 13*a*b^6)*c)*x)*sqrt(a^2 + b^2 - 2*a*c + c^2))*sqrt(c*x^2
+ b*x + a)*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*sqrt((a^2 + b^2 - 2*a*
c + c^2 - sqrt(a^2 + b^2 - 2*a*c + c^2)*(a - c))/(2*a^2 + b^2 - 4
*a*c + 2*c^2 - 2*sqrt(a^2 + b^2 - 2*a*c + c^2)*(a - c))) - 2*(96*
a^7*b^2 + 152*a^5*b^4 + 68*a^3*b^6 + 9*a*b^8)*c + 4*(8*a^7*b^3 +
16*a^5*b^5 + 9*a^3*b^7 + a*b^9 - 40*a*b^3*c^6 + 8*b^3*c^7 + 8*(9*
a^2*b^3 + 2*b^5)*c^5 - 8*(5*a^3*b^3 + 6*a*b^5)*c^4 - (40*a^4*b^3
- 32*a^2*b^5 - 9*b^7)*c^3 + (72*a^5*b^3 + 32*a^3*b^5 - 9*a*b^7)*c
^2 - (40*a^6*b^3 + 48*a^4*b^5 + 9*a^2*b^7 - b^9)*c)*x + 4*(8*a^7*
b^2 + 16*a^5*b^4 + 9*a^3*b^6 + a*b^8 - 48*a^2*b^2*c^5 + 8*a*b^2*c
^6 + 8*(15*a^3*b^2 + 2*a*b^4)*c^4 - 32*(5*a^4*b^2 + 2*a^2*b^4)*c^
3 + 3*(40*a^5*b^2 + 32*a^3*b^4 + 3*a*b^6)*c^2 - (48*a*b^2*c^6 - 8
*b^2*c^7 - 8*(15*a^2*b^2 + 2*b^4)*c^5 + 32*(5*a^3*b^2 + 2*a*b^4)*
c^4 - 3*(40*a^4*b^2 + 32*a^2*b^4 + 3*b^6)*c^3 + 2*(24*a^5*b^2 + 3
2*a^3*b^4 + 9*a*b^6)*c^2 - (8*a^6*b^2 + 16*a^4*b^4 + 9*a^2*b^6 +
b^8)*c)*x^2 - 2*(24*a^6*b^2 + 32*a^4*b^4 + 9*a^2*b^6)*c + (8*a^6*
b^3 + 16*a^4*b^5 + 9*a^2*b^7 + b^9 - 48*a*b^3*c^5 + 8*b^3*c^6 + 8
*(15*a^2*b^3 + 2*b^5)*c^4 - 32*(5*a^3*b^3 + 2*a*b^5)*c^3 + 3*(40*
a^4*b^3 + 32*a^2*b^5 + 3*b^7)*c^2 - 2*(24*a^5*b^3 + 32*a^3*b^5 +
9*a*b^7)*c)*x - 4*(2*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 + 10*a^2*b^2*c
^4 - 2*a*b^2*c^5 - (20*a^3*b^2 + 3*a*b^4)*c^3 + (20*a^4*b^2 + 9*a
^2*b^4)*c^2 + (10*a*b^2*c^5 - 2*b^2*c^6 - (20*a^2*b^2 + 3*b^4)*c^
4 + (20*a^3*b^2 + 9*a*b^4)*c^3 - (10*a^4*b^2 + 9*a^2*b^4 + b^6)*c
^2 + (2*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c)*x^2 - (10*a^5*b^2 + 9*a^3
*b^4 + a*b^6)*c + (2*a^5*b^3 + 3*a^3*b^5 + a*b^7 + 10*a*b^3*c^4 -
 2*b^3*c^5 - (20*a^2*b^3 + 3*b^5)*c^3 + (20*a^3*b^3 + 9*a*b^5)*c^
2 - (10*a^4*b^3 + 9*a^2*b^5 + b^7)*c)*x)*sqrt(a^2 + b^2 - 2*a*c +
 c^2))*sqrt(a^2 + b^2 - 2*a*c + c^2) - 4*(8*a^7*b^2 + 14*a^5*b^4
+ 7*a^3*b^6 + a*b^8 - 2*(4*a^2*b^2 + b^4)*c^5 + 10*(4*a^3*b^2 + a
*b^4)*c^4 - (80*a^4*b^2 + 32*a^2*b^4 + 3*b^6)*c^3 + (80*a^5*b^2 +
 56*a^3*b^4 + 9*a*b^6)*c^2 + (2*a^5*b^4 + 3*a^3*b^6 + a*b^8 + 40*
a*b^2*c^6 - 8*b^2*c^7 - 2*(40*a^2*b^2 + 7*b^4)*c^5 + 2*(40*a^3*b^
2 + 23*a*b^4)*c^4 - (40*a^4*b^2 + 56*a^2*b^4 + 7*b^6)*c^3 + (8*a^
5*b^2 + 32*a^3*b^4 + 13*a*b^6)*c^2 - (10*a^4*b^4 + 9*a^2*b^6 + b^
8)*c)*x^2 - (40*a^6*b^2 + 46*a^4*b^4 + 13*a^2*b^6 + b^8)*c + 4*(2
*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 + 6*a*b^5*c^3 + 8*a*b^3*c^5 - 2*b^
3*c^6 - (10*a^2*b^3 + 3*b^5)*c^4 + (10*a^4*b^3 - b^7)*c^2 - 2*(4*
a^5*b^3 + 3*a^3*b^5)*c)*x)*sqrt(a^2 + b^2 - 2*a*c + c^2))/(4*(2*a
^5 + 3*a^3*b^2 + a*b^4 + 10*a*c^4 - 2*c^5 - (20*a^2 + 3*b^2)*c^3
+ (20*a^3 + 9*a*b^2)*c^2 - (10*a^4 + 9*a^2*b^2 + b^4)*c)*sqrt(a^2
 + b^2 - 2*a*c + c^2)*x^2 - (8*a^6 + 16*a^4*b^2 + 9*a^2*b^4 + b^6
 - 48*a*c^5 + 8*c^6 + 8*(15*a^2 + 2*b^2)*c^4 - 32*(5*a^3 + 2*a*b^
2)*c^3 + 3*(40*a^4 + 32*a^2*b^2 + 3*b^4)*c^2 - 2*(24*a^5 + 32*a^3
*b^2 + 9*a*b^4)*c)*x^2))*sqrt((a^2 + b^2 - 2*a*c + c^2 - sqrt(a^2
 + b^2 - 2*a*c + c^2)*(a - c))/(2*a^2 + b^2 - 4*a*c + 2*c^2 - 2*s
qrt(a^2 + b^2 - 2*a*c + c^2)*(a - c))) + sqrt(2)*(a*b^2 - b^2*c +
 2*(a*b*c - b*c^2)*x - (2*b*c*x + b^2)*sqrt(a^2 + b^2 - 2*a*c + c
^2))*sqrt((a^2 + b^2 - 2*a*c + c^2 - sqrt(a^2 + b^2 - 2*a*c + c^2
)*(a - c))/(2*a^2 + b^2 - 4*a*c + 2*c^2 - 2*sqrt(a^2 + b^2 - 2*a*
c + c^2)*(a - c))) + 2*sqrt(c*x^2 + b*x + a)*(a^2 + b^2 - 2*a*c +
 c^2)^(1/4)*(a*b - b*c - sqrt(a^2 + b^2 - 2*a*c + c^2)*b))) - 4*(
a^2 + b^2 - 2*a*c + c^2)^(1/4)*sqrt(b^2)*arctan((2*sqrt(c*x^2 + b
```

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*x + a)*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*sqrt(b^2)*b - sqrt(2)*(sq
rt(a^2 + b^2 - 2*a*c + c^2)*sqrt(b^2)*(b*x + 2*a) - (2*a^2 - 2*a*
c + (a*b - b*c)*x)*sqrt(b^2))*sqrt((a^2 + b^2 - 2*a*c + c^2 - sqr
t(a^2 + b^2 - 2*a*c + c^2)*(a - c))/(2*a^2 + b^2 - 4*a*c + 2*c^2
 - 2*sqrt(a^2 + b^2 - 2*a*c + c^2)*(a - c))))/(sqrt(2)*((a - c)*x
 - sqrt(a^2 + b^2 - 2*a*c + c^2)*x)*sqrt(-(32*a^8*b^2 + 72*a^6*b^4
 + 52*a^4*b^6 + 13*a^2*b^8 + b^10 + 8*(4*a^2*b^2 + b^4)*c^6 - 48*
(4*a^3*b^2 + a*b^4)*c^5 + 8*(60*a^4*b^2 + 23*a^2*b^4 + 2*b^6)*c^4
 - 32*(20*a^5*b^2 + 13*a^3*b^4 + 2*a*b^6)*c^3 + 3*(160*a^6*b^2 +
168*a^4*b^4 + 44*a^2*b^6 + 3*b^8)*c^2 + (8*a^6*b^4 + 16*a^4*b^6 +
 9*a^2*b^8 + b^10 - 192*a*b^2*c^7 + 32*b^2*c^8 + 24*(20*a^2*b^2 +
 3*b^4)*c^6 - 16*(40*a^3*b^2 + 19*a*b^4)*c^5 + 4*(120*a^4*b^2 + 1
26*a^2*b^4 + 13*b^6)*c^4 - 8*(24*a^5*b^2 + 52*a^3*b^4 + 17*a*b^6)
*c^3 + (32*a^6*b^2 + 184*a^4*b^4 + 132*a^2*b^6 + 13*b^8)*c^2 - 2*
(24*a^5*b^4 + 32*a^3*b^6 + 9*a*b^8)*c)*x^2 - 2*sqrt(2)*(4*a^4*b^5
 + 5*a^2*b^7 + b^9 - 80*a*b^3*c^5 + 16*b^3*c^6 + 4*(40*a^2*b^3 +
7*b^5)*c^4 - 8*(20*a^3*b^3 + 11*a*b^5)*c^3 + (80*a^4*b^3 + 96*a^2
*b^5 + 13*b^7)*c^2 - 2*(8*a^5*b^3 + 20*a^3*b^5 + 9*a*b^7)*c - 2*(
4*a^5*b^4 + 6*a^3*b^6 + 2*a*b^8 + 96*a*b^2*c^6 - 16*b^2*c^7 - 16*
(15*a^2*b^2 + 2*b^4)*c^5 + 4*(80*a^3*b^2 + 33*a*b^4)*c^4 - (240*a
^4*b^2 + 208*a^2*b^4 + 19*b^6)*c^3 + 4*(24*a^5*b^2 + 38*a^3*b^4 +
 11*a*b^6)*c^2 - (16*a^6*b^2 + 48*a^4*b^4 + 31*a^2*b^6 + 3*b^8)*c
)*x - (4*a^3*b^5 + 3*a*b^7 + 64*a*b^3*c^4 - 16*b^3*c^5 - 4*(24*a^
2*b^3 + 5*b^5)*c^3 + 4*(16*a^3*b^3 + 11*a*b^5)*c^2 - (16*a^4*b^3
+ 28*a^2*b^5 + 5*b^7)*c - (8*a^4*b^4 + 8*a^2*b^6 + b^8 - 160*a*b^
2*c^5 + 32*b^2*c^6 + 16*(20*a^2*b^2 + 3*b^4)*c^4 - 8*(40*a^3*b^2
+ 19*a*b^4)*c^3 + 2*(80*a^4*b^2 + 84*a^2*b^4 + 9*b^6)*c^2 - 2*(16
*a^5*b^2 + 36*a^3*b^4 + 13*a*b^6)*c)*x)*sqrt(a^2 + b^2 - 2*a*c +
c^2))*sqrt(c*x^2 + b*x + a)*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*sqrt(
(a^2 + b^2 - 2*a*c + c^2 - sqrt(a^2 + b^2 - 2*a*c + c^2)*(a - c))
/(2*a^2 + b^2 - 4*a*c + 2*c^2 - 2*sqrt(a^2 + b^2 - 2*a*c + c^2)*(
a - c))) - 2*(96*a^7*b^2 + 152*a^5*b^4 + 68*a^3*b^6 + 9*a*b^8)*c
+ 4*(8*a^7*b^3 + 16*a^5*b^5 + 9*a^3*b^7 + a*b^9 - 40*a*b^3*c^6 +
8*b^3*c^7 + 8*(9*a^2*b^3 + 2*b^5)*c^5 - 8*(5*a^3*b^3 + 6*a*b^5)*c
^4 - (40*a^4*b^3 - 32*a^2*b^5 - 9*b^7)*c^3 + (72*a^5*b^3 + 32*a^3
*b^5 - 9*a*b^7)*c^2 - (40*a^6*b^3 + 48*a^4*b^5 + 9*a^2*b^7 - b^9)
*c)*x + 4*(8*a^7*b^2 + 16*a^5*b^4 + 9*a^3*b^6 + a*b^8 - 48*a^2*b^
2*c^5 + 8*a*b^2*c^6 + 8*(15*a^3*b^2 + 2*a*b^4)*c^4 - 32*(5*a^4*b^
2 + 2*a^2*b^4)*c^3 + 3*(40*a^5*b^2 + 32*a^3*b^4 + 3*a*b^6)*c^2 -
(48*a*b^2*c^6 - 8*b^2*c^7 - 8*(15*a^2*b^2 + 2*b^4)*c^5 + 32*(5*a^
3*b^2 + 2*a*b^4)*c^4 - 3*(40*a^4*b^2 + 32*a^2*b^4 + 3*b^6)*c^3 +
2*(24*a^5*b^2 + 32*a^3*b^4 + 9*a*b^6)*c^2 - (8*a^6*b^2 + 16*a^4*b
^4 + 9*a^2*b^6 + b^8)*c)*x^2 - 2*(24*a^6*b^2 + 32*a^4*b^4 + 9*a^2
*b^6)*c + (8*a^6*b^3 + 16*a^4*b^5 + 9*a^2*b^7 + b^9 - 48*a*b^3*c^
5 + 8*b^3*c^6 + 8*(15*a^2*b^3 + 2*b^5)*c^4 - 32*(5*a^3*b^3 + 2*a*
b^5)*c^3 + 3*(40*a^4*b^3 + 32*a^2*b^5 + 3*b^7)*c^2 - 2*(24*a^5*b^
3 + 32*a^3*b^5 + 9*a*b^7)*c)*x - 4*(2*a^6*b^2 + 3*a^4*b^4 + a^2*b
^6 + 10*a^2*b^2*c^4 - 2*a*b^2*c^5 - (20*a^3*b^2 + 3*a*b^4)*c^3 +
(20*a^4*b^2 + 9*a^2*b^4)*c^2 + (10*a*b^2*c^5 - 2*b^2*c^6 - (20*a^
2*b^2 + 3*b^4)*c^4 + (20*a^3*b^2 + 9*a*b^4)*c^3 - (10*a^4*b^2 + 9
*a^2*b^4 + b^6)*c^2 + (2*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c)*x^2 - (1
0*a^5*b^2 + 9*a^3*b^4 + a*b^6)*c + (2*a^5*b^3 + 3*a^3*b^5 + a*b^7
 + 10*a*b^3*c^4 - 2*b^3*c^5 - (20*a^2*b^3 + 3*b^5)*c^3 + (20*a^3*
b^3 + 9*a*b^5)*c^2 - (10*a^4*b^3 + 9*a^2*b^5 + b^7)*c)*x)*sqrt(a^
2 + b^2 - 2*a*c + c^2))*sqrt(a^2 + b^2 - 2*a*c + c^2) - 4*(8*a^7*
b^2 + 14*a^5*b^4 + 7*a^3*b^6 + a*b^8 - 2*(4*a^2*b^2 + b^4)*c^5 +
10*(4*a^3*b^2 + a*b^4)*c^4 - (80*a^4*b^2 + 32*a^2*b^4 + 3*b^6)*c^
3 + (80*a^5*b^2 + 56*a^3*b^4 + 9*a*b^6)*c^2 + (2*a^5*b^4 + 3*a^3*
b^6 + a*b^8 + 40*a*b^2*c^6 - 8*b^2*c^7 - 2*(40*a^2*b^2 + 7*b^4)*c
^5 + 2*(40*a^3*b^2 + 23*a*b^4)*c^4 - (40*a^4*b^2 + 56*a^2*b^4 + 7
*b^6)*c^3 + (8*a^5*b^2 + 32*a^3*b^4 + 13*a*b^6)*c^2 - (10*a^4*b^4
 + 9*a^2*b^6 + b^8)*c)*x^2 - (40*a^6*b^2 + 46*a^4*b^4 + 13*a^2*b^
6 + b^8)*c + 4*(2*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 + 6*a*b^5*c^3 + 8
*a*b^3*c^5 - 2*b^3*c^6 - (10*a^2*b^3 + 3*b^5)*c^4 + (10*a^4*b^3 -
 b^7)*c^2 - 2*(4*a^5*b^3 + 3*a^3*b^5)*c)*x)*sqrt(a^2 + b^2 - 2*a*
c + c^2))/(4*(2*a^5 + 3*a^3*b^2 + a*b^4 + 10*a*c^4 - 2*c^5 - (20*
a^2 + 3*b^2)*c^3 + (20*a^3 + 9*a*b^2)*c^2 - (10*a^4 + 9*a^2*b^2 +
 b^4)*c)*sqrt(a^2 + b^2 - 2*a*c + c^2)*x^2 - (8*a^6 + 16*a^4*b^2
+ 9*a^2*b^4 + b^6 - 48*a*c^5 + 8*c^6 + 8*(15*a^2 + 2*b^2)*c^4 - 3
2*(5*a^3 + 2*a*b^2)*c^3 + 3*(40*a^4 + 32*a^2*b^2 + 3*b^4)*c^2 - 2
*(24*a^5 + 32*a^3*b^2 + 9*a*b^4)*c)*x^2))*sqrt((a^2 + b^2 - 2*a*c
 + c^2 - sqrt(a^2 + b^2 - 2*a*c + c^2)*(a - c))/(2*a^2 + b^2 - 4*
```

```
a*c + 2*c^2 - 2*sqrt(a^2 + b^2 - 2*a*c + c^2)*(a - c))) + sqrt(2)
*(a*b^2 - b^2*c + 2*(a*b*c - b*c^2)*x - (2*b*c*x + b^2)*sqrt(a^2
+ b^2 - 2*a*c + c^2))*sqrt((a^2 + b^2 - 2*a*c + c^2 - sqrt(a^2 +
b^2 - 2*a*c + c^2)*(a - c))/(2*a^2 + b^2 - 4*a*c + 2*c^2 - 2*sqrt
(a^2 + b^2 - 2*a*c + c^2)*(a - c))) - 2*sqrt(c*x^2 + b*x + a)*(a^
2 + b^2 - 2*a*c + c^2)^(1/4)*(a*b - b*c - sqrt(a^2 + b^2 - 2*a*c
+ c^2)*b))))/((a - c - sqrt(a^2 + b^2 - 2*a*c + c^2))*sqrt((a^2 +
 b^2 - 2*a*c + c^2 - sqrt(a^2 + b^2 - 2*a*c + c^2)*(a - c))/(2*a^
2 + b^2 - 4*a*c + 2*c^2 - 2*sqrt(a^2 + b^2 - 2*a*c + c^2)*(a - c)
)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx - c}{(x^2 + 1)\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a-c)/(x**2+1)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((a + b*x - c)/((x**2 + 1)*sqrt(a + b*x + c*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x + a - c)/(sqrt(c*x^2 + b*x + a)*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.13 $\int \frac{(A+Bx)(a+bx+cx^2)}{d+ex+fx^2}\,dx$

Optimal. Leaf size=184

$$-\frac{\log\left(d+ex+fx^2\right)\left(Af(ce-bf)-B\left(af^2-bef-cdf+ce^2\right)\right)}{2f^3}-\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(Af\left(2af^2-bef-2cdf+ce^2\right)+B\left(f\left(-aef-2bdf+be^2\right)-c\left(e^3-3def\right)\right)\right)}{f^3\sqrt{e^2-4df}}-\frac{x(-Acf-bBf+Bce)}{f^2}+\frac{Bcx^2}{2f}$$

```
[Out] -(((B*c*e - b*B*f - A*c*f)*x)/f^2) + (B*c*x^2)/(2*f) - ((A*f*(c*e
^2 - 2*c*d*f - b*e*f + 2*a*f^2) + B*(f*(b*e^2 - 2*b*d*f - a*e*f)
- c*(e^3 - 3*d*e*f)))*ArcTanh[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(f^
3*Sqrt[e^2 - 4*d*f]) - ((A*f*(c*e - b*f) - B*(c*e^2 - c*d*f - b*e
*f + a*f^2))*Log[d + e*x + f*x^2])/(2*f^3)
```

Rubi [A] time = 0.6998, antiderivative size = 182, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$

$$-\frac{\log\left(d+ex+fx^2\right)\left(Af(ce-bf)-B\left(af^2-bef-cdf+ce^2\right)\right)}{2f^3}-\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(Af\left(2af^2-bef-2cdf+ce^2\right)+Bf\left(-aef-2bdf+be^2\right)-Bc\left(e^3-3def\right)\right)}{f^3\sqrt{e^2-4df}}-\frac{x(-Acf-bBf+Bce)}{f^2}+\frac{Bcx^2}{2f}$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2),x]
```

```
[Out] -(((B*c*e - b*B*f - A*c*f)*x)/f^2) + (B*c*x^2)/(2*f) - ((B*f*(b*e
^2 - 2*b*d*f - a*e*f) - B*c*(e^3 - 3*d*e*f) + A*f*(c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2))*ArcTanh[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(f^
3*Sqrt[e^2 - 4*d*f]) - ((A*f*(c*e - b*f) - B*(c*e^2 - c*d*f - b*e
*f + a*f^2))*Log[d + e*x + f*x^2])/(2*f^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bc\int x\,dx}{f}+(Acf+Bbf-Bce)\int\frac{1}{f^2}\,dx+\frac{\left(Abf^2-Acef+Baf^2-Bbef-Bcdf+Bce^2\right)\log\left(d+ex+fx^2\right)}{2f^3}+\frac{\left(e\left(Abf^2-Acef+Baf^2-Bbef-Bcdf+Bce^2\right)-2f\left(Aaf^2-Acdf-Bbdf+Bcde\right)\right)\operatorname{atanh}\left(\frac{e+2fx}{\sqrt{-4df+e^2}}\right)}{f^3\sqrt{-4df+e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)*(c*x**2+b*x+a)/(f*x**2+e*x+d),x)
```

```
[Out] B*c*Integral(x, x)/f + (A*c*f + B*b*f - B*c*e)*Integral(f**(-2),
x) + (A*b*f**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**
2)*log(d + e*x + f*x**2)/(2*f**3) + (e*(A*b*f**2 - A*c*e*f + B*a*
f**2 - B*b*e*f - B*c*d*f + B*c*e**2) - 2*f*(A*a*f**2 - A*c*d*f -
```

```
B*b*d*f + B*c*d*e))*atanh((e + 2*f*x)/sqrt(-4*d*f + e**2))/(f**3*
sqrt(-4*d*f + e**2))
```

Mathematica [A] time = 0.363237, size = 175, normalized size = 0.95

$$\frac{\log(d+x(e+fx))\left(Bf(af-be)+Af(bf-ce)+Bc\left(e^2-df\right)\right)-\frac{2\tan^{-1}\left(\frac{e+2fx}{\sqrt{4df-e^2}}\right)\left(Af\left(-2af^2+bef+2cdf-ce^2\right)+Bf\left(aef+2bdf-be^2\right)+\right.}{\sqrt{4df-e^2}}}{2f^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2),x]
```

```
[Out] (2*f*(-(B*c*e) + b*B*f + A*c*f)*x + B*c*f^2*x^2 - (2*(B*f*(-(b*e^
2) + 2*b*d*f + a*e*f) + B*c*(e^3 - 3*d*e*f) + A*f*(-(c*e^2) + 2*c
*d*f + b*e*f - 2*a*f^2))*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]])/
Sqrt[-e^2 + 4*d*f] + (B*f*(-(b*e) + a*f) + A*f*(-(c*e) + b*f) + B
*c*(e^2 - d*f))*Log[d + x*(e + f*x)])/(2*f^3)
```

Maple [B] time = 0.01, size = 510, normalized size = 2.8

$$\frac{Bcx^2}{2f}+\frac{Acx}{f}+\frac{bBx}{f}-\frac{Bcex}{f^2}+\frac{\ln\left(fx^2+ex+d\right)Ab}{2f}-\frac{\ln\left(fx^2+ex+d\right)Ace}{2f^2}$$
$$+\frac{\ln\left(fx^2+ex+d\right)Ba}{2f}-\frac{\ln\left(fx^2+ex+d\right)Bbe}{2f^2}-\frac{\ln\left(fx^2+ex+d\right)Bcd}{2f^2}$$
$$+\frac{\ln\left(fx^2+ex+d\right)Bce^2}{2f^3}+2\frac{Aa}{\sqrt{4df-e^2}}\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)$$
$$-2\frac{Acd}{f\sqrt{4df-e^2}}\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)-2\frac{bBd}{f\sqrt{4df-e^2}}\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)$$
$$+3\frac{Bcde}{f^2\sqrt{4df-e^2}}\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)-\frac{Aeb}{f}\arctan\left((2fx+e)\frac{1}{\sqrt{4df-e^2}}\right)\frac{1}{\sqrt{4df-e^2}}$$
$$+\frac{e^2Ac}{f^2}\arctan\left((2fx+e)\frac{1}{\sqrt{4df-e^2}}\right)\frac{1}{\sqrt{4df-e^2}}$$
$$-\frac{aBe}{f}\arctan\left((2fx+e)\frac{1}{\sqrt{4df-e^2}}\right)\frac{1}{\sqrt{4df-e^2}}$$
$$+\frac{e^2Bb}{f^2}\arctan\left((2fx+e)\frac{1}{\sqrt{4df-e^2}}\right)\frac{1}{\sqrt{4df-e^2}}$$
$$-\frac{Bce^3}{f^3}\arctan\left((2fx+e)\frac{1}{\sqrt{4df-e^2}}\right)\frac{1}{\sqrt{4df-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x)
```

```
[Out] 1/2*B*c*x^2/f+1/f*A*c*x+1/f*b*B*x-1/f^2*B*c*e*x+1/2/f*ln(f*x^2+e*
x+d)*A*b-1/2/f^2*ln(f*x^2+e*x+d)*A*c*e+1/2/f*ln(f*x^2+e*x+d)*B*a-
1/2/f^2*ln(f*x^2+e*x+d)*B*b*e-1/2/f^2*ln(f*x^2+e*x+d)*B*c*d+1/2/f
^3*ln(f*x^2+e*x+d)*B*c*e^2+2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(
4*d*f-e^2)^(1/2))*A*a-2/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d
*f-e^2)^(1/2))*A*c*d-2/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*
f-e^2)^(1/2))*B*b*d+3/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d
*f-e^2)^(1/2))*B*c*d*e-1/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*
d*f-e^2)^(1/2))*e*A*b+1/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4
*d*f-e^2)^(1/2))*e^2*A*c-1/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(
```

```
4*d*f-e^2)^(1/2))*e*B*a+1/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/
(4*d*f-e^2)^(1/2))*e^2*B*b-1/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f*x+
e)/(4*d*f-e^2)^(1/2))*e^3*B*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)*(B*x + A)/(f*x^2 + e*x + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.295823, size = 1, normalized size = 0.01

$$\left[-\frac{\left(Bce^3 - 2\,Aaf^3 + (2\,(Bb + Ac)d + (Ba + Ab)e)f^2 - \left(3\,Bcde + (Bb + Ac)e^2\right)f\right)\log\left(-\frac{e^3 - 4\,def + 2\,(e^2f - 4\,df^2)x - (2\,f^2x^2 + 2\,efx + e^2 -}{fx^2 + ex + d}\right.}{2}\right.$$

$$-\frac{2\left(Bce^3 - 2\,Aaf^3 + (2\,(Bb + Ac)d + (Ba + Ab)e)f^2 - \left(3\,Bcde + (Bb + Ac)e^2\right)f\right)\arctan\left(-\frac{\sqrt{-e^2 + 4\,df}(2\,fx + e)}{e^2 - 4\,df}\right) - \left(Bcf^2x^2 - 2\right.}{2\,\sqrt{-e^2 + 4\,df}\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)*(B*x + A)/(f*x^2 + e*x + d),x, algorithm="fricas")
```

```
[Out] [-1/2*((B*c*e^3 - 2*A*a*f^3 + (2*(B*b + A*c)*d + (B*a + A*b)*e)*f
^2 - (3*B*c*d*e + (B*b + A*c)*e^2)*f)*log(-(e^3 - 4*d*e*f + 2*(e^
2*f - 4*d*f^2)*x - (2*f^2*x^2 + 2*e*f*x + e^2 - 2*d*f)*sqrt(e^2 -
 4*d*f))/(f*x^2 + e*x + d)) - (B*c*f^2*x^2 - 2*(B*c*e*f - (B*b +
A*c)*f^2)*x + (B*c*e^2 + (B*a + A*b)*f^2 - (B*c*d + (B*b + A*c)*e
)*f)*log(f*x^2 + e*x + d))*sqrt(e^2 - 4*d*f))/(sqrt(e^2 - 4*d*f)*
f^3), -1/2*(2*(B*c*e^3 - 2*A*a*f^3 + (2*(B*b + A*c)*d + (B*a + A*
b)*e)*f^2 - (3*B*c*d*e + (B*b + A*c)*e^2)*f)*arctan(-sqrt(-e^2 +
4*d*f)*(2*f*x + e)/(e^2 - 4*d*f)) - (B*c*f^2*x^2 - 2*(B*c*e*f - (
B*b + A*c)*f^2)*x + (B*c*e^2 + (B*a + A*b)*f^2 - (B*c*d + (B*b +
A*c)*e)*f)*log(f*x^2 + e*x + d))*sqrt(-e^2 + 4*d*f))/(sqrt(-e^2 +
 4*d*f)*f^3)]
```

Sympy [A] time = 29.3022, size = 1260, normalized size = 6.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)*(c*x**2+b*x+a)/(f*x**2+e*x+d),x)
```

```
[Out] B*c*x**2/(2*f) + (-sqrt(-4*d*f + e**2)*(-2*A*a*f**3 + A*b*e*f**2
+ 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e**
2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*f**2
 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3))*l
og(x + (-A*a*e*f**2 + 2*A*b*d*f**2 - A*c*d*e*f + 2*B*a*d*f**2 - B
*b*d*e*f - 2*B*c*d**2*f + B*c*d*e**2 - 4*d*f**3*(-sqrt(-4*d*f + e
**2)*(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*
```

```
e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f
**3*(4*d*f - e**2)) + (A*b*f**2 - A*c*e*f + B*a*f**2 - B*b*e*f -
B*c*d*f + B*c*e**2)/(2*f**3)) + e**2*f**2*(-sqrt(-4*d*f + e**2)*(
-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2
 + 2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4
*d*f - e**2)) + (A*b*f**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*
f + B*c*e**2)/(2*f**3)))/(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2
 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*
e*f + B*c*e**3)) + (sqrt(-4*d*f + e**2)*(-2*A*a*f**3 + A*b*e*f**2
 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e*
*2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*f**
2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3))*
log(x + (-A*a*e*f**2 + 2*A*b*d*f**2 - A*c*d*e*f + 2*B*a*d*f**2 -
B*b*d*e*f - 2*B*c*d**2*f + B*c*d*e**2 - 4*d*f**3*(sqrt(-4*d*f + e
**2)*(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*
e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f
**3*(4*d*f - e**2)) + (A*b*f**2 - A*c*e*f + B*a*f**2 - B*b*e*f -
B*c*d*f + B*c*e**2)/(2*f**3)) + e**2*f**2*(sqrt(-4*d*f + e**2)*(-
2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2
+ 2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*
d*f - e**2)) + (A*b*f**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f
 + B*c*e**2)/(2*f**3)))/(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2
- A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e
*f + B*c*e**3)) + x*(A*c*f + B*b*f - B*c*e)/f**2
```

GIAC/XCAS [A] time = 0.268485, size = 258, normalized size = 1.4

$$\frac{Bcfx^2 + 2\,Bbfx + 2\,Acfx - 2\,Bcxe}{2\,f^2} - \frac{\left(Bcdf - Baf^2 - Abf^2 + Bbfe + Acfe - Bce^2\right)\ln\left(fx^2 + xe + d\right)}{2\,f^3} - \frac{\left(2\,Bbdf^2 + 2\,Acdf^2 - 2\,Aaf^3 - 3\,Bcdfe + Baf^2e + Abf^2e - Bbfe^2 - Acfe^2 + Bce^3\right)\arctan\left(\frac{2\,fx+e}{\sqrt{4\,df-e^2}}\right)}{\sqrt{4\,df-e^2}\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)*(B*x + A)/(f*x^2 + e*x + d),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*f*x^2 + 2*B*b*f*x + 2*A*c*f*x - 2*B*c*x*e)/f^2 - 1/2*(B*
c*d*f - B*a*f^2 - A*b*f^2 + B*b*f*e + A*c*f*e - B*c*e^2)*ln(f*x^2
 + x*e + d)/f^3 - (2*B*b*d*f^2 + 2*A*c*d*f^2 - 2*A*a*f^3 - 3*B*c*
d*f*e + B*a*f^2*e + A*b*f^2*e - B*b*f*e^2 - A*c*f*e^2 + B*c*e^3)*
arctan((2*f*x + e)/sqrt(4*d*f - e^2))/(sqrt(4*d*f - e^2)*f^3)
```

3.14 $\int \frac{(A+Bx)(a+bx+cx^2)^2}{d+ex+fx^2}\,dx$

Optimal. Leaf size=542

$$\frac{\log\left(d+ex+fx^2\right)\left(B\left(-f^2\left(-a^2f^2+2abef+b^2\left(-\left(e^2-df\right)\right)\right)+2cf\left(af\left(e^2-df\right)-b\left(e^3-2def\right)\right)+c^2\left(d^2f^2-3de^2\right.\right.\right.}{2f^5}$$

$$-\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(Af\left(-f^2\left(-2a^2f^2+2abef+b^2\left(-\left(e^2-2df\right)\right)\right)+2cf\left(af\left(e^2-2df\right)-b\left(e^3-3def\right)\right)+c^2\left(2d^2f^2\right.\right.\right.}{}$$

$$+\frac{x\left(Af\left(-2cf(be-af)+b^2f^2+c^2\left(e^2-df\right)\right)+B(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)\right)}{f^4}$$

$$-\frac{x^2\left(Acf(ce-2bf)-B\left(-2cf(be-af)+b^2f^2+c^2\left(e^2-df\right)\right)\right)}{2f^3}-\frac{cx^3(-Acf-2bBf+Bce)}{3f^2}+\frac{Bc^2x^4}{4f}$$

```
[Out] ((B*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + A*f*(b^2*f^
2 - 2*c*f*(b*e - a*f) + c^2*(e^2 - d*f)))*x)/f^4 - ((A*c*f*(c*e -
 2*b*f) - B*(b^2*f^2 - 2*c*f*(b*e - a*f) + c^2*(e^2 - d*f)))*x^2)
/(2*f^3) - (c*(B*c*e - 2*b*B*f - A*c*f)*x^3)/(3*f^2) + (B*c^2*x^4
)/(4*f) - ((A*f*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) - f^2*(2*a*b*e
*f - 2*a^2*f^2 - b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(e^2 - 2*d*f) -
b*(e^3 - 3*d*e*f))) - B*(c^2*(e^5 - 5*d*e^3*f + 5*d^2*e*f^2) + f^
2*(a^2*e*f^2 - 2*a*b*f*(e^2 - 2*d*f) + b^2*(e^3 - 3*d*e*f)) + 2*c
*f*(a*e*f*(e^2 - 3*d*f) - b*(e^4 - 4*d*e^2*f + 2*d^2*f^2))))*ArcT
anh[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(f^5*Sqrt[e^2 - 4*d*f]) + ((A
*f*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + B*(c^2*(e^4
- 3*d*e^2*f + d^2*f^2) - f^2*(2*a*b*e*f - a^2*f^2 - b^2*(e^2 - d*
f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(e^3 - 2*d*e*f))))*Log[d + e*x +
 f*x^2])/(2*f^5)
```

Rubi [A] time = 2.96122, antiderivative size = 542, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\log\left(d+ex+fx^2\right)\left(B\left(-f^2\left(-a^2f^2+2abef+b^2\left(-\left(e^2-df\right)\right)\right)+2cf\left(af\left(e^2-df\right)-b\left(e^3-2def\right)\right)+c^2\left(d^2f^2-3de^2\right.\right.\right.}{2f^5}$$

$$-\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(Af\left(-f^2\left(-2a^2f^2+2abef+b^2\left(-\left(e^2-2df\right)\right)\right)+2cf\left(af\left(e^2-2df\right)-b\left(e^3-3def\right)\right)+c^2\left(2d^2f^2\right.\right.\right.}{}$$

$$-\frac{x\left(B(ce-bf)\left(2af^2-bef-2cdf+ce^2\right)-Af\left(-2cf(be-af)+b^2f^2+c^2\left(e^2-df\right)\right)\right)}{f^4}$$

$$-\frac{x^2\left(Acf(ce-2bf)-B\left(-2cf(be-af)+b^2f^2+c^2\left(e^2-df\right)\right)\right)}{2f^3}-\frac{cx^3(-Acf-2bBf+Bce)}{3f^2}+\frac{Bc^2x^4}{4f}$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2),x]
```

```
[Out] -(((B*(c*e - b*f)*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2) - A*f*(b^2*
f^2 - 2*c*f*(b*e - a*f) + c^2*(e^2 - d*f)))*x)/f^4) - ((A*c*f*(c*
e - 2*b*f) - B*(b^2*f^2 - 2*c*f*(b*e - a*f) + c^2*(e^2 - d*f)))*x
^2)/(2*f^3) - (c*(B*c*e - 2*b*B*f - A*c*f)*x^3)/(3*f^2) + (B*c^2*
x^4)/(4*f) - ((A*f*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) - f^2*(2*a*
b*e*f - 2*a^2*f^2 - b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(e^2 - 2*d*f)
 - b*(e^3 - 3*d*e*f))) - B*(c^2*(e^5 - 5*d*e^3*f + 5*d^2*e*f^2) +
 f^2*(a^2*e*f^2 - 2*a*b*f*(e^2 - 2*d*f) + b^2*(e^3 - 3*d*e*f)) +
2*c*f*(a*e*f*(e^2 - 3*d*f) - b*(e^4 - 4*d*e^2*f + 2*d^2*f^2))))*A
rcTanh[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(f^5*Sqrt[e^2 - 4*d*f]) +
((A*f*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + B*(c^2*(e
^4 - 3*d*e^2*f + d^2*f^2) - f^2*(2*a*b*e*f - a^2*f^2 - b^2*(e^2 -
 d*f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(e^3 - 2*d*e*f))))*Log[d + e*
```

```
x + f*x^2])/(2*f^5)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)*(c*x**2+b*x+a)**2/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.8531, size = 535, normalized size = 0.99

$$6\log(d+x(e+fx))\left(B\left(f^2\left(a^2f^2-2abef+b^2\left(e^2-df\right)\right)-2cf\left(af\left(df-e^2\right)+b\left(e^3-2def\right)\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2),x]
```

```
[Out] (12*f*(-(B*(c*e - b*f)*(f*(-(b*e) + 2*a*f) + c*(e^2 - 2*d*f))) +
A*f*(b^2*f^2 + 2*c*f*(-(b*e) + a*f) + c^2*(e^2 - d*f)))*x + 6*f^2
*(A*c*f*(-(c*e) + 2*b*f) + B*(b^2*f^2 + 2*c*f*(-(b*e) + a*f) + c^
2*(e^2 - d*f)))*x^2 + 4*c*f^3*(-(B*c*e) + 2*b*B*f + A*c*f)*x^3 +
3*B*c^2*f^4*x^4 - (12*(-(A*f*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) +
 f^2*(-2*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(e
^2 - 2*d*f) - b*(e^3 - 3*d*e*f)))) + B*(c^2*(e^5 - 5*d*e^3*f + 5*
d^2*e*f^2) + f^2*(a^2*e*f^2 + 2*a*b*f*(-e^2 + 2*d*f) + b^2*(e^3 -
 3*d*e*f)) - 2*c*f*(-(a*e*f*(e^2 - 3*d*f)) + b*(e^4 - 4*d*e^2*f +
 2*d^2*f^2))))*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2
+ 4*d*f] + 6*(A*f*(-(c*e) + b*f)*(f*(-(b*e) + 2*a*f) + c*(e^2 - 2
*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f + d^2*f^2) + f^2*(-2*a*b*e*f + a
^2*f^2 + b^2*(e^2 - d*f)) - 2*c*f*(a*f*(-e^2 + d*f) + b*(e^3 - 2*
d*e*f))))*Log[d + x*(e + f*x)])/(12*f^5)
```

Maple [B] time = 0.013, size = 1672, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x)
```

```
[Out] 6/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*a*c
*d*e+2/3/f*B*x^3*b*c+1/f*A*x^2*b*c+1/2/f*ln(f*x^2+e*x+d)*B*a^2+2/
(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*a^2-2/f^2
*B*b*c*d*x+1/2/f*B*x^2*b^2+1/f*A*b^2*x+1/3/f*A*x^3*c^2+1/f*B*x^2*
a*c-1/2/f^2*B*x^2*c^2*d+2/f*A*c*a*x-1/f^2*A*c^2*d*x+2/f*b*B*a*x-4
/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*a*c*d+
1/2/f^3*B*x^2*c^2*e^2-1/2/f^2*A*x^2*c^2*e-1/2/f^2*ln(f*x^2+e*x+d)
*B*b^2*d+1/2/f^3*ln(f*x^2+e*x+d)*B*b^2*e^2+1/2/f^3*ln(f*x^2+e*x+d
)*B*c^2*d^2+1/f*ln(f*x^2+e*x+d)*A*a*b-1/2/f^4*ln(f*x^2+e*x+d)*A*c
^2*e^3-1/2/f^2*ln(f*x^2+e*x+d)*A*b^2*e+1/2/f^5*ln(f*x^2+e*x+d)*B*
c^2*e^4+2/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2
))*A*c^2*d^2+1/f^4/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)
```

```
^(1/2))*e^4*A*c^2+1/f^3*ln(f*x^2+e*x+d)*B*a*c*e^2-1/f^4*ln(f*x^2+
e*x+d)*B*b*c*e^3-3/2/f^4*ln(f*x^2+e*x+d)*B*c^2*d*e^2-2/f/(4*d*f-e
^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*b^2*d-1/f/(4*d*f-
e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e*B*a^2-1/f^3/(4*d
*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^3*B*b^2-1/f^5
/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^5*B*c^2+
1/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^2*A
*b^2-1/f^2*ln(f*x^2+e*x+d)*B*a*b*e-1/f^2*ln(f*x^2+e*x+d)*B*a*c*d-
1/f^2*ln(f*x^2+e*x+d)*A*a*c*e-1/f^2*ln(f*x^2+e*x+d)*A*b*c*d+1/f^3
*ln(f*x^2+e*x+d)*A*b*c*e^2+2/f^3*b*c*B*e^2*x+2/f^3*B*c^2*d*e*x-1/
f^2*B*x^2*b*c*e-2/f^2*b*A*c*e*x-2/f^2*c*B*a*e*x+4/f^2/(4*d*f-e^2)
^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*b*c*d^2+2/f^4/(4*d*f
-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^4*B*b*c-5/f^3/(
4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*c^2*d^2*e-
4/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*c^2
*d*e^2-4/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*
B*a*b*d+3/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2
))*B*b^2*d*e+5/f^4/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)
^(1/2))*B*c^2*d*e^3-2/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f
-e^2)^(1/2))*e*A*a*b+2/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*
d*f-e^2)^(1/2))*e^2*A*a*c-2/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e
)/(4*d*f-e^2)^(1/2))*e^3*A*b*c+2/f^2/(4*d*f-e^2)^(1/2)*arctan((2*
f*x+e)/(4*d*f-e^2)^(1/2))*e^2*B*a*b-2/f^3/(4*d*f-e^2)^(1/2)*arcta
n((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^3*B*a*c+2/f^3*ln(f*x^2+e*x+d)*B*
b*c*d*e-8/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2
))*B*b*c*d*e^2+6/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^
2)^(1/2))*A*b*c*d*e+1/f^3*ln(f*x^2+e*x+d)*A*c^2*d*e+1/f^3*A*c^2*e
^2*x-1/f^2*b^2*B*e*x-1/f^4*B*c^2*e^3*x-1/3/f^2*B*x^3*c^2*e+1/4*B*
c^2*x^4/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)^2*(B*x + A)/(f*x^2 + e*x + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.425817, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)^2*(B*x + A)/(f*x^2 + e*x + d),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(B*c^2*e^5 - 2*A*a^2*f^5 + (2*(2*B*a*b + A*b^2 + 2*A*a*
c)*d + (B*a^2 + 2*A*a*b)*e)*f^4 - (2*(2*B*b*c + A*c^2)*d^2 + 3*(B
*b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2 + 2*A*a*c)*e^2)*f^
3 + (5*B*c^2*d^2*e + 4*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 + 2*(B*a
+ A*b)*c)*e^3)*f^2 - (5*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f)*l
og(-(e^3 - 4*d*e*f + 2*(e^2*f - 4*d*f^2)*x - (2*f^2*x^2 + 2*e*f*x
 + e^2 - 2*d*f)*sqrt(e^2 - 4*d*f))/(f*x^2 + e*x + d)) - (3*B*c^2*
f^4*x^4 - 4*(B*c^2*e*f^3 - (2*B*b*c + A*c^2)*f^4)*x^3 + 6*(B*c^2*
e^2*f^2 + (B*b^2 + 2*(B*a + A*b)*c)*f^4 - (B*c^2*d + (2*B*b*c + A
*c^2)*e)*f^3)*x^2 - 12*(B*c^2*e^3*f - (2*B*a*b + A*b^2 + 2*A*a*c)
*f^4 + ((2*B*b*c + A*c^2)*d + (B*b^2 + 2*(B*a + A*b)*c)*e)*f^3 -
(2*B*c^2*d*e + (2*B*b*c + A*c^2)*e^2)*f^2)*x + 6*(B*c^2*e^4 + (B*
a^2 + 2*A*a*b)*f^4 - ((B*b^2 + 2*(B*a + A*b)*c)*d + (2*B*a*b + A*
b^2 + 2*A*a*c)*e)*f^3 + (B*c^2*d^2 + 2*(2*B*b*c + A*c^2)*d*e + (B
*b^2 + 2*(B*a + A*b)*c)*e^2)*f^2 - (3*B*c^2*d*e^2 + (2*B*b*c + A*
```

```
c^2)*e^3)*f)*log(f*x^2 + e*x + d))*sqrt(e^2 - 4*d*f))/(sqrt(e^2 -
 4*d*f)*f^5), -1/12*(12*(B*c^2*e^5 - 2*A*a^2*f^5 + (2*(2*B*a*b +
A*b^2 + 2*A*a*c)*d + (B*a^2 + 2*A*a*b)*e)*f^4 - (2*(2*B*b*c + A*c
^2)*d^2 + 3*(B*b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2 + 2*
A*a*c)*e^2)*f^3 + (5*B*c^2*d^2*e + 4*(2*B*b*c + A*c^2)*d*e^2 + (B
*b^2 + 2*(B*a + A*b)*c)*e^3)*f^2 - (5*B*c^2*d*e^3 + (2*B*b*c + A*
c^2)*e^4)*f)*arctan(-sqrt(-e^2 + 4*d*f)*(2*f*x + e)/(e^2 - 4*d*f)
) - (3*B*c^2*f^4*x^4 - 4*(B*c^2*e*f^3 - (2*B*b*c + A*c^2)*f^4)*x^
3 + 6*(B*c^2*e^2*f^2 + (B*b^2 + 2*(B*a + A*b)*c)*f^4 - (B*c^2*d +
 (2*B*b*c + A*c^2)*e)*f^3)*x^2 - 12*(B*c^2*e^3*f - (2*B*a*b + A*b
^2 + 2*A*a*c)*f^4 + ((2*B*b*c + A*c^2)*d + (B*b^2 + 2*(B*a + A*b)
*c)*e)*f^3 - (2*B*c^2*d*e + (2*B*b*c + A*c^2)*e^2)*f^2)*x + 6*(B*
c^2*e^4 + (B*a^2 + 2*A*a*b)*f^4 - ((B*b^2 + 2*(B*a + A*b)*c)*d +
(2*B*a*b + A*b^2 + 2*A*a*c)*e)*f^3 + (B*c^2*d^2 + 2*(2*B*b*c + A*
c^2)*d*e + (B*b^2 + 2*(B*a + A*b)*c)*e^2)*f^2 - (3*B*c^2*d*e^2 +
(2*B*b*c + A*c^2)*e^3)*f)*log(f*x^2 + e*x + d))*sqrt(-e^2 + 4*d*f
))/(sqrt(-e^2 + 4*d*f)*f^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)*(c*x**2+b*x+a)**2/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.277663, size = 996, normalized size = 1.84

$$\frac{3\,Bc^2f^3x^4+8\,Bbcf^3x^3+4\,Ac^2f^3x^3-4\,Bc^2f^2x^3e-6\,Bc^2df^2x^2+6\,Bb^2f^3x^2+12\,Bacf^3x^2+12\,Abcf^3x^2-12\,Bbcf^2x^2e-\cdots}{\cdots}$$
$$+\frac{\left(Bc^2d^2f^2-Bb^2df^3-2\,Bacdf^3-2\,Abcdf^3+Ba^2f^4+2\,Aabf^4+4\,Bbcdf^2e+2\,Ac^2df^2e-2\,Babf^3e-Ab^2f^3e-2\,Aacf\cdots\right.}{2\,f^5}$$
$$+\frac{\left(4\,Bbcd^2f^3+2\,Ac^2d^2f^3-4\,Babdf^4-2\,Ab^2df^4-4\,Aacdf^4+2\,Aa^2f^5-5\,Bc^2d^2f^2e+3\,Bb^2df^3e+6\,Bacdf^3e+6\,Abcdf\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)^2*(B*x + A)/(f*x^2 + e*x + d),x, algorithm="giac")
```

```
[Out] 1/12*(3*B*c^2*f^3*x^4 + 8*B*b*c*f^3*x^3 + 4*A*c^2*f^3*x^3 - 4*B*c
^2*f^2*x^3*e - 6*B*c^2*d*f^2*x^2 + 6*B*b^2*f^3*x^2 + 12*B*a*c*f^3
*x^2 + 12*A*b*c*f^3*x^2 - 12*B*b*c*f^2*x^2*e - 6*A*c^2*f^2*x^2*e
- 24*B*b*c*d*f^2*x - 12*A*c^2*d*f^2*x + 24*B*a*b*f^3*x + 12*A*b^2
*f^3*x + 24*A*a*c*f^3*x + 6*B*c^2*f*x^2*e^2 + 24*B*c^2*d*f*x*e -
12*B*b^2*f^2*x*e - 24*B*a*c*f^2*x*e - 24*A*b*c*f^2*x*e + 24*B*b*c
*f*x*e^2 + 12*A*c^2*f*x*e^2 - 12*B*c^2*x*e^3)/f^4 + 1/2*(B*c^2*d^
2*f^2 - B*b^2*d*f^3 - 2*B*a*c*d*f^3 - 2*A*b*c*d*f^3 + B*a^2*f^4 +
 2*A*a*b*f^4 + 4*B*b*c*d*f^2*e + 2*A*c^2*d*f^2*e - 2*B*a*b*f^3*e
- A*b^2*f^3*e - 2*A*a*c*f^3*e - 3*B*c^2*d*f*e^2 + B*b^2*f^2*e^2 +
 2*B*a*c*f^2*e^2 + 2*A*b*c*f^2*e^2 - 2*B*b*c*f*e^3 - A*c^2*f*e^3
+ B*c^2*e^4)*ln(f*x^2 + x*e + d)/f^5 + (4*B*b*c*d^2*f^3 + 2*A*c^2
*d^2*f^3 - 4*B*a*b*d*f^4 - 2*A*b^2*d*f^4 - 4*A*a*c*d*f^4 + 2*A*a^
2*f^5 - 5*B*c^2*d^2*f^2*e + 3*B*b^2*d*f^3*e + 6*B*a*c*d*f^3*e + 6
*A*b*c*d*f^3*e - B*a^2*f^4*e - 2*A*a*b*f^4*e - 8*B*b*c*d*f^2*e^2
- 4*A*c^2*d*f^2*e^2 + 2*B*a*b*f^3*e^2 + A*b^2*f^3*e^2 + 2*A*a*c*f
^3*e^2 + 5*B*c^2*d*f*e^3 - B*b^2*f^2*e^3 - 2*B*a*c*f^2*e^3 - 2*A*
b*c*f^2*e^3 + 2*B*b*c*f*e^4 + A*c^2*f*e^4 - B*c^2*e^5)*arctan((2*
f*x + e)/sqrt(4*d*f - e^2))/(sqrt(4*d*f - e^2)*f^5)
```

3.15 $\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx$

Optimal. Leaf size=406

$$\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(B(aef-2bdf+cde)-A\left(2af^2-bef-2cdf+ce^2\right)\right)}{\sqrt{e^2-4df}\,\left(f\left(a^2f-abe+b^2d\right)-c\left(bde-a\left(e^2-2df\right)\right)+c^2d^2\right)}+\frac{\log\left(a+bx+cx^2\right)(-aBf+Abf-Ace+Bcd)}{2\left(f\left(a^2f-abe+b^2d\right)-c\left(bde-a\left(e^2-2df\right)\right)+c^2d^2\right)}-\frac{\log\left(d+ex+fx^2\right)(-aBf+Abf-Ace+Bcd)}{2\left(f\left(a^2f-abe+b^2d\right)-c\left(bde-a\left(e^2-2df\right)\right)+c^2d^2\right)}-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-b(aBf+Ace+Bcd)+2c(-aAf+aBe+Acd)+Ab^2f\right)}{\sqrt{b^2-4ac}\,\left(f\left(a^2f-abe+b^2d\right)-c\left(bde-a\left(e^2-2df\right)\right)+c^2d^2\right)}$$

```
[Out] -(((A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*
B*f))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*
(c^2*d^2 + f*(b^2*d - a*b*e + a^2*f) - c*(b*d*e - a*(e^2 - 2*d*f)
)))) + ((B*(c*d*e - 2*b*d*f + a*e*f) - A*(c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2))*ArcTanh[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(Sqrt[e^2 -
4*d*f]*(c^2*d^2 + f*(b^2*d - a*b*e + a^2*f) - c*(b*d*e - a*(e^2 -
 2*d*f)))) + ((B*c*d - A*c*e + A*b*f - a*B*f)*Log[a + b*x + c*x^2
])/(2*(c^2*d^2 + f*(b^2*d - a*b*e + a^2*f) - c*(b*d*e - a*(e^2 -
2*d*f)))) - ((B*c*d - A*c*e + A*b*f - a*B*f)*Log[d + e*x + f*x^2]
)/(2*(c^2*d^2 + f*(b^2*d - a*b*e + a^2*f) - c*(b*d*e - a*(e^2 - 2
*d*f))))
```

Rubi [A] time = 1.12677, antiderivative size = 398, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(B(aef-2bdf+cde)-A\left(2af^2-bef-2cdf+ce^2\right)\right)}{\sqrt{e^2-4df}\,\left(f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2\right)}+\frac{\log\left(a+bx+cx^2\right)(-aBf+Abf-Ace+Bcd)}{2\left(f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2\right)}-\frac{\log\left(d+ex+fx^2\right)(-aBf+Abf-Ace+Bcd)}{2\left(f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2\right)}-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-b(aBf+Ace+Bcd)+2c(-aAf+aBe+Acd)+Ab^2f\right)}{\sqrt{b^2-4ac}\,\left(f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x]
```

```
[Out] -(((A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*
B*f))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*
(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f
)))) + ((B*(c*d*e - 2*b*d*f + a*e*f) - A*(c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2))*ArcTanh[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(Sqrt[e^2 -
4*d*f]*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2
- 2*d*f))) + ((B*c*d - A*c*e + A*b*f - a*B*f)*Log[a + b*x + c*x^2
])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 -
 2*d*f))) - ((B*c*d - A*c*e + A*b*f - a*B*f)*Log[d + e*x + f*x^2]
)/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 -
2*d*f)))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.06166, size = 267, normalized size = 0.66

$$\frac{\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)(-b(aBf+Ace+Bcd)+2c(-aAf+aBe+Acd)+Ab^2f)}{\sqrt{4ac-b^2}}-\frac{2\tan^{-1}\left(\frac{e+2fx}{\sqrt{4df-e^2}}\right)\left(A\left(-2af^2+bef+2cdf-ce^2\right)+B(aef-2bdf+cde)\right)}{\sqrt{4df-e^2}}+\log(a+x}{2\left(f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+\right.}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*(A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a
*B*f))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
 - (2*(B*(c*d*e - 2*b*d*f + a*e*f) + A*(-(c*e^2) + 2*c*d*f + b*e*
f - 2*a*f^2))*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 +
 4*d*f] + (B*c*d - A*c*e + A*b*f - a*B*f)*Log[a + x*(b + c*x)] +
(-(B*c*d) + A*c*e - A*b*f + a*B*f)*Log[d + x*(e + f*x)])/(2*(c^2*
d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f)))
```

Maple [B] time = 0.014, size = 1698, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x)
```

```
[Out] -1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*
f*ln(f*x^2+e*x+d)*A*b+1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*
d*f-b*c*d*e+c^2*d^2)*ln(f*x^2+e*x+d)*A*c*e+1/2/(a^2*f^2-a*b*e*f-2
*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*f*ln(f*x^2+e*x+d)*B*a-1
/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*ln
(f*x^2+e*x+d)*B*c*d+2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-
b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(
1/2))*A*a*f^2-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*
e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*
A*b*e*f-2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*
d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*c*d*
f+1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(
4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*c*e^2-1/(a
^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-
e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*a*e*f+2/(a^2*f^2
-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(
1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*b*d*f-1/(a^2*f^2-a*b*e
*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*a
rctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*c*d*e+1/2/(a^2*f^2-a*b*e*f-2
*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*ln(c*x^2+b*x+a)*A*b*f-1
/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*c*
ln(c*x^2+b*x+a)*A*e-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*
f-b*c*d*e+c^2*d^2)*ln(c*x^2+b*x+a)*B*a*f+1/2/(a^2*f^2-a*b*e*f-2*a
```

```
*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*c*ln(c*x^2+b*x+a)*B*d-2/(
a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c
-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*a*c*f+1/(a^2*f^
2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^
(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*b^2*f-1/(a^2*f^2-a*b*
e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*
arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*b*c*e+2/(a^2*f^2-a*b*e*f-2*
a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*c^2*d-1/(a^2*f^2-a*b*e*f-2*a*c*d*
f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))*B*a*b*f+2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*
e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(
4*a*c-b^2)^(1/2))*B*a*c*e-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^
2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-
b^2)^(1/2))*B*b*c*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + b*x + a)*(f*x^2 + e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + b*x + a)*(f*x^2 + e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx + A}{(cx^2 + bx + a)(fx^2 + ex + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + b*x + a)*(f*x^2 + e*x + d)),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)/((c*x^2 + b*x + a)*(f*x^2 + e*x + d)), x)
```

3.16 $\int \frac{A+Bx}{(a+bx+cx^2)^2(d+ex+fx^2)}\,dx$

Optimal. Leaf size=1075

result too large to display

```
[Out] -((A*c*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f -
 c*(b*e + 2*a*f)) + c*(A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*
(B*c*d + A*c*e + a*B*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f))*(a + b*x + c*x^2))) - ((b^5*(B*d - A*e)*f^2 -
 2*b^4*f*(B*c*d*e - A*(c*e^2 - c*d*f + a*f^2)) - 4*c^2*(A*(c^3*d^
3 - 3*a^3*f^3 - a^2*c*f*(e^2 - 7*d*f) + a*c^2*d*(3*e^2 - 5*d*f))
- a*B*e*(c^2*d^2 - 3*a^2*f^2 - a*c*(e^2 - 2*d*f))) - 4*b^2*c*(B*c
^2*d^2*e + A*f*(2*c^2*d^2 + 3*a^2*f^2 + 3*a*c*(e^2 - d*f))) + 2*b
*c*(B*(c^3*d^3 + 3*a^3*f^3 + a*c^2*d*(e^2 - 7*d*f) + 3*a^2*c*f*(e
^2 + d*f)) + A*c*e*(3*c^2*d^2 + 3*a^2*f^2 + a*c*(3*e^2 + 2*d*f)))
 - b^3*(A*c*e*(c*e^2 - 2*c*d*f - 4*a*f^2) + B*(4*a*c*d*f^2 + a^2*
f^3 - c^2*d*(e^2 + 5*d*f))))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/((b^2 - 4*a*c)^(3/2)*(c^2*d^2 + f*(b^2*d - a*b*e + a^2*f) - c
*(b*d*e - a*(e^2 - 2*d*f)))^2) + ((B*(c^2*d*e*(e^2 - 3*d*f) - 2*c
*d*f*(b*e^2 - 2*b*d*f - a*e*f) + f^2*(b^2*d*e - 4*a*b*d*f + a^2*e
*f)) - A*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) - f^2*(2*a*b*e*f - 2*
a^2*f^2 - b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3
- 3*d*e*f))))*ArcTanh[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(Sqrt[e^2 -
 4*d*f]*(c^2*d^2 + f*(b^2*d - a*b*e + a^2*f) - c*(b*d*e - a*(e^2
- 2*d*f)))^2) + ((A*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f
)) - B*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*d*(e^2 -
d*f)))*Log[a + b*x + c*x^2])/(2*(c^2*d^2 + f*(b^2*d - a*b*e + a^2
*f) - c*(b*d*e - a*(e^2 - 2*d*f)))^2) - ((A*(c*e - b*f)*(f*(b*e -
 2*a*f) - c*(e^2 - 2*d*f)) - B*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d
- a^2*f) - c^2*d*(e^2 - d*f)))*Log[d + e*x + f*x^2])/(2*(c^2*d^2
+ f*(b^2*d - a*b*e + a^2*f) - c*(b*d*e - a*(e^2 - 2*d*f)))^2)
```

Rubi [A] time = 11.0869, antiderivative size = 1067, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aBe-aAf)\right)\ldots}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)(cx^2+bx+a)}$$

$$-\frac{\left((Bd-Ae)f^2b^5-2f\left(-aAf^2+Bcde-Ac\left(e^2-df\right)\right)b^4-\left(Ace\left(ce^2-4af^2-2cdf\right)+B\left(a^2f^3+4acdf^2-c^2d\left(e^2+5d\ldots\right.\right.\right.\right.}{\ldots}$$

$$+\frac{\left(B\left(de\left(e^2-3df\right)c^2-2df\left(be^2-afe-2bdf\right)c+f^2\left(efa^2-4bdfa+b^2de\right)\right)-A\left(\left(e^4-4dfe^2+2d^2f^2\right)c^2+2f\left(af\left(\ldots\right.\right.\right.\right.}{\sqrt{e^2-4df}\left(c^2d^2-bced+f\left(fa^2-bea+b^2d\right)\ldots\right.}$$

$$+\frac{\left(A(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-B\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)\right)\log\left(cx^2+bx+a\right)}{2\left(c^2d^2-bced+f\left(fa^2-bea+b^2d\right)+ac\left(e^2-2df\right)\right)^2}$$

$$-\frac{\left(A(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-B\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)\right)\log\left(fx^2+ex+d\right)}{2\left(c^2d^2-bced+f\left(fa^2-bea+b^2d\right)+ac\left(e^2-2df\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x]
```

```
[Out] -((A*c*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f -
 c*(b*e + 2*a*f)) + c*(A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*
(B*c*d + A*c*e + a*B*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f))*(a + b*x + c*x^2))) - ((b^5*(B*d - A*e)*f^2 -
 2*b^4*f*(B*c*d*e - a*A*f^2 - A*c*(e^2 - d*f)) - 4*c^2*(A*(c^3*d^
3 - 3*a^3*f^3 - a^2*c*f*(e^2 - 7*d*f) + a*c^2*d*(3*e^2 - 5*d*f))
- a*B*e*(c^2*d^2 - 3*a^2*f^2 - a*c*(e^2 - 2*d*f))) - 4*b^2*(B*c^3
*d^2*e + A*c*f*(2*c^2*d^2 + 3*a^2*f^2 + 3*a*c*(e^2 - d*f))) + 2*b
*c*(B*(c^3*d^3 + 3*a^3*f^3 + a*c^2*d*(e^2 - 7*d*f) + 3*a^2*c*f*(e
^2 + d*f)) + A*c*e*(3*c^2*d^2 + 3*a^2*f^2 + a*c*(3*e^2 + 2*d*f)))
```

```
 - b^3*(A*c*e*(c*e^2 - 2*c*d*f - 4*a*f^2) + B*(4*a*c*d*f^2 + a^2*
f^3 - c^2*d*(e^2 + 5*d*f))))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/((b^2 - 4*a*c)^(3/2)*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e +
a^2*f) + a*c*(e^2 - 2*d*f))^2) + ((B*(c^2*d*e*(e^2 - 3*d*f) - 2*c
*d*f*(b*e^2 - 2*b*d*f - a*e*f) + f^2*(b^2*d*e - 4*a*b*d*f + a^2*e
*f)) - A*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) - f^2*(2*a*b*e*f - 2*
a^2*f^2 - b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3
- 3*d*e*f))))*ArcTanh[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(Sqrt[e^2 -
 4*d*f]*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2
 - 2*d*f))^2) + ((A*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f
)) - B*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*d*(e^2 -
d*f)))*Log[a + b*x + c*x^2])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a
*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2) - ((A*(c*e - b*f)*(f*(b*e -
 2*a*f) - c*(e^2 - 2*d*f)) - B*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d
- a^2*f) - c^2*d*(e^2 - d*f)))*Log[d + e*x + f*x^2])/(2*(c^2*d^2
- b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/(c*x**2+b*x+a)**2/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 8.88929, size = 1376, normalized size = 1.28

$$\frac{-Afb^3+Aceb^2+aBfb^2-Acfxb^2-Ac^2db-aBceb+3aAcfb+Bc^2dxb+Ac^2exb+aBcfxb+2aBc^2d-2aAc^2e-2a^2Bc\ldots}{(b^2-4ac)(dfb^2-cdeb-aefb+c^2d^2+ace^2+a^2f^2-2acdf)(cx^2+bx+a)}$$
$$+\frac{\left(Bdf^2b^5-Aef^2b^5+2aAf^3b^4-2Acdf^2b^4+2Ace^2fb^4-2Bcdefb^4-Ac^2e^3b^3-a^2Bf^3b^3+Bc^2de^2b^3-4aBcdf^2b^3+4aA\ldots\right.}{\ldots}$$
$$+\frac{\left(Ac^2e^4-Bc^2de^3-2Abcfe^3+Ab^2f^2e^2+2aAcf^2e^2-4Ac^2dfe^2+2bBcdfe^2-2aAbf^3e-a^2Bf^3e-b^2Bdf^2e+6Abcdf^2\ldots\right.}{\sqrt{4df-e^2}\,(dfb^2-cdeb-aefb+c\ldots}$$
$$+\frac{\left(-Ac^2e^3+Bc^2de^2+2Abcfe^2-Ab^2f^2e-2aAcf^2e+2Ac^2dfe-2bBcdfe+2aAbf^3-a^2Bf^3+b^2Bdf^2-2Abcdf^2+2aB\ldots\right.}{2\left(dfb^2-cdeb-aefb+c^2d^2+ace^2+a^2f^2-2acdf\right)^2}$$
$$+\frac{\left(Ac^2e^3-Bc^2de^2-2Abcfe^2+Ab^2f^2e+2aAcf^2e-2Ac^2dfe+2bBcdfe-2aAbf^3+a^2Bf^3-b^2Bdf^2+2Abcdf^2-2aBc\ldots\right.}{2\left(dfb^2-cdeb-aefb+c^2d^2+ace^2+a^2f^2-2acdf\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x]
```

```
[Out] (-(A*b*c^2*d) + 2*a*B*c^2*d + A*b^2*c*e - a*b*B*c*e - 2*a*A*c^2*e
 - A*b^3*f + a*b^2*B*f + 3*a*A*b*c*f - 2*a^2*B*c*f + b*B*c^2*d*x
- 2*A*c^3*d*x + A*b*c^2*e*x - 2*a*B*c^2*e*x - A*b^2*c*f*x + a*b*B
*c*f*x + 2*a*A*c^2*f*x)/((b^2 - 4*a*c)*(c^2*d^2 - b*c*d*e + a*c*e
^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)*(a + b*x + c*x^2))
+ ((2*b*B*c^4*d^3 - 4*A*c^5*d^3 - 4*b^2*B*c^3*d^2*e + 6*A*b*c^4*d
^2*e + 4*a*B*c^4*d^2*e + b^3*B*c^2*d*e^2 + 2*a*b*B*c^3*d*e^2 - 12
*a*A*c^4*d*e^2 - A*b^3*c^2*e^3 + 6*a*A*b*c^3*e^3 - 4*a^2*B*c^3*e^
3 + 5*b^3*B*c^2*d^2*f - 8*A*b^2*c^3*d^2*f - 14*a*b*B*c^3*d^2*f +
20*a*A*c^4*d^2*f - 2*b^4*B*c*d*e*f + 2*A*b^3*c^2*d*e*f + 4*a*A*b*
c^3*d*e*f + 8*a^2*B*c^3*d*e*f + 2*A*b^4*c*e^2*f - 12*a*A*b^2*c^2*
e^2*f + 6*a^2*b*B*c^2*e^2*f + 4*a^2*A*c^3*e^2*f + b^5*B*d*f^2 - 2
*A*b^4*c*d*f^2 - 4*a*b^3*B*c*d*f^2 + 12*a*A*b^2*c^2*d*f^2 + 6*a^2
*b*B*c^2*d*f^2 - 28*a^2*A*c^3*d*f^2 - A*b^5*e*f^2 + 4*a*A*b^3*c*e
```

```
*f^2 + 6*a^2*A*b*c^2*e*f^2 - 12*a^3*B*c^2*e*f^2 + 2*a*A*b^4*f^3 -
 a^2*b^3*B*f^3 - 12*a^2*A*b^2*c*f^3 + 6*a^3*b*B*c*f^3 + 12*a^3*A*
c^2*f^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/((b^2 - 4*a*c)*S
qrt[-b^2 + 4*a*c]*(c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*
d*f - a*b*e*f + a^2*f^2)^2) + ((-(B*c^2*d*e^3) + A*c^2*e^4 + 3*B*
c^2*d^2*e*f + 2*b*B*c*d*e^2*f - 4*A*c^2*d*e^2*f - 2*A*b*c*e^3*f -
 4*b*B*c*d^2*f^2 + 2*A*c^2*d^2*f^2 - b^2*B*d*e*f^2 + 6*A*b*c*d*e*
f^2 - 2*a*B*c*d*e*f^2 + A*b^2*e^2*f^2 + 2*a*A*c*e^2*f^2 - 2*A*b^2
*d*f^3 + 4*a*b*B*d*f^3 - 4*a*A*c*d*f^3 - 2*a*A*b*e*f^3 - a^2*B*e*
f^3 + 2*a^2*A*f^4)*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]])/(Sqrt[
-e^2 + 4*d*f]*(c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f
- a*b*e*f + a^2*f^2)^2) + ((B*c^2*d*e^2 - A*c^2*e^3 - B*c^2*d^2*f
 - 2*b*B*c*d*e*f + 2*A*c^2*d*e*f + 2*A*b*c*e^2*f + b^2*B*d*f^2 -
2*A*b*c*d*f^2 + 2*a*B*c*d*f^2 - A*b^2*e*f^2 - 2*a*A*c*e*f^2 + 2*a
*A*b*f^3 - a^2*B*f^3)*Log[a + b*x + c*x^2])/(2*(c^2*d^2 - b*c*d*e
 + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)^2) + ((-(B*
c^2*d*e^2) + A*c^2*e^3 + B*c^2*d^2*f + 2*b*B*c*d*e*f - 2*A*c^2*d*
e*f - 2*A*b*c*e^2*f - b^2*B*d*f^2 + 2*A*b*c*d*f^2 - 2*a*B*c*d*f^2
 + A*b^2*e*f^2 + 2*a*A*c*e*f^2 - 2*a*A*b*f^3 + a^2*B*f^3)*Log[d +
 e*x + f*x^2])/(2*(c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*
d*f - a*b*e*f + a^2*f^2)^2)
```

Maple [B] time = 0.054, size = 54204, normalized size = 50.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + b*x + a)^2*(f*x^2 + e*x + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + b*x + a)^2*(f*x^2 + e*x + d)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/(c*x**2+b*x+a)**2/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.38843, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + b*x + a)^2*(f*x^2 + e*x + d)),x, algorithm="giac")
```

```
[Out] Done
```

3.17 $\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}\,dx$

Optimal. Leaf size=140

$$\frac{3(b+2cx)(2cg-bh)}{2d^2(b^2-4ac)^2(a+bx+cx^2)}-\frac{-2ah+x(2cg-bh)+bg}{2d^2(b^2-4ac)(a+bx+cx^2)^2}-\frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^2(b^2-4ac)^{5/2}}$$

```
[Out] -(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*(b^2 - 4*a*c)*d^2*(a + b*x +
c*x^2)^2) + (3*(2*c*g - b*h)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*d^2*
(a + b*x + c*x^2)) - (6*c*(2*c*g - b*h)*ArcTanh[(b + 2*c*x)/Sqrt[
b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*d^2)
```

Rubi [A] time = 0.307764, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$

$$\frac{3(b+2cx)(2cg-bh)}{2d^2(b^2-4ac)^2(a+bx+cx^2)}-\frac{-2ah+x(2cg-bh)+bg}{2d^2(b^2-4ac)(a+bx+cx^2)^2}-\frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^2(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)/((a + b*x + c*x^2)*(a*d + b*d*x + c*d*x^2)^2),x]
```

```
[Out] -(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*(b^2 - 4*a*c)*d^2*(a + b*x +
c*x^2)^2) + (3*(2*c*g - b*h)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*d^2*
(a + b*x + c*x^2)) - (6*c*(2*c*g - b*h)*ArcTanh[(b + 2*c*x)/Sqrt[
b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*d^2)
```

Rubi in Sympy [A] time = 49.7295, size = 131, normalized size = 0.94

$$\frac{6c\,(bh-2cg)\,\text{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{d^2\,(-4ac+b^2)^{\frac{5}{2}}}-\frac{3\,(b+2cx)\left(\frac{bh}{2}-cg\right)}{d^2\,(-4ac+b^2)^2\,(a+bx+cx^2)}+\frac{2ah-bg+x\,(bh-2cg)}{2d^2\,(-4ac+b^2)\,(a+bx+cx^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x+g)/(c*x**2+b*x+a)/(c*d*x**2+b*d*x+a*d)**2,x)
```

```
[Out] 6*c*(b*h - 2*c*g)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(d**2*(-
4*a*c + b**2)**(5/2)) - 3*(b + 2*c*x)*(b*h/2 - c*g)/(d**2*(-4*a*c
 + b**2)**2*(a + b*x + c*x**2)) + (2*a*h - b*g + x*(b*h - 2*c*g))
/(2*d**2*(-4*a*c + b**2)*(a + b*x + c*x**2)**2)
```

Mathematica [A] time = 0.263056, size = 131, normalized size = 0.94

$$\frac{\frac{(b^2-4ac)(2ah-bg+bhx-2cgx)}{(a+x(b+cx))^2}-\frac{12c(bh-2cg)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{3(b+2cx)(2cg-bh)}{a+x(b+cx)}}{2d^2(b^2-4ac)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)/((a + b*x + c*x^2)*(a*d + b*d*x + c*d*x^2)^2),x]
```

```
[Out] (((b^2 - 4*a*c)*(-(b*g) + 2*a*h - 2*c*g*x + b*h*x))/(a + x*(b + c
*x))^2 + (3*(2*c*g - b*h)*(b + 2*c*x))/(a + x*(b + c*x)) - (12*c*
(-2*c*g + b*h)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2
+ 4*a*c])/(2*(b^2 - 4*a*c)^2*d^2)
```

Maple [B] time = 0.007, size = 340, normalized size = 2.4

$$
\begin{aligned}
&-\frac{bxh}{2\,d^2\,(4\,ac-b^2)\,(cx^2+bx+a)^2}+\frac{cxg}{d^2\,(4\,ac-b^2)\,(cx^2+bx+a)^2}-\frac{ah}{d^2\,(4\,ac-b^2)\,(cx^2+bx+a)^2}\\
&+\frac{bg}{2\,d^2\,(4\,ac-b^2)\,(cx^2+bx+a)^2}-3\,\frac{cxbh}{d^2\,(4\,ac-b^2)^2\,(cx^2+bx+a)}+6\,\frac{c^2xg}{d^2\,(4\,ac-b^2)^2\,(cx^2+bx+a)}\\
&-\frac{3\,b^2h}{2\,d^2\,(4\,ac-b^2)^2\,(cx^2+bx+a)}+3\,\frac{bcg}{d^2\,(4\,ac-b^2)^2\,(cx^2+bx+a)}\\
&-6\,\frac{bch}{d^2\,(4\,ac-b^2)^{5/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)+12\,\frac{c^2g}{d^2\,(4\,ac-b^2)^{5/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x)
```

```
[Out] -1/2/d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^2*x*b*h+1/d^2/(4*a*c-b^2)/(c*x
^2+b*x+a)^2*x*c*g-1/d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^2*a*h+1/2/d^2/(
4*a*c-b^2)/(c*x^2+b*x+a)^2*b*g-3/d^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)*
c*x*b*h+6/d^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)*c^2*x*g-3/2/d^2/(4*a*c-
b^2)^2/(c*x^2+b*x+a)*b^2*h+3/d^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b*c*
g-6/d^2/(4*a*c-b^2)^(5/2)*c*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b
*h+12/d^2/(4*a*c-b^2)^(5/2)*c^2*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))*g
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x + g)/((c*d*x^2 + b*d*x + a*d)^2*(c*x^2 + b*x + a)),x, algorithm="ma
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.305622, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x + g)/((c*d*x^2 + b*d*x + a*d)^2*(c*x^2 + b*x + a)),x, algorithm="fr
```

```
[Out] [-1/2*(6*(2*a^2*c^2*g - a^2*b*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(
2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^2 + 2*a*c^3)*g - (b^3*c +
2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x)*log((b^3 - 4*a
*b*c + 2*(b^2*c - 4*a*c^2)*x + (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c
)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) - (6*(2*c^3*g - b*c^2*h)*
x^3 + 9*(2*b*c^2*g - b^2*c*h)*x^2 - (b^3 - 10*a*b*c)*g - (a*b^2 +
 8*a^2*c)*h + 2*(2*(b^2*c + 5*a*c^2)*g - (b^3 + 5*a*b*c)*h)*x)*sq
rt(b^2 - 4*a*c))/(((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2*x^4 +
 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2*x^3 + (b^6 - 6*a*b^4*
```

```
c + 32*a^3*c^3)*d^2*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*
d^2*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*d^2)*sqrt(b^2 - 4*a*
c)), 1/2*(12*(2*a^2*c^2*g - a^2*b*c*h + (2*c^4*g - b*c^3*h)*x^4 +
 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^2 + 2*a*c^3)*g - (b^3*
c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x)*arctan(-sq
rt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (6*(2*c^3*g - b*c^2
*h)*x^3 + 9*(2*b*c^2*g - b^2*c*h)*x^2 - (b^3 - 10*a*b*c)*g - (a*b
^2 + 8*a^2*c)*h + 2*(2*(b^2*c + 5*a*c^2)*g - (b^3 + 5*a*b*c)*h)*x
)*sqrt(-b^2 + 4*a*c))/(((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2*
x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2*x^3 + (b^6 - 6*a
*b^4*c + 32*a^3*c^3)*d^2*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*
c^2)*d^2*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*d^2)*sqrt(-b^2
+ 4*a*c))]
```

Sympy [A] time = 4.81789, size = 709, normalized size = 5.06

$$\frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)\log\left(x+\frac{-192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)+144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)-36ab^4c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)+3b^6c\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdots}{6bc^2h-12c^3g}\right)}{d^2}-\frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)\log\left(x+\frac{192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)-144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)+36ab^4c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)-3b^6c\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdots}{6bc^2h-12c^3g}\right)}{d^2}-\frac{8a^2ch+ab^2h-10abcg+b^3g+x^3\left(6bc^2h-12c^3g\right)+x^2\left(9b^2ch-18bc^2g\right)+x\left(10abc\cdots\right.}{32a^4c^2d^2-16a^3b^2cd^2+2a^2b^4d^2+x^4\left(32a^2c^4d^2-16ab^2c^3d^2+2b^4c^2d^2\right)+x^3\left(64a^2bc^3d^2-32ab^3c^2d^2+4b^5cd^2\right)+x^2\left(64a^3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x+g)/(c*x**2+b*x+a)/(c*d*x**2+b*d*x+a*d)**2,x)
```

```
[Out] 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g)*log(x + (-192*a**3*c
**4*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 144*a**2*b**2*c**3
*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) - 36*a*b**4*c**2*sqrt(-
1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 3*b**6*c*sqrt(-1/(4*a*c - b*
*2)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b*c**2*g)/(6*b*c**2*h - 12
*c**3*g))/d**2 - 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g)*log
(x + (192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) - 14
4*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 36*a*
b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) - 3*b**6*c*sqr
t(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b*c**2*g)/
(6*b*c**2*h - 12*c**3*g))/d**2 - (8*a**2*c*h + a*b**2*h - 10*a*b*
c*g + b**3*g + x**3*(6*b*c**2*h - 12*c**3*g) + x**2*(9*b**2*c*h -
 18*b*c**2*g) + x*(10*a*b*c*h - 20*a*c**2*g + 2*b**3*h - 4*b**2*c
*g))/(32*a**4*c**2*d**2 - 16*a**3*b**2*c*d**2 + 2*a**2*b**4*d**2
+ x**4*(32*a**2*c**4*d**2 - 16*a*b**2*c**3*d**2 + 2*b**4*c**2*d**
2) + x**3*(64*a**2*b*c**3*d**2 - 32*a*b**3*c**2*d**2 + 4*b**5*c*d
**2) + x**2*(64*a**3*c**3*d**2 - 12*a*b**4*c*d**2 + 2*b**6*d**2)
+ x*(64*a**3*b*c**2*d**2 - 32*a**2*b**3*c*d**2 + 4*a*b**5*d**2))
```

GIAC/XCAS [A] time = 0.277088, size = 296, normalized size = 2.11

$$\frac{6\left(2c^2g-bch\right)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\left(b^4d^2-8ab^2cd^2+16a^2c^2d^2\right)\sqrt{-b^2+4ac}}+\frac{12c^3gx^3-6bc^2hx^3+18bc^2gx^2-9b^2chx^2+4b^2cgx+20ac^2gx-2b^3hx-10abchx-b^3g+10abcg-ab^2h-8a^2ch}{2\left(b^4d^2-8ab^2cd^2+16a^2c^2d^2\right)\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x + g)/((c*d*x^2 + b*d*x + a*d)^2*(c*x^2 + b*x + a)),x, algorithm="gi
```

```
[Out] 6*(2*c^2*g - b*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4*
d^2 - 8*a*b^2*c*d^2 + 16*a^2*c^2*d^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(
12*c^3*g*x^3 - 6*b*c^2*h*x^3 + 18*b*c^2*g*x^2 - 9*b^2*c*h*x^2 + 4
*b^2*c*g*x + 20*a*c^2*g*x - 2*b^3*h*x - 10*a*b*c*h*x - b^3*g + 10
*a*b*c*g - a*b^2*h - 8*a^2*c*h)/((b^4*d^2 - 8*a*b^2*c*d^2 + 16*a^
2*c^2*d^2)*(c*x^2 + b*x + a)^2)
```

3.18 $\int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)}\,dx$

Optimal. Leaf size=140

$$\frac{3(b+2cx)(2cg-bh)}{2d\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{-2ah+x(2cg-bh)+bg}{2d\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d\left(b^2-4ac\right)^{5/2}}$$

```
[Out] -(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*(b^2 - 4*a*c)*d*(a + b*x + c*
x^2)^2) + (3*(2*c*g - b*h)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*d*(a +
 b*x + c*x^2)) - (6*c*(2*c*g - b*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^2
- 4*a*c]])/((b^2 - 4*a*c)^(5/2)*d)
```

Rubi [A] time = 0.241466, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$

$$\frac{3(b+2cx)(2cg-bh)}{2d\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{-2ah+x(2cg-bh)+bg}{2d\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(g + h*x)/((a + b*x + c*x^2)^2*(a*d + b*d*x + c*d*x^2)),x]
```

```
[Out] -(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*(b^2 - 4*a*c)*d*(a + b*x + c*
x^2)^2) + (3*(2*c*g - b*h)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*d*(a +
 b*x + c*x^2)) - (6*c*(2*c*g - b*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^2
- 4*a*c]])/((b^2 - 4*a*c)^(5/2)*d)
```

Rubi in Sympy [A] time = 50.043, size = 126, normalized size = 0.9

$$\frac{6c\left(bh-2cg\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{d\left(-4ac+b^2\right)^{\frac{5}{2}}}-\frac{3\left(b+2cx\right)\left(\frac{bh}{2}-cg\right)}{d\left(-4ac+b^2\right)^2\left(a+bx+cx^2\right)}+\frac{2ah-bg+x\left(bh-2cg\right)}{2d\left(-4ac+b^2\right)\left(a+bx+cx^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((h*x+g)/(c*x**2+b*x+a)**2/(c*d*x**2+b*d*x+a*d),x)
```

```
[Out] 6*c*(b*h - 2*c*g)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(d*(-4*a
*c + b**2)**(5/2)) - 3*(b + 2*c*x)*(b*h/2 - c*g)/(d*(-4*a*c + b**
2)**2*(a + b*x + c*x**2)) + (2*a*h - b*g + x*(b*h - 2*c*g))/(2*d*
(-4*a*c + b**2)*(a + b*x + c*x**2)**2)
```

Mathematica [A] time = 0.0491449, size = 131, normalized size = 0.94

$$\frac{\frac{\left(b^2-4ac\right)(2ah-bg+bhx-2cgx)}{(a+x(b+cx))^2}-\frac{12c(bh-2cg)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{3(b+2cx)(2cg-bh)}{a+x(b+cx)}}{2d\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(g + h*x)/((a + b*x + c*x^2)^2*(a*d + b*d*x + c*d*x^2)),x]
```

```
[Out] (((b^2 - 4*a*c)*(-(b*g) + 2*a*h - 2*c*g*x + b*h*x))/(a + x*(b + c
*x))^2 + (3*(2*c*g - b*h)*(b + 2*c*x))/(a + x*(b + c*x)) - (12*c*
(-2*c*g + b*h)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2
+ 4*a*c])/(2*(b^2 - 4*a*c)^2*d)
```

Maple [B] time = 0.006, size = 340, normalized size = 2.4

$$-\frac{bxh}{2\,d\,(4\,ac-b^2)\,(cx^2+bx+a)^2}+\frac{cxg}{d\,(4\,ac-b^2)\,(cx^2+bx+a)^2}-\frac{ah}{d\,(4\,ac-b^2)\,(cx^2+bx+a)^2}+\frac{bg}{2\,d\,(4\,ac-b^2)\,(cx^2+bx+a)^2}-3\,\frac{cxbh}{d\,(4\,ac-b^2)^2\,(cx^2+bx+a)}+6\,\frac{c^2xg}{d\,(4\,ac-b^2)^2\,(cx^2+bx+a)}-\frac{3\,b^2h}{2\,d\,(4\,ac-b^2)^2\,(cx^2+bx+a)}+3\,\frac{bcg}{d\,(4\,ac-b^2)^2\,(cx^2+bx+a)}-6\,\frac{bch}{d\,(4\,ac-b^2)^{5/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)+12\,\frac{c^2g}{d\,(4\,ac-b^2)^{5/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x)
```

```
[Out] -1/2/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*x*b*h+1/d/(4*a*c-b^2)/(c*x^2+b
*x+a)^2*x*c*g-1/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*a*h+1/2/d/(4*a*c-b^
2)/(c*x^2+b*x+a)^2*b*g-3/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*c*x*b*h+6/
d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*c^2*x*g-3/2/d/(4*a*c-b^2)^2/(c*x^2+
b*x+a)*b^2*h+3/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b*c*g-6/d/(4*a*c-b^2
)^(5/2)*c*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*h+12/d/(4*a*c-b^2
)^(5/2)*c^2*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*g
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x + g)/((c*d*x^2 + b*d*x + a*d)*(c*x^2 + b*x + a)^2),x, algorithm="ma
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.301746, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x + g)/((c*d*x^2 + b*d*x + a*d)*(c*x^2 + b*x + a)^2),x, algorithm="fr
```

```
[Out] [-1/2*(6*(2*a^2*c^2*g - a^2*b*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(
2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^2 + 2*a*c^3)*g - (b^3*c +
2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x)*log((b^3 - 4*a
*b*c + 2*(b^2*c - 4*a*c^2)*x + (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c
)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) - (6*(2*c^3*g - b*c^2*h)*
x^3 + 9*(2*b*c^2*g - b^2*c*h)*x^2 - (b^3 - 10*a*b*c)*g - (a*b^2 +
 8*a^2*c)*h + 2*(2*(b^2*c + 5*a*c^2)*g - (b^3 + 5*a*b*c)*h)*x)*sq
rt(b^2 - 4*a*c))/(((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*x^4 + 2
*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*x^3 + (b^6 - 6*a*b^4*c +
32*a^3*c^3)*d*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d*x +
```

```
(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*d)*sqrt(b^2 - 4*a*c)), 1/2*(
12*(2*a^2*c^2*g - a^2*b*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^
3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^2 + 2*a*c^3)*g - (b^3*c + 2*a*b*
c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x)*arctan(-sqrt(-b^2 +
4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (6*(2*c^3*g - b*c^2*h)*x^3 +
9*(2*b*c^2*g - b^2*c*h)*x^2 - (b^3 - 10*a*b*c)*g - (a*b^2 + 8*a^2
*c)*h + 2*(2*(b^2*c + 5*a*c^2)*g - (b^3 + 5*a*b*c)*h)*x)*sqrt(-b^
2 + 4*a*c))/(((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*x^4 + 2*(b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*x^3 + (b^6 - 6*a*b^4*c + 32*a^
3*c^3)*d*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d*x + (a^2*
b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*d)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 4.75154, size = 680, normalized size = 4.86

$$\frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(bh-2cg)\log\left(x+\frac{-192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)+144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)-36ab^4c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)+3b^6c\sqrt{-\cdots}}{6bc^2h-12c^3g}\right.}{d}$$
$$-\frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(bh-2cg)\log\left(x+\frac{192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)-144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)+36ab^4c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)-3b^6c\sqrt{-\cdots}}{6bc^2h-12c^3g}\right.}{d}$$
$$-\frac{8a^2ch+ab^2h-10abcg+b^3g+x^3\left(6bc^2h-12c^3g\right)+x^2\left(9b^2ch-18bc^2g\right)+x\left(10abch-20a\cdots\right.}{32a^4c^2d-16a^3b^2cd+2a^2b^4d+x^4\left(32a^2c^4d-16ab^2c^3d+2b^4c^2d\right)+x^3\left(64a^2bc^3d-32ab^3c^2d+4b^5cd\right)+x^2\left(64a^3c^3d-12\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x+g)/(c*x**2+b*x+a)**2/(c*d*x**2+b*d*x+a*d),x)
```

```
[Out] 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g)*log(x + (-192*a**3*c
**4*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 144*a**2*b**2*c**3
*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) - 36*a*b**4*c**2*sqrt(-
1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 3*b**6*c*sqrt(-1/(4*a*c - b*
*2)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b*c**2*g)/(6*b*c**2*h - 12
*c**3*g))/d - 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g)*log(x
+ (192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) - 144*a
**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 36*a*b**
4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) - 3*b**6*c*sqrt(-
1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b*c**2*g)/(6*
b*c**2*h - 12*c**3*g))/d - (8*a**2*c*h + a*b**2*h - 10*a*b*c*g +
b**3*g + x**3*(6*b*c**2*h - 12*c**3*g) + x**2*(9*b**2*c*h - 18*b*
c**2*g) + x*(10*a*b*c*h - 20*a*c**2*g + 2*b**3*h - 4*b**2*c*g))/(
32*a**4*c**2*d - 16*a**3*b**2*c*d + 2*a**2*b**4*d + x**4*(32*a**2
*c**4*d - 16*a*b**2*c**3*d + 2*b**4*c**2*d) + x**3*(64*a**2*b*c**
3*d - 32*a*b**3*c**2*d + 4*b**5*c*d) + x**2*(64*a**3*c**3*d - 12*
a*b**4*c*d + 2*b**6*d) + x*(64*a**3*b*c**2*d - 32*a**2*b**3*c*d +
 4*a*b**5*d))
```

GIAC/XCAS [A] time = 0.276455, size = 279, normalized size = 1.99

$$\frac{6\left(2\,c^2g-bch\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4d-8\,ab^2cd+16\,a^2c^2d\right)\sqrt{-b^2+4\,ac}}+\frac{12\,c^3gx^3-6\,bc^2hx^3+18\,bc^2gx^2-9\,b^2chx^2+4\,b^2cgx+20\,ac^2gx-2\,b^3hx-10\,abchx-b^3g+10\,abcg-ab^2h-8\,a^2ch}{2\left(b^4d-8\,ab^2cd+16\,a^2c^2d\right)\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x + g)/((c*d*x^2 + b*d*x + a*d)*(c*x^2 + b*x + a)^2),x, algorithm="gi
```

```
[Out] 6*(2*c^2*g - b*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4*
d - 8*a*b^2*c*d + 16*a^2*c^2*d)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3
*g*x^3 - 6*b*c^2*h*x^3 + 18*b*c^2*g*x^2 - 9*b^2*c*h*x^2 + 4*b^2*c
*g*x + 20*a*c^2*g*x - 2*b^3*h*x - 10*a*b*c*h*x - b^3*g + 10*a*b*c
*g - a*b^2*h - 8*a^2*c*h)/((b^4*d - 8*a*b^2*c*d + 16*a^2*c^2*d)*(
c*x^2 + b*x + a)^2)
```

3.19 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=617

$$\frac{\left(2f(Af(cd-af)-Bd(ce-bf))-\left(e-\sqrt{e^2-4df}\right)\left(B\left(f(be-af)-c\left(e^2-df\right)\right)+Af(ce-bf)\right)\right)\tanh^{-1}\left(\frac{4af+2x(}{2\sqrt{2}\sqrt{a+bx+cx}}\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

$$-\frac{\left(2f(Af(cd-af)-Bd(ce-bf))-\left(\sqrt{e^2-4df}+e\right)\left(B\left(f(be-af)-c\left(e^2-df\right)\right)+Af(ce-bf)\right)\right)\tanh^{-1}\left(\frac{4af+2x}{2\sqrt{2}\sqrt{a+bx+c}}\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-2Acf-bBf+2Bce)}{2\sqrt{c}f^2}+\frac{B\sqrt{a+bx+cx^2}}{f}$$

```
[Out] (B*Sqrt[a + b*x + c*x^2])/f - ((2*B*c*e - b*B*f - 2*A*c*f)*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2)
 + ((2*f*(A*f*(c*d - a*f) - B*d*(c*e - b*f)) - (e - Sqrt[e^2 - 4*
d*f])*(A*f*(c*e - b*f) + B*(f*(b*e - a*f) - c*(e^2 - d*f))))*ArcT
anh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2
 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
- (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2
]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 -
(c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*f*(A*f*(c*d - a*f) - B*d*(c
*e - b*f)) - (e + Sqrt[e^2 - 4*d*f])*(A*f*(c*e - b*f) + B*(f*(b*e
 - a*f) - c*(e^2 - d*f))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d
*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e
^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*S
qrt[a + b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2
 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 18.5359, antiderivative size = 615, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{\left(2f(Af(cd-af)-Bd(ce-bf))-\left(e-\sqrt{e^2-4df}\right)\left(Af(ce-bf)-B\left(af^2-bef-cdf+ce^2\right)\right)\right)\tanh^{-1}\left(\frac{4af+2x(bf}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{}}\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

$$-\frac{\left(2f(Af(cd-af)-Bd(ce-bf))-\left(\sqrt{e^2-4df}+e\right)\left(Bf(be-af)+Af(ce-bf)-Bc\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{4af+2x(}{2\sqrt{2}\sqrt{a+bx+cx}}\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-2Acf-bBf+2Bce)}{2\sqrt{c}f^2}+\frac{B\sqrt{a+bx+cx^2}}{f}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] (B*Sqrt[a + b*x + c*x^2])/f - ((2*B*c*e - b*B*f - 2*A*c*f)*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2)
 + ((2*f*(A*f*(c*d - a*f) - B*d*(c*e - b*f)) - (e - Sqrt[e^2 - 4*
d*f])*(A*f*(c*e - b*f) - B*(c*e^2 - c*d*f - b*e*f + a*f^2)))*ArcT
anh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2
 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
- (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2
]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 -
(c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*f*(A*f*(c*d - a*f) - B*d*(c
```

```
*e - b*f)) - (e + Sqrt[e^2 - 4*d*f])*(B*f*(b*e - a*f) + A*f*(c*e
- b*f) - B*c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d
*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e
^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*S
qrt[a + b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2
 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [B] time = 6.24853, size = 1629, normalized size = 2.64

$$\frac{\sqrt{a+x(b+cx)}B}{f}$$
$$+\frac{\left(-Bce^3+bBfe^2+Acfe^2+Bc\sqrt{e^2-4df}e^2-Abf^2e-aBf^2e+3Bcdfe-bBf\sqrt{e^2-4df}e-Acf\sqrt{e^2-4df}e+2aAf^3-\cdots\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{ce^2-bfe-c\sqrt{e^2-4df}e\cdots}}$$
$$+\frac{\left(Bce^3-bBfe^2-Acfe^2+Bc\sqrt{e^2-4df}e^2+Abf^2e+aBf^2e-3Bcdfe-bBf\sqrt{e^2-4df}e-Acf\sqrt{e^2-4df}e-2aAf^3+\cdots\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{ce^2-bfe+c\sqrt{e^2-4df}e+\cdots}}$$
$$-\frac{(2Bce-bBf-2Acf)\sqrt{a+x(b+cx)}\log\left(b+2cx+2\sqrt{c}\sqrt{cx^2+bx+a}\right)}{2\sqrt{c}f^2\sqrt{cx^2+bx+a}}$$
$$-\frac{\left(Bce^3-bBfe^2-Acfe^2+Bc\sqrt{e^2-4df}e^2+Abf^2e+aBf^2e-3Bcdfe-bBf\sqrt{e^2-4df}e-Acf\sqrt{e^2-4df}e-2aAf^3+\cdots\right.}{\cdots}$$
$$-\frac{\left(-Bce^3+bBfe^2+Acfe^2+Bc\sqrt{e^2-4df}e^2-Abf^2e-aBf^2e+3Bcdfe-bBf\sqrt{e^2-4df}e-Acf\sqrt{e^2-4df}e+2aAf^3-\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] (B*Sqrt[a + x*(b + c*x)])/f + ((-(B*c*e^3) + 3*B*c*d*e*f + b*B*e^
2*f + A*c*e^2*f - 2*b*B*d*f^2 - 2*A*c*d*f^2 - A*b*e*f^2 - a*B*e*f
^2 + 2*a*A*f^3 + B*c*e^2*Sqrt[e^2 - 4*d*f] - B*c*d*f*Sqrt[e^2 - 4
*d*f] - b*B*e*f*Sqrt[e^2 - 4*d*f] - A*c*e*f*Sqrt[e^2 - 4*d*f] + A
*b*f^2*Sqrt[e^2 - 4*d*f] + a*B*f^2*Sqrt[e^2 - 4*d*f])*Sqrt[a + x*
(b + c*x)]*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x])/(Sqrt[2]*f^2*Sqrt
[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e
^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]) + ((B
*c*e^3 - 3*B*c*d*e*f - b*B*e^2*f - A*c*e^2*f + 2*b*B*d*f^2 + 2*A*
c*d*f^2 + A*b*e*f^2 + a*B*e*f^2 - 2*a*A*f^3 + B*c*e^2*Sqrt[e^2 -
4*d*f] - B*c*d*f*Sqrt[e^2 - 4*d*f] - b*B*e*f*Sqrt[e^2 - 4*d*f] -
A*c*e*f*Sqrt[e^2 - 4*d*f] + A*b*f^2*Sqrt[e^2 - 4*d*f] + a*B*f^2*S
qrt[e^2 - 4*d*f])*Sqrt[a + x*(b + c*x)]*Log[e + Sqrt[e^2 - 4*d*f]
 + 2*f*x])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f -
b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*
Sqrt[a + b*x + c*x^2]) - ((2*B*c*e - b*B*f - 2*A*c*f)*Sqrt[a + x*
```

```
(b + c*x)]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(2*S
qrt[c]*f^2*Sqrt[a + b*x + c*x^2]) - ((B*c*e^3 - 3*B*c*d*e*f - b*B
*e^2*f - A*c*e^2*f + 2*b*B*d*f^2 + 2*A*c*d*f^2 + A*b*e*f^2 + a*B*
e*f^2 - 2*a*A*f^3 + B*c*e^2*Sqrt[e^2 - 4*d*f] - B*c*d*f*Sqrt[e^2
- 4*d*f] - b*B*e*f*Sqrt[e^2 - 4*d*f] - A*c*e*f*Sqrt[e^2 - 4*d*f]
+ A*b*f^2*Sqrt[e^2 - 4*d*f] + a*B*f^2*Sqrt[e^2 - 4*d*f])*Sqrt[a +
 x*(b + c*x)]*Log[-(b*e^2) + 4*b*d*f - b*e*Sqrt[e^2 - 4*d*f] + 4*
a*f*Sqrt[e^2 - 4*d*f] - 2*c*e^2*x + 8*c*d*f*x - 2*c*e*Sqrt[e^2 -
4*d*f]*x + 2*b*f*Sqrt[e^2 - 4*d*f]*x + 2*Sqrt[2]*Sqrt[e^2 - 4*d*f
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f]
- b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]])/(Sqrt[2]*f^2*Sqr
t[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[
e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]) - ((
-(B*c*e^3) + 3*B*c*d*e*f + b*B*e^2*f + A*c*e^2*f - 2*b*B*d*f^2 -
2*A*c*d*f^2 - A*b*e*f^2 - a*B*e*f^2 + 2*a*A*f^3 + B*c*e^2*Sqrt[e^
2 - 4*d*f] - B*c*d*f*Sqrt[e^2 - 4*d*f] - b*B*e*f*Sqrt[e^2 - 4*d*f
] - A*c*e*f*Sqrt[e^2 - 4*d*f] + A*b*f^2*Sqrt[e^2 - 4*d*f] + a*B*f
^2*Sqrt[e^2 - 4*d*f])*Sqrt[a + x*(b + c*x)]*Log[b*e^2 - 4*b*d*f -
 b*e*Sqrt[e^2 - 4*d*f] + 4*a*f*Sqrt[e^2 - 4*d*f] + 2*c*e^2*x - 8*
c*d*f*x - 2*c*e*Sqrt[e^2 - 4*d*f]*x + 2*b*f*Sqrt[e^2 - 4*d*f]*x +
 2*Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x
+ c*x^2]])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f -
b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*
Sqrt[a + b*x + c*x^2])
```

Maple [B] time = 0.026, size = 16209, normalized size = 26.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)*(B*x + A)/(f*x^2 + e*x + d),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)*(B*x + A)/(f*x^2 + e*x + d),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral((A + B*x)*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)*(B*x + A)/(f*x^2 + e*x + d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.20 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=1092

result too large to display

```
[Out] -((2*A*c*f*(4*c*e - 5*b*f) - B*(b^2*f^2 - 2*c*f*(5*b*e - 4*a*f) +
 8*c^2*(e^2 - d*f)) + 2*c*f*(2*B*c*e - b*B*f - 2*A*c*f)*x)*Sqrt[a
 + b*x + c*x^2])/(8*c*f^3) + (B*(a + b*x + c*x^2)^(3/2))/(3*f) +
((2*A*c*f*(3*b^2*f^2 - 12*c*f*(b*e - a*f) + 8*c^2*(e^2 - d*f)) -
B*(b^3*f^3 + 6*b*c*f^2*(b*e - 2*a*f) - 24*c^2*f*(b*e^2 - b*d*f -
a*e*f) + 16*c^3*(e^3 - 2*d*e*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*f^4) - ((2*c*f*(B*d*(c*e - b
*f)*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2) + A*f*(2*c*d*f*(b*e - a*f
) - f^2*(b^2*d - a^2*f) - c^2*d*(e^2 - d*f))) - c*(e - Sqrt[e^2 -
 4*d*f])*(A*f*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + B
*(c^2*(e^4 - 3*d*e^2*f + d^2*f^2) - f^2*(2*a*b*e*f - a^2*f^2 - b^
2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(e^3 - 2*d*e*f)))))*A
rcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[
e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqr
t[2]*c*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((2*f*(B*d*(c*e - b*f)*(c*
e^2 - 2*c*d*f - b*e*f + 2*a*f^2) + A*f*(2*c*d*f*(b*e - a*f) - f^2
*(b^2*d - a^2*f) - c^2*d*(e^2 - d*f))) - (e + Sqrt[e^2 - 4*d*f])*
(A*f*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + B*(c^2*(e^
4 - 3*d*e^2*f + d^2*f^2) - f^2*(2*a*b*e*f - a^2*f^2 - b^2*(e^2 -
d*f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(e^3 - 2*d*e*f)))))*ArcTanh[(4
*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d
*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e
 - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^4*
Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e -
 b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 32.0558, antiderivative size = 1092, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$

$$\frac{B\left(cx^2+bx+a\right)^{3/2}}{3f}$$
$$-\frac{\left(2Acf(4ce-5bf)-B\left(8\left(e^2-df\right)c^2-2f(5be-4af)c+b^2f^2\right)+2cf(2Bce-bBf-2Acf)x\right)\sqrt{cx^2+bx+a}}{8cf^3}$$
$$+\frac{\left(2Acf\left(8\left(e^2-df\right)c^2-12f(be-af)c+3b^2f^2\right)-B\left(16\left(e^3-2def\right)c^3-24f\left(be^2-afe-bdf\right)c^2+6bf^2(be-2af)c\cdots\right.\right.}{16c^{3/2}f^4}$$
$$-\frac{\left(2cf\left(Bd(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)+Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)\right)-c\left(e-\sqrt{e^2\cdots}\right.\right.}{\cdots}$$
$$+\frac{\left(2f\left(Bd(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)+Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)\right)-\left(e+\sqrt{e^2-4\cdots}\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] -((2*A*c*f*(4*c*e - 5*b*f) - B*(b^2*f^2 - 2*c*f*(5*b*e - 4*a*f) +
 8*c^2*(e^2 - d*f)) + 2*c*f*(2*B*c*e - b*B*f - 2*A*c*f)*x)*Sqrt[a
 + b*x + c*x^2])/(8*c*f^3) + (B*(a + b*x + c*x^2)^(3/2))/(3*f) +
((2*A*c*f*(3*b^2*f^2 - 12*c*f*(b*e - a*f) + 8*c^2*(e^2 - d*f)) -
B*(b^3*f^3 + 6*b*c*f^2*(b*e - 2*a*f) - 24*c^2*f*(b*e^2 - b*d*f -
```

```
a*e*f) + 16*c^3*(e^3 - 2*d*e*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*f^4) - ((2*c*f*(B*d*(c*e - b
*f)*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2) + A*f*(2*c*d*f*(b*e - a*f
) - f^2*(b^2*d - a^2*f) - c^2*d*(e^2 - d*f))) - c*(e - Sqrt[e^2 -
 4*d*f])*(A*f*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + B
*(c^2*(e^4 - 3*d*e^2*f + d^2*f^2) - f^2*(2*a*b*e*f - a^2*f^2 - b^
2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(e^3 - 2*d*e*f)))))*A
rcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[
e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqr
t[2]*c*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((2*f*(B*d*(c*e - b*f)*(c*
e^2 - 2*c*d*f - b*e*f + 2*a*f^2) + A*f*(2*c*d*f*(b*e - a*f) - f^2
*(b^2*d - a^2*f) - c^2*d*(e^2 - d*f))) - (e + Sqrt[e^2 - 4*d*f])*
(A*f*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + B*(c^2*(e^
4 - 3*d*e^2*f + d^2*f^2) - f^2*(2*a*b*e*f - a^2*f^2 - b^2*(e^2 -
d*f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(e^3 - 2*d*e*f)))))*ArcTanh[(4
*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d
*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e
 - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^4*
Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e -
 b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)*(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [B] time = 6.52031, size = 3733, normalized size = 3.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] ((-(-24*B*c^2*e^2 + 24*B*c^2*d*f + 30*b*B*c*e*f + 24*A*c^2*e*f -
3*b^2*B*f^2 - 30*A*b*c*f^2 - 32*a*B*c*f^2)/(24*c*f^3) + ((-6*B*c*
e + 7*b*B*f + 6*A*c*f)*x)/(12*f^2) + (B*c*x^2)/(3*f))*(a + x*(b +
 c*x))^(3/2))/(a + b*x + c*x^2) + ((-(B*c^2*e^5) + 5*B*c^2*d*e^3*
f + 2*b*B*c*e^4*f + A*c^2*e^4*f - 5*B*c^2*d^2*e*f^2 - 8*b*B*c*d*e
^2*f^2 - 4*A*c^2*d*e^2*f^2 - b^2*B*e^3*f^2 - 2*A*b*c*e^3*f^2 - 2*
a*B*c*e^3*f^2 + 4*b*B*c*d^2*f^3 + 2*A*c^2*d^2*f^3 + 3*b^2*B*d*e*f
^3 + 6*A*b*c*d*e*f^3 + 6*a*B*c*d*e*f^3 + A*b^2*e^2*f^3 + 2*a*b*B*
e^2*f^3 + 2*a*A*c*e^2*f^3 - 2*A*b^2*d*f^4 - 4*a*b*B*d*f^4 - 4*a*A
*c*d*f^4 - 2*a*A*b*e*f^4 - a^2*B*e*f^4 + 2*a^2*A*f^5 + B*c^2*e^4*
Sqrt[e^2 - 4*d*f] - 3*B*c^2*d*e^2*f*Sqrt[e^2 - 4*d*f] - 2*b*B*c*e
^3*f*Sqrt[e^2 - 4*d*f] - A*c^2*e^3*f*Sqrt[e^2 - 4*d*f] + B*c^2*d^
2*f^2*Sqrt[e^2 - 4*d*f] + 4*b*B*c*d*e*f^2*Sqrt[e^2 - 4*d*f] + 2*A
*c^2*d*e*f^2*Sqrt[e^2 - 4*d*f] + b^2*B*e^2*f^2*Sqrt[e^2 - 4*d*f]
+ 2*A*b*c*e^2*f^2*Sqrt[e^2 - 4*d*f] + 2*a*B*c*e^2*f^2*Sqrt[e^2 -
4*d*f] - b^2*B*d*f^3*Sqrt[e^2 - 4*d*f] - 2*A*b*c*d*f^3*Sqrt[e^2 -
 4*d*f] - 2*a*B*c*d*f^3*Sqrt[e^2 - 4*d*f] - A*b^2*e*f^3*Sqrt[e^2
- 4*d*f] - 2*a*b*B*e*f^3*Sqrt[e^2 - 4*d*f] - 2*a*A*c*e*f^3*Sqrt[e
^2 - 4*d*f] + 2*a*A*b*f^4*Sqrt[e^2 - 4*d*f] + a^2*B*f^4*Sqrt[e^2
- 4*d*f])*(a + x*(b + c*x))^(3/2)*Log[-e + Sqrt[e^2 - 4*d*f] - 2*
f*x])/(Sqrt[2]*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(a +
```

```
b*x + c*x^2)^(3/2)) + ((B*c^2*e^5 - 5*B*c^2*d*e^3*f - 2*b*B*c*e^4
*f - A*c^2*e^4*f + 5*B*c^2*d^2*e*f^2 + 8*b*B*c*d*e^2*f^2 + 4*A*c^
2*d*e^2*f^2 + b^2*B*e^3*f^2 + 2*A*b*c*e^3*f^2 + 2*a*B*c*e^3*f^2 -
 4*b*B*c*d^2*f^3 - 2*A*c^2*d^2*f^3 - 3*b^2*B*d*e*f^3 - 6*A*b*c*d*
e*f^3 - 6*a*B*c*d*e*f^3 - A*b^2*e^2*f^3 - 2*a*b*B*e^2*f^3 - 2*a*A
*c*e^2*f^3 + 2*A*b^2*d*f^4 + 4*a*b*B*d*f^4 + 4*a*A*c*d*f^4 + 2*a*
A*b*e*f^4 + a^2*B*e*f^4 - 2*a^2*A*f^5 + B*c^2*e^4*Sqrt[e^2 - 4*d*
f] - 3*B*c^2*d*e^2*f*Sqrt[e^2 - 4*d*f] - 2*b*B*c*e^3*f*Sqrt[e^2 -
 4*d*f] - A*c^2*e^3*f*Sqrt[e^2 - 4*d*f] + B*c^2*d^2*f^2*Sqrt[e^2
- 4*d*f] + 4*b*B*c*d*e*f^2*Sqrt[e^2 - 4*d*f] + 2*A*c^2*d*e*f^2*Sq
rt[e^2 - 4*d*f] + b^2*B*e^2*f^2*Sqrt[e^2 - 4*d*f] + 2*A*b*c*e^2*f
^2*Sqrt[e^2 - 4*d*f] + 2*a*B*c*e^2*f^2*Sqrt[e^2 - 4*d*f] - b^2*B*
d*f^3*Sqrt[e^2 - 4*d*f] - 2*A*b*c*d*f^3*Sqrt[e^2 - 4*d*f] - 2*a*B
*c*d*f^3*Sqrt[e^2 - 4*d*f] - A*b^2*e*f^3*Sqrt[e^2 - 4*d*f] - 2*a*
b*B*e*f^3*Sqrt[e^2 - 4*d*f] - 2*a*A*c*e*f^3*Sqrt[e^2 - 4*d*f] + 2
*a*A*b*f^4*Sqrt[e^2 - 4*d*f] + a^2*B*f^4*Sqrt[e^2 - 4*d*f])*(a +
x*(b + c*x))^(3/2)*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[2]*f
^4*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e
*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(a + b*x + c*x^2)^(3/
2)) - ((16*B*c^3*e^3 - 32*B*c^3*d*e*f - 24*b*B*c^2*e^2*f - 16*A*c
^3*e^2*f + 24*b*B*c^2*d*f^2 + 16*A*c^3*d*f^2 + 6*b^2*B*c*e*f^2 +
24*A*b*c^2*e*f^2 + 24*a*B*c^2*e*f^2 + b^3*B*f^3 - 6*A*b^2*c*f^3 -
 12*a*b*B*c*f^3 - 24*a*A*c^2*f^3)*(a + x*(b + c*x))^(3/2)*Log[b +
 2*c*x + 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(16*c^(3/2)*f^4*(a + b
*x + c*x^2)^(3/2)) - ((B*c^2*e^5 - 5*B*c^2*d*e^3*f - 2*b*B*c*e^4*
f - A*c^2*e^4*f + 5*B*c^2*d^2*e*f^2 + 8*b*B*c*d*e^2*f^2 + 4*A*c^2
*d*e^2*f^2 + b^2*B*e^3*f^2 + 2*A*b*c*e^3*f^2 + 2*a*B*c*e^3*f^2 -
4*b*B*c*d^2*f^3 - 2*A*c^2*d^2*f^3 - 3*b^2*B*d*e*f^3 - 6*A*b*c*d*e
*f^3 - 6*a*B*c*d*e*f^3 - A*b^2*e^2*f^3 - 2*a*b*B*e^2*f^3 - 2*a*A*
c*e^2*f^3 + 2*A*b^2*d*f^4 + 4*a*b*B*d*f^4 + 4*a*A*c*d*f^4 + 2*a*A
*b*e*f^4 + a^2*B*e*f^4 - 2*a^2*A*f^5 + B*c^2*e^4*Sqrt[e^2 - 4*d*f
] - 3*B*c^2*d*e^2*f*Sqrt[e^2 - 4*d*f] - 2*b*B*c*e^3*f*Sqrt[e^2 -
4*d*f] - A*c^2*e^3*f*Sqrt[e^2 - 4*d*f] + B*c^2*d^2*f^2*Sqrt[e^2 -
 4*d*f] + 4*b*B*c*d*e*f^2*Sqrt[e^2 - 4*d*f] + 2*A*c^2*d*e*f^2*Sqr
t[e^2 - 4*d*f] + b^2*B*e^2*f^2*Sqrt[e^2 - 4*d*f] + 2*A*b*c*e^2*f^
2*Sqrt[e^2 - 4*d*f] + 2*a*B*c*e^2*f^2*Sqrt[e^2 - 4*d*f] - b^2*B*d
*f^3*Sqrt[e^2 - 4*d*f] - 2*A*b*c*d*f^3*Sqrt[e^2 - 4*d*f] - 2*a*B*
c*d*f^3*Sqrt[e^2 - 4*d*f] - A*b^2*e*f^3*Sqrt[e^2 - 4*d*f] - 2*a*b
*B*e*f^3*Sqrt[e^2 - 4*d*f] - 2*a*A*c*e*f^3*Sqrt[e^2 - 4*d*f] + 2*
a*A*b*f^4*Sqrt[e^2 - 4*d*f] + a^2*B*f^4*Sqrt[e^2 - 4*d*f])*(a + x
*(b + c*x))^(3/2)*Log[-(b*e^2) + 4*b*d*f - b*e*Sqrt[e^2 - 4*d*f]
+ 4*a*f*Sqrt[e^2 - 4*d*f] - 2*c*e^2*x + 8*c*d*f*x - 2*c*e*Sqrt[e^
2 - 4*d*f]*x + 2*b*f*Sqrt[e^2 - 4*d*f]*x + 2*Sqrt[2]*Sqrt[e^2 - 4
*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d
*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]])/(Sqrt[2]*f^4
*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*S
qrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(a + b*x + c*x^2)^(3/2)
) - ((-(B*c^2*e^5) + 5*B*c^2*d*e^3*f + 2*b*B*c*e^4*f + A*c^2*e^4*
f - 5*B*c^2*d^2*e*f^2 - 8*b*B*c*d*e^2*f^2 - 4*A*c^2*d*e^2*f^2 - b
^2*B*e^3*f^2 - 2*A*b*c*e^3*f^2 - 2*a*B*c*e^3*f^2 + 4*b*B*c*d^2*f^
3 + 2*A*c^2*d^2*f^3 + 3*b^2*B*d*e*f^3 + 6*A*b*c*d*e*f^3 + 6*a*B*c
*d*e*f^3 + A*b^2*e^2*f^3 + 2*a*b*B*e^2*f^3 + 2*a*A*c*e^2*f^3 - 2*
A*b^2*d*f^4 - 4*a*b*B*d*f^4 - 4*a*A*c*d*f^4 - 2*a*A*b*e*f^4 - a^2
*B*e*f^4 + 2*a^2*A*f^5 + B*c^2*e^4*Sqrt[e^2 - 4*d*f] - 3*B*c^2*d*
e^2*f*Sqrt[e^2 - 4*d*f] - 2*b*B*c*e^3*f*Sqrt[e^2 - 4*d*f] - A*c^2
*e^3*f*Sqrt[e^2 - 4*d*f] + B*c^2*d^2*f^2*Sqrt[e^2 - 4*d*f] + 4*b*
B*c*d*e*f^2*Sqrt[e^2 - 4*d*f] + 2*A*c^2*d*e*f^2*Sqrt[e^2 - 4*d*f]
 + b^2*B*e^2*f^2*Sqrt[e^2 - 4*d*f] + 2*A*b*c*e^2*f^2*Sqrt[e^2 - 4
*d*f] + 2*a*B*c*e^2*f^2*Sqrt[e^2 - 4*d*f] - b^2*B*d*f^3*Sqrt[e^2
- 4*d*f] - 2*A*b*c*d*f^3*Sqrt[e^2 - 4*d*f] - 2*a*B*c*d*f^3*Sqrt[e
^2 - 4*d*f] - A*b^2*e*f^3*Sqrt[e^2 - 4*d*f] - 2*a*b*B*e*f^3*Sqrt[
e^2 - 4*d*f] - 2*a*A*c*e*f^3*Sqrt[e^2 - 4*d*f] + 2*a*A*b*f^4*Sqrt
[e^2 - 4*d*f] + a^2*B*f^4*Sqrt[e^2 - 4*d*f])*(a + x*(b + c*x))^(3
/2)*Log[b*e^2 - 4*b*d*f - b*e*Sqrt[e^2 - 4*d*f] + 4*a*f*Sqrt[e^2
- 4*d*f] + 2*c*e^2*x - 8*c*d*f*x - 2*c*e*Sqrt[e^2 - 4*d*f]*x + 2*
b*f*Sqrt[e^2 - 4*d*f]*x + 2*Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^
2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]])/(Sqrt[2]*f^4*Sqrt[e^2 - 4*d*f
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f]
+ b*f*Sqrt[e^2 - 4*d*f]]*(a + b*x + c*x^2)^(3/2))
```

Maple [B] time = 0.04, size = 59465, normalized size = 54.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)^(3/2)*(B*x + A)/(f*x^2 + e*x + d),x, algorithm="maxima
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)^(3/2)*(B*x + A)/(f*x^2 + e*x + d),x, algorithm="fricas
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)*(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)^(3/2)*(B*x + A)/(f*x^2 + e*x + d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.21 $\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx$

Optimal. Leaf size=416

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2x\left(ce-f\left(b-\sqrt{b^2-4ac}\right)\right)-e\left(b-\sqrt{b^2-4ac}\right)+4cd}{2\sqrt{2}\sqrt{d+ex+fx^2}\sqrt{\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\tanh^{-1}\left(\frac{2x\left(ce-f\left(\sqrt{b^2-4ac}+b\right)\right)-e\left(\sqrt{b^2-4ac}+b\right)+4cd}{2\sqrt{2}\sqrt{d+ex+fx^2}\sqrt{-\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}$$

```
[Out] ((b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*ArcTanh[(4*c*d - (b - Sqrt[b
^2 - 4*a*c])*e + 2*(c*e - (b - Sqrt[b^2 - 4*a*c])*f)*x)/(2*Sqrt[2
]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f + Sqrt[b^2 - 4*a*c]*(c*e
 - b*f)]*Sqrt[d + e*x + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt
[2*c^2*d - b*c*e + b^2*f - 2*a*c*f + Sqrt[b^2 - 4*a*c]*(c*e - b*f
)]) + ((2*A*c - B*(b + Sqrt[b^2 - 4*a*c]))*ArcTanh[(4*c*d - (b +
Sqrt[b^2 - 4*a*c])*e + 2*(c*e - (b + Sqrt[b^2 - 4*a*c])*f)*x)/(2*
Sqrt[2]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c
]*(c*e - b*f)]*Sqrt[d + e*x + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c
]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e
 - b*f)])
```

Rubi [A] time = 6.3233, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2x\left(ce-f\left(b-\sqrt{b^2-4ac}\right)\right)-e\left(b-\sqrt{b^2-4ac}\right)+4cd}{2\sqrt{2}\sqrt{d+ex+fx^2}\sqrt{\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\tanh^{-1}\left(\frac{2x\left(ce-f\left(\sqrt{b^2-4ac}+b\right)\right)-e\left(\sqrt{b^2-4ac}+b\right)+4cd}{2\sqrt{2}\sqrt{d+ex+fx^2}\sqrt{-\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] ((b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*ArcTanh[(4*c*d - (b - Sqrt[b
^2 - 4*a*c])*e + 2*(c*e - (b - Sqrt[b^2 - 4*a*c])*f)*x)/(2*Sqrt[2
]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f + Sqrt[b^2 - 4*a*c]*(c*e
 - b*f)]*Sqrt[d + e*x + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt
[2*c^2*d - b*c*e + b^2*f - 2*a*c*f + Sqrt[b^2 - 4*a*c]*(c*e - b*f
)]) + ((2*A*c - B*(b + Sqrt[b^2 - 4*a*c]))*ArcTanh[(4*c*d - (b +
Sqrt[b^2 - 4*a*c])*e + 2*(c*e - (b + Sqrt[b^2 - 4*a*c])*f)*x)/(2*
Sqrt[2]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c
]*(c*e - b*f)]*Sqrt[d + e*x + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c
]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e
 - b*f)])
```

Rubi in Sympy [A] time = 141.626, size = 394, normalized size = 0.95

$$-\frac{\sqrt{2}\left(2Ac-B\left(b-\sqrt{-4ac+b^2}\right)\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(-be+4cd+e\sqrt{-4ac+b^2}+x\left(2ce-2f\left(b-\sqrt{-4ac+b^2}\right)\right)\right)}{4\sqrt{d+ex+fx^2}\sqrt{-2acf+b^2f-bce+2c^2d-\sqrt{-4ac+b^2}(bf-ce)}}\right)}{2\sqrt{-4ac+b^2}\sqrt{-2acf+b^2f-bce+2c^2d-\sqrt{-4ac+b^2}\,(bf-ce)}}+\frac{\sqrt{2}\left(2Ac-B\left(b+\sqrt{-4ac+b^2}\right)\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(4cd-e\left(b+\sqrt{-4ac+b^2}\right)+x\left(2ce-2f\left(b+\sqrt{-4ac+b^2}\right)\right)\right)}{4\sqrt{d+ex+fx^2}\sqrt{-2acf+b^2f-bce+2c^2d-\sqrt{-4ac+b^2}(-bf+ce)}}\right)}{2\sqrt{-4ac+b^2}\sqrt{-2acf+b^2f-bce+2c^2d-\sqrt{-4ac+b^2}\,(-bf+ce)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] -sqrt(2)*(2*A*c - B*(b - sqrt(-4*a*c + b**2)))*atanh(sqrt(2)*(-b*
e + 4*c*d + e*sqrt(-4*a*c + b**2) + x*(2*c*e - 2*f*(b - sqrt(-4*a
*c + b**2))))/(4*sqrt(d + e*x + f*x**2)*sqrt(-2*a*c*f + b**2*f -
b*c*e + 2*c**2*d - sqrt(-4*a*c + b**2)*(b*f - c*e))))/(2*sqrt(-4*
a*c + b**2)*sqrt(-2*a*c*f + b**2*f - b*c*e + 2*c**2*d - sqrt(-4*a
*c + b**2)*(b*f - c*e))) + sqrt(2)*(2*A*c - B*(b + sqrt(-4*a*c +
b**2)))*atanh(sqrt(2)*(4*c*d - e*(b + sqrt(-4*a*c + b**2)) + x*(2
*c*e - 2*f*(b + sqrt(-4*a*c + b**2))))/(4*sqrt(d + e*x + f*x**2)*
sqrt(-2*a*c*f + b**2*f - b*c*e + 2*c**2*d - sqrt(-4*a*c + b**2)*(
-b*f + c*e))))/(2*sqrt(-4*a*c + b**2)*sqrt(-2*a*c*f + b**2*f - b*
c*e + 2*c**2*d - sqrt(-4*a*c + b**2)*(-b*f + c*e)))
```

Mathematica [A] time = 1.7273, size = 696, normalized size = 1.67

$$\frac{\frac{\left(B\sqrt{b^2-4ac}+2Ac-bB\right)\log\left(\sqrt{b^2-4ac}-b-2cx\right)}{\sqrt{c\left(e\sqrt{b^2-4ac}-2af-be\right)+bf\left(b-\sqrt{b^2-4ac}\right)+2c^2d}}+\frac{\left(B\sqrt{b^2-4ac}-2Ac+bB\right)\log\left(\sqrt{b^2-4ac}+b+2cx\right)}{\sqrt{-c\left(e\sqrt{b^2-4ac}+2af+be\right)+bf\left(\sqrt{b^2-4ac}+b\right)+2c^2d}}-\frac{\left(B\sqrt{b^2-4ac}-2Ac+bB\right)\log\left(2\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+\ldots\right.}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] (((-(b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*
c] - 2*c*x])/Sqrt[2*c^2*d + b*(b - Sqrt[b^2 - 4*a*c])*f + c*(-(b*
e) + Sqrt[b^2 - 4*a*c]*e - 2*a*f)] + ((b*B - 2*A*c + B*Sqrt[b^2 -
 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x])/Sqrt[2*c^2*d + b*(b
+ Sqrt[b^2 - 4*a*c])*f - c*(b*e + Sqrt[b^2 - 4*a*c]*e + 2*a*f)] -
 ((b*B - 2*A*c + B*Sqrt[b^2 - 4*a*c])*Log[-(b^2*(e + 2*f*x)) - b*
Sqrt[b^2 - 4*a*c]*(e + 2*f*x) + 2*c*(2*Sqrt[b^2 - 4*a*c]*d + 2*a*
e + Sqrt[b^2 - 4*a*c]*e*x + 4*a*f*x) + 2*Sqrt[2]*Sqrt[b^2 - 4*a*c
]*Sqrt[2*c^2*d + b*(b + Sqrt[b^2 - 4*a*c])*f - c*(b*e + Sqrt[b^2
- 4*a*c]*e + 2*a*f)]*Sqrt[d + x*(e + f*x)]])/Sqrt[2*c^2*d + b*(b
+ Sqrt[b^2 - 4*a*c])*f - c*(b*e + Sqrt[b^2 - 4*a*c]*e + 2*a*f)] -
 ((-(b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*Log[b^2*(e + 2*f*x) - b*
Sqrt[b^2 - 4*a*c]*(e + 2*f*x) + 2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[
2*c^2*d + b*(b - Sqrt[b^2 - 4*a*c])*f + c*(-(b*e) + Sqrt[b^2 - 4*
a*c]*e - 2*a*f)]*Sqrt[d + x*(e + f*x)] + 2*c*(2*Sqrt[b^2 - 4*a*c]
*d + Sqrt[b^2 - 4*a*c]*e*x - 2*a*(e + 2*f*x))])/Sqrt[2*c^2*d + b*
(b - Sqrt[b^2 - 4*a*c])*f + c*(-(b*e) + Sqrt[b^2 - 4*a*c]*e - 2*a
*f)])/(Sqrt[2]*Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.079, size = 2269, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] 2/(-4*a*c+b^2)^(1/2)/(-2*(-b*f*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1
/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*ln((-(-b*f*(-4*a*
c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/
c^2-(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2)
)/c)+1/2*(-2*(-b*f*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c*e+2*a*
c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*(4*(x+1/2*(b+(-4*a*c+b^2)^(1/
2))/c)^2*f-4*(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*a*c+b
^2)^(1/2))/c)-2*(-b*f*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c*e+2
*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2))/(x+1/2*(b+(-4*a*c+b^2)^(1
/2))/c))*A-1/c/(-2*(-b*f*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c*
e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*ln((-(-b*f*(-4*a*c+b^2)
^(1/2)+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2-(f
*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)+1
/2*(-2*(-b*f*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^
2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*(4*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
^2*f-4*(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*a*c+b^2)^(1
/2))/c)-2*(-b*f*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f
-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2))/(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c
))*B-1/(-4*a*c+b^2)^(1/2)/c/(-2*(-b*f*(-4*a*c+b^2)^(1/2)+(-4*a*c+
b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*ln((-(-b*f
*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*
c^2*d)/c^2-(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*a*c+b^2
)^(1/2))/c)+1/2*(-2*(-b*f*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c
*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*(4*(x+1/2*(b+(-4*a*c+b
^2)^(1/2))/c)^2*f-4*(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-
4*a*c+b^2)^(1/2))/c)-2*(-b*f*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2
)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2))/(x+1/2*(b+(-4*a*c+
b^2)^(1/2))/c))*b*B-2/(-4*a*c+b^2)^(1/2)/(-2*(b*f*(-4*a*c+b^2)^(1
/2)-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2
)*ln((-(b*f*(-4*a*c+b^2)^(1/2)-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2
*f+b*c*e-2*c^2*d)/c^2-(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x-1/2/c*
(-b+(-4*a*c+b^2)^(1/2)))+1/2*(-2*(b*f*(-4*a*c+b^2)^(1/2)-(-4*a*c+
b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*(4*(x-1/2/
c*(-b+(-4*a*c+b^2)^(1/2)))^2*f-4*(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/
c*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))-2*(b*f*(-4*a*c+b^2)^(1/2)-(-4
*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2))/(x-1
/2/c*(-b+(-4*a*c+b^2)^(1/2))))*A-1/c/(-2*(b*f*(-4*a*c+b^2)^(1/2)-
(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*ln
((-(b*f*(-4*a*c+b^2)^(1/2)-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b
*c*e-2*c^2*d)/c^2-(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x-1/2/c*(-b+
(-4*a*c+b^2)^(1/2)))+1/2*(-2*(b*f*(-4*a*c+b^2)^(1/2)-(-4*a*c+b^2)
^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*(4*(x-1/2/c*(-
b+(-4*a*c+b^2)^(1/2)))^2*f-4*(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x
-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))-2*(b*f*(-4*a*c+b^2)^(1/2)-(-4*a*c
+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2))/(x-1/2/c
*(-b+(-4*a*c+b^2)^(1/2))))*B+1/(-4*a*c+b^2)^(1/2)/c/(-2*(b*f*(-4*
a*c+b^2)^(1/2)-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d
)/c^2)^(1/2)*ln((-(b*f*(-4*a*c+b^2)^(1/2)-(-4*a*c+b^2)^(1/2)*c*e+
2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2-(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/
c*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))+1/2*(-2*(b*f*(-4*a*c+b^2)^(1/
2)-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)
*(4*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))^2*f-4*(-f*(-4*a*c+b^2)^(1/2
)+b*f-c*e)/c*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))-2*(b*f*(-4*a*c+b^2
)^(1/2)-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^
(1/2))/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))*b*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + b*x + a)*sqrt(f*x^2 + e*x + d)),x, algorithm="maxima
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + b*x + a)*sqrt(f*x^2 + e*x + d)),x, algorithm="fricas
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] Integral((A + B*x)/((a + b*x + c*x**2)*sqrt(d + e*x + f*x**2)), x
)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + b*x + a)*sqrt(f*x^2 + e*x + d)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.22 $\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx$

Optimal. Leaf size=780

$$\frac{\sqrt{A\left(-\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)+aBe}\sqrt{B\left(\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)-Ace}\tanh^{-1}\left(\frac{\sqrt{\cdots}}{\sqrt{2}\sqrt{a}\cdots}\right.}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2\cdots}}$$

$$-\frac{\sqrt{B\left(-\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)-Ace}\sqrt{A\left(\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)+aBe}\tanh^{-1}\left(\frac{\cdots}{\sqrt{2}\sqrt{\cdots}}\right.}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{a^2f^2+ac\left(e^2-2df\right)+\cdots}}$$

```
[Out] (Sqrt[a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 -
2*d*f)])]*Sqrt[-(A*c*e) + B*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 +
 a*c*(e^2 - 2*d*f)])]*ArcTanh[(Sqrt[e]*(a*(A*c*e - B*(c*d - a*f +
 Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])) - c*(a*B*e + A*(c*
d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)]))*x))/(Sqrt
[2]*Sqrt[a]*Sqrt[c]*Sqrt[a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^
2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[-(A*c*e) + B*(c*d - a*f + Sqrt[
c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[d + e*x + f*x^2])])
/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[e]*Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e
^2 - 2*d*f)]) - (Sqrt[-(A*c*e) + B*(c*d - a*f - Sqrt[c^2*d^2 + a^
2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[a*B*e + A*(c*d - a*f + Sqrt[c^2
*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*ArcTanh[(Sqrt[e]*(a*(A*c*e
- B*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])) -
c*(a*B*e + A*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d
*f)]))*x))/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[-(A*c*e) + B*(c*d - a*f
- Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[a*B*e + A*(c
*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[d +
 e*x + f*x^2])])/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[e]*Sqrt[c^2*d^2 +
a^2*f^2 + a*c*(e^2 - 2*d*f)])
```

Rubi [A] time = 10.709, antiderivative size = 780, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$\frac{\sqrt{A\left(-\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)+aBe}\sqrt{B\left(\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)-Ace}\tanh^{-1}\left(\frac{\sqrt{\cdots}}{\sqrt{2}\sqrt{a}\cdots}\right.}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2\cdots}}$$

$$-\frac{\sqrt{B\left(-\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)-Ace}\sqrt{A\left(\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)+aBe}\tanh^{-1}\left(\frac{\cdots}{\sqrt{2}\sqrt{\cdots}}\right.}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{a^2f^2+ac\left(e^2-2df\right)+\cdots}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] (Sqrt[a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 -
2*d*f)])]*Sqrt[-(A*c*e) + B*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 +
 a*c*(e^2 - 2*d*f)])]*ArcTanh[(Sqrt[e]*(a*(A*c*e - B*(c*d - a*f +
 Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])) - c*(a*B*e + A*(c*
d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)]))*x))/(Sqrt
[2]*Sqrt[a]*Sqrt[c]*Sqrt[a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^
2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[-(A*c*e) + B*(c*d - a*f + Sqrt[
c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[d + e*x + f*x^2])])
/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[e]*Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e
^2 - 2*d*f)]) - (Sqrt[-(A*c*e) + B*(c*d - a*f - Sqrt[c^2*d^2 + a^
2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[a*B*e + A*(c*d - a*f + Sqrt[c^2
*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*ArcTanh[(Sqrt[e]*(a*(A*c*e
```

```
- B*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])) -
c*(a*B*e + A*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d
*f)]))*x))/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[-(A*c*e) + B*(c*d - a*f
- Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[a*B*e + A*(c
*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[d +
 e*x + f*x^2])])/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[e]*Sqrt[c^2*d^2 +
a^2*f^2 + a*c*(e^2 - 2*d*f)])
```

Rubi in Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/(c*x**2+a)/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C]   time = 2.73241, size = 411, normalized size = 0.53

$$\frac{\frac{\left(-\sqrt{a}B+iA\sqrt{c}\right)\log\left(\frac{i\sqrt{a}\sqrt{c}\left(2\sqrt{d+x(e+fx)}\sqrt{i\sqrt{a}\sqrt{c}e-af+cd}+i\sqrt{a}(e+2fx)+\sqrt{c}(2d+ex)\right)}{\left(\sqrt{a}+i\sqrt{c}x\right)\left(\sqrt{a}B-iA\sqrt{c}\right)\sqrt{i\sqrt{a}\sqrt{c}e-af+cd}}\right)}{\sqrt{i\sqrt{a}\sqrt{c}e-af+cd}}-\frac{\left(\sqrt{a}B+iA\sqrt{c}\right)\log\left(-\frac{\sqrt{a}\sqrt{c}\left(2i\sqrt{d+x(e+fx)}\sqrt{-i\sqrt{a}\sqrt{c}e-af+cd}+\sqrt{a}(e+2fx)+i\sqrt{c}(2d+ex)\right)}{\left(\sqrt{a}-i\sqrt{c}x\right)\left(\sqrt{a}B+iA\sqrt{c}\right)\sqrt{-i\sqrt{a}\sqrt{c}e-af+cd}}\right)}{\sqrt{-i\sqrt{a}\sqrt{c}e-af+cd}}}{2\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] (-(((Sqrt[a]*B + I*A*Sqrt[c])*Log[-((Sqrt[a]*Sqrt[c]*(I*Sqrt[c]*(
2*d + e*x) + Sqrt[a]*(e + 2*f*x) + (2*I)*Sqrt[c*d - I*Sqrt[a]*Sqr
t[c]*e - a*f]*Sqrt[d + x*(e + f*x)]))/((Sqrt[a]*B + I*A*Sqrt[c])*
Sqrt[c*d - I*Sqrt[a]*Sqrt[c]*e - a*f]*(Sqrt[a] - I*Sqrt[c]*x)))])
/Sqrt[c*d - I*Sqrt[a]*Sqrt[c]*e - a*f]) + ((-(Sqrt[a]*B) + I*A*Sq
rt[c])*Log[(I*Sqrt[a]*Sqrt[c]*(Sqrt[c]*(2*d + e*x) + I*Sqrt[a]*(e
 + 2*f*x) + 2*Sqrt[c*d + I*Sqrt[a]*Sqrt[c]*e - a*f]*Sqrt[d + x*(e
 + f*x)]))/((Sqrt[a]*B - I*A*Sqrt[c])*Sqrt[c*d + I*Sqrt[a]*Sqrt[c
]*e - a*f]*(Sqrt[a] + I*Sqrt[c]*x))])/Sqrt[c*d + I*Sqrt[a]*Sqrt[c
]*e - a*f])/(2*Sqrt[a]*Sqrt[c])
```

Maple [A]   time = 0.065, size = 784, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] 1/2/(-a*c)^(1/2)/(-((-a*c)^(1/2)*e+f*a-c*d)/c)^(1/2)*ln((-2*((-a*
c)^(1/2)*e+f*a-c*d)/c+1/c*(-2*f*(-a*c)^(1/2)+c*e)*(x+(-a*c)^(1/2)
/c)+2*(-((-a*c)^(1/2)*e+f*a-c*d)/c)^(1/2)*((x+(-a*c)^(1/2)/c)^2*f
+1/c*(-2*f*(-a*c)^(1/2)+c*e)*(x+(-a*c)^(1/2)/c)-((-a*c)^(1/2)*e+f
*a-c*d)/c)^(1/2))/(x+(-a*c)^(1/2)/c))*A-1/2/c/(-((-a*c)^(1/2)*e+f
*a-c*d)/c)^(1/2)*ln((-2*((-a*c)^(1/2)*e+f*a-c*d)/c+1/c*(-2*f*(-a*
c)^(1/2)+c*e)*(x+(-a*c)^(1/2)/c)+2*(-((-a*c)^(1/2)*e+f*a-c*d)/c)^
(1/2)*((x+(-a*c)^(1/2)/c)^2*f+1/c*(-2*f*(-a*c)^(1/2)+c*e)*(x+(-a*
c)^(1/2)/c)-((-a*c)^(1/2)*e+f*a-c*d)/c)^(1/2))/(x+(-a*c)^(1/2)/c)
```

```
)*B-1/2/(-a*c)^(1/2)/(-(-(-a*c)^(1/2)*e+f*a-c*d)/c)^(1/2)*ln((-2*
(-(-a*c)^(1/2)*e+f*a-c*d)/c+(2*f*(-a*c)^(1/2)+c*e)/c*(x-(-a*c)^(1
/2)/c)+2*(-(-(-a*c)^(1/2)*e+f*a-c*d)/c)^(1/2)*((x-(-a*c)^(1/2)/c)
^2*f+(2*f*(-a*c)^(1/2)+c*e)/c*(x-(-a*c)^(1/2)/c)-(-(-a*c)^(1/2)*e
+f*a-c*d)/c)^(1/2))/(x-(-a*c)^(1/2)/c))*A-1/2/c/(-(-(-a*c)^(1/2)*
e+f*a-c*d)/c)^(1/2)*ln((-2*(-(-a*c)^(1/2)*e+f*a-c*d)/c+(2*f*(-a*c
)^(1/2)+c*e)/c*(x-(-a*c)^(1/2)/c)+2*(-(-(-a*c)^(1/2)*e+f*a-c*d)/c
)^(1/2)*((x-(-a*c)^(1/2)/c)^2*f+(2*f*(-a*c)^(1/2)+c*e)/c*(x-(-a*c
)^(1/2)/c)-(-(-a*c)^(1/2)*e+f*a-c*d)/c)^(1/2))/(x-(-a*c)^(1/2)/c)
)*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + a)*sqrt(f*x^2 + e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 45.8573, size = 9262, normalized size = 11.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + a)*sqrt(f*x^2 + e*x + d)),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 - A^2*
a*c)*f + (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*sq
rt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B
*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^
2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^
2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*
d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)))/(a*
c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*log(-(2*(A*B^
3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 - A^4*c^2)*e^2 - 2*(A*B^3*a^2 +
 A^3*B*a*c)*e*f - 2*(2*(A*B^3*a^2 + A^3*B*a*c)*f^2 - (2*(A*B^3*a*
c + A^3*B*c^2)*d + (B^4*a^2 - A^4*c^2)*e)*f)*x + 2*(2*A^2*B*c^3*d
^2 + 2*A^2*B*a^2*c*f^2 + (3*A*B^2*a*c^2 - A^3*c^3)*d*e + (B^3*a^2
*c - A^2*B*a*c^2)*e^2 - (4*A^2*B*a*c^2*d + (3*A*B^2*a^2*c - A^3*a
*c^2)*e)*f - (B*a*c^4*d^3 - A*a*c^4*d^2*e + B*a^2*c^3*d*e^2 - A*a
^2*c^3*e^3 - B*a^4*c*f^3 + (3*B*a^3*c^2*d - A*a^3*c^2*e)*f^2 - (3
*B*a^2*c^3*d^2 - 2*A*a^2*c^3*d*e + B*a^3*c^2*e^2)*f)*sqrt(-(4*A^2
*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e
+ (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d +
(A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^
3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*
c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)))*sqrt(f*x^2 +
e*x + d)*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 -
A^2*a*c)*f + (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2
)*sqrt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A
^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^
2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^
4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*
c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)))
/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)) - (2*(B^2
*a*c^3 + A^2*c^4)*d^3 + 2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e^2 - 4*(B^
2*a^2*c^2 + A^2*a*c^3)*d^2*f + 2*(B^2*a^3*c + A^2*a^2*c^2)*d*f^2
+ ((B^2*a*c^3 + A^2*c^4)*d^2*e + (B^2*a^2*c^2 + A^2*a*c^3)*e^3 -
2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e*f + (B^2*a^3*c + A^2*a^2*c^2)*e*f
^2)*x)*sqrt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*
```

```
c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*
(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a
^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3
*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)
*f)))/x) + 1/4*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*
a^2 - A^2*a*c)*f + (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3
*c*f^2)*sqrt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a
*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4
*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*
a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(
3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2
)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*log
(-(2*(A*B^3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 - A^4*c^2)*e^2 - 2*(A
*B^3*a^2 + A^3*B*a*c)*e*f - 2*(2*(A*B^3*a^2 + A^3*B*a*c)*f^2 - (2
*(A*B^3*a*c + A^3*B*c^2)*d + (B^4*a^2 - A^4*c^2)*e)*f)*x - 2*(2*A
^2*B*c^3*d^2 + 2*A^2*B*a^2*c*f^2 + (3*A*B^2*a*c^2 - A^3*c^3)*d*e
+ (B^3*a^2*c - A^2*B*a*c^2)*e^2 - (4*A^2*B*a*c^2*d + (3*A*B^2*a^2
*c - A^3*a*c^2)*e)*f - (B*a*c^4*d^3 - A*a*c^4*d^2*e + B*a^2*c^3*d
*e^2 - A*a^2*c^3*e^3 - B*a^4*c*f^3 + (3*B*a^3*c^2*d - A*a^3*c^2*e
)*f^2 - (3*B*a^2*c^3*d^2 - 2*A*a^2*c^3*d*e + B*a^3*c^2*e^2)*f)*sq
rt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B
*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^
2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^
2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*
d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)))*sqr
t(f*x^2 + e*x + d)*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (
B^2*a^2 - A^2*a*c)*f + (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f +
 a^3*c*f^2)*sqrt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B
^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2
 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4
+ 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 +
 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d
*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))
 - (2*(B^2*a*c^3 + A^2*c^4)*d^3 + 2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e
^2 - 4*(B^2*a^2*c^2 + A^2*a*c^3)*d^2*f + 2*(B^2*a^3*c + A^2*a^2*c
^2)*d*f^2 + ((B^2*a*c^3 + A^2*c^4)*d^2*e + (B^2*a^2*c^2 + A^2*a*c
^3)*e^3 - 2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e*f + (B^2*a^3*c + A^2*a^
2*c^2)*e*f^2)*x)*sqrt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4
*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2
)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5
*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*
f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*
c^3*d*e^2)*f)))/x) - 1/4*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)
*d + (B^2*a^2 - A^2*a*c)*f - (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2
*d*f + a^3*c*f^2)*sqrt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 +
4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^
2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^
5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c
*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3
*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c
*f^2))*log(-(2*(A*B^3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 - A^4*c^2)*
e^2 - 2*(A*B^3*a^2 + A^3*B*a*c)*e*f - 2*(2*(A*B^3*a^2 + A^3*B*a*c
)*f^2 - (2*(A*B^3*a*c + A^3*B*c^2)*d + (B^4*a^2 - A^4*c^2)*e)*f)*
x + 2*(2*A^2*B*c^3*d^2 + 2*A^2*B*a^2*c*f^2 + (3*A*B^2*a*c^2 - A^3
*c^3)*d*e + (B^3*a^2*c - A^2*B*a*c^2)*e^2 - (4*A^2*B*a*c^2*d + (3
*A*B^2*a^2*c - A^3*a*c^2)*e)*f + (B*a*c^4*d^3 - A*a*c^4*d^2*e + B
*a^2*c^3*d*e^2 - A*a^2*c^3*e^3 - B*a^4*c*f^3 + (3*B*a^3*c^2*d - A
*a^3*c^2*e)*f^2 - (3*B*a^2*c^3*d^2 - 2*A*a^2*c^3*d*e + B*a^3*c^2*
e^2)*f)*sqrt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a
*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4
*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*
a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(
3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2
)*f)))*sqrt(f*x^2 + e*x + d)*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*
c^2)*d + (B^2*a^2 - A^2*a*c)*f - (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2
*c^2*d*f + a^3*c*f^2)*sqrt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^
2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^
4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(
a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a
^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 +
 a^3*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a
^3*c*f^2)) + (2*(B^2*a*c^3 + A^2*c^4)*d^3 + 2*(B^2*a^2*c^2 + A^2*
```

```
a*c^3)*d*e^2 - 4*(B^2*a^2*c^2 + A^2*a*c^3)*d^2*f + 2*(B^2*a^3*c +
 A^2*a^2*c^2)*d*f^2 + ((B^2*a*c^3 + A^2*c^4)*d^2*e + (B^2*a^2*c^2
 + A^2*a*c^3)*e^3 - 2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e*f + (B^2*a^3*
c + A^2*a^2*c^2)*e*f^2)*x)*sqrt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a
^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c
 + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)
*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^
3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*
d^3 + a^3*c^3*d*e^2)*f)))/x) + 1/4*sqrt(-(2*A*B*a*c*e - (B^2*a*c
- A^2*c^2)*d + (B^2*a^2 - A^2*a*c)*f - (a*c^3*d^2 + a^2*c^2*e^2 -
 2*a^2*c^2*d*f + a^3*c*f^2)*sqrt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*
a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*
c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e
)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f
^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4
*d^3 + a^3*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d
*f + a^3*c*f^2))*log(-(2*(A*B^3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 -
 A^4*c^2)*e^2 - 2*(A*B^3*a^2 + A^3*B*a*c)*e*f - 2*(2*(A*B^3*a^2 +
 A^3*B*a*c)*f^2 - (2*(A*B^3*a*c + A^3*B*c^2)*d + (B^4*a^2 - A^4*c
^2)*e)*f)*x - 2*(2*A^2*B*c^3*d^2 + 2*A^2*B*a^2*c*f^2 + (3*A*B^2*a
*c^2 - A^3*c^3)*d*e + (B^3*a^2*c - A^2*B*a*c^2)*e^2 - (4*A^2*B*a*
c^2*d + (3*A*B^2*a^2*c - A^3*a*c^2)*e)*f + (B*a*c^4*d^3 - A*a*c^4
*d^2*e + B*a^2*c^3*d*e^2 - A*a^2*c^3*e^3 - B*a^4*c*f^3 + (3*B*a^3
*c^2*d - A*a^3*c^2*e)*f^2 - (3*B*a^2*c^3*d^2 - 2*A*a^2*c^3*d*e +
B*a^3*c^2*e^2)*f)*sqrt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 +
4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^
2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^
5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c
*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3
*c^3*d*e^2)*f)))*sqrt(f*x^2 + e*x + d)*sqrt(-(2*A*B*a*c*e - (B^2*
a*c - A^2*c^2)*d + (B^2*a^2 - A^2*a*c)*f - (a*c^3*d^2 + a^2*c^2*e
^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*sqrt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*
B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^
2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*
c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2
*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2
*c^4*d^3 + a^3*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c
^2*d*f + a^3*c*f^2)) + (2*(B^2*a*c^3 + A^2*c^4)*d^3 + 2*(B^2*a^2*
c^2 + A^2*a*c^3)*d*e^2 - 4*(B^2*a^2*c^2 + A^2*a*c^3)*d^2*f + 2*(B
^2*a^3*c + A^2*a^2*c^2)*d*f^2 + ((B^2*a*c^3 + A^2*c^4)*d^2*e + (B
^2*a^2*c^2 + A^2*a*c^3)*e^3 - 2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e*f +
 (B^2*a^3*c + A^2*a^2*c^2)*e*f^2)*x)*sqrt(-(4*A^2*B^2*c^2*d^2 + 4
*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A
^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3
*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^
4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4
*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx}{(a + cx^2)\sqrt{d + ex + fx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/(c*x**2+a)/(f*x**2+e*x+d)**(1/2),x)

[Out] Integral((A + B*x)/((a + c*x**2)*sqrt(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + a)*sqrt(f*x^2 + e*x + d)),x, algorithm="giac")

[Out] Exception raised: RuntimeError
```

3.23 $\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}}\,dx$

Optimal. Leaf size=302

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2cd-fx\left(b-\sqrt{b^2-4ac}\right)}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\tanh^{-1}\left(\frac{2cd-fx\left(\sqrt{b^2-4ac}+b\right)}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf\left(\sqrt{b^2-4ac}+b\right)-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf\left(\sqrt{b^2-4ac}+b\right)-2acf+2c^2d}}$$

```
[Out] ((b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*d - (b - Sqrt[b
^2 - 4*a*c])*f*x)/(Sqrt[2]*Sqrt[2*c^2*d - 2*a*c*f + b*(b - Sqrt[b
^2 - 4*a*c])*f]*Sqrt[d + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqr
t[2*c^2*d - 2*a*c*f + b*(b - Sqrt[b^2 - 4*a*c])*f]) + ((2*A*c - B
*(b + Sqrt[b^2 - 4*a*c]))*ArcTanh[(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*f*x)/(Sqrt[2]*Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c]
)*f]*Sqrt[d + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d -
 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f])
```

Rubi [A] time = 2.14318, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2cd-fx\left(b-\sqrt{b^2-4ac}\right)}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\tanh^{-1}\left(\frac{2cd-fx\left(\sqrt{b^2-4ac}+b\right)}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf\left(\sqrt{b^2-4ac}+b\right)-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf\left(\sqrt{b^2-4ac}+b\right)-2acf+2c^2d}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] ((b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*d - (b - Sqrt[b
^2 - 4*a*c])*f*x)/(Sqrt[2]*Sqrt[2*c^2*d - 2*a*c*f + b*(b - Sqrt[b
^2 - 4*a*c])*f]*Sqrt[d + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqr
t[2*c^2*d - 2*a*c*f + b*(b - Sqrt[b^2 - 4*a*c])*f]) + ((2*A*c - B
*(b + Sqrt[b^2 - 4*a*c]))*ArcTanh[(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*f*x)/(Sqrt[2]*Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c]
)*f]*Sqrt[d + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d -
 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f])
```

Rubi in Sympy [A] time = 64.4208, size = 308, normalized size = 1.02

$$-\frac{\sqrt{2}\left(2Ac - B\left(b - \sqrt{-4ac + b^2}\right)\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(2cd - fx\left(b - \sqrt{-4ac + b^2}\right)\right)}{2\sqrt{d + fx^2}\sqrt{-2acf + b^2f - bf\sqrt{-4ac + b^2} + 2c^2d}}\right)}{2\sqrt{-4ac + b^2}\sqrt{-2acf + b^2f - bf\sqrt{-4ac + b^2} + 2c^2d}} + \frac{\sqrt{2}\left(2Ac - B\left(b + \sqrt{-4ac + b^2}\right)\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(2cd - fx\left(b + \sqrt{-4ac + b^2}\right)\right)}{2\sqrt{d + fx^2}\sqrt{-2acf + b^2f + bf\sqrt{-4ac + b^2} + 2c^2d}}\right)}{2\sqrt{-4ac + b^2}\sqrt{-2acf + b^2f + bf\sqrt{-4ac + b^2} + 2c^2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+d)**(1/2),x)
```

```
[Out] -sqrt(2)*(2*A*c - B*(b - sqrt(-4*a*c + b**2)))*atanh(sqrt(2)*(2*c
*d - f*x*(b - sqrt(-4*a*c + b**2)))/(2*sqrt(d + f*x**2)*sqrt(-2*a
*c*f + b**2*f - b*f*sqrt(-4*a*c + b**2) + 2*c**2*d)))/(2*sqrt(-4*
a*c + b**2)*sqrt(-2*a*c*f + b**2*f - b*f*sqrt(-4*a*c + b**2) + 2*
c**2*d)) + sqrt(2)*(2*A*c - B*(b + sqrt(-4*a*c + b**2)))*atanh(sq
rt(2)*(2*c*d - f*x*(b + sqrt(-4*a*c + b**2)))/(2*sqrt(d + f*x**2)
*sqrt(-2*a*c*f + b**2*f + b*f*sqrt(-4*a*c + b**2) + 2*c**2*d)))/(
2*sqrt(-4*a*c + b**2)*sqrt(-2*a*c*f + b**2*f + b*f*sqrt(-4*a*c +
b**2) + 2*c**2*d))
```

Mathematica [A] time = 1.20744, size = 512, normalized size = 1.7

$$-\frac{\left(B\sqrt{b^2-4ac}+2Ac-bB\right)\log\left(\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+fx^2}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}+2cd\sqrt{b^2-4ac}-bfx\sqrt{b^2-4ac}-4acfx+b^2fx\right)}{\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}} - \frac{\left(B\sqrt{b^2-4ac}-2Ac+bB\right)\ldots}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] (((-(b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*
c] - 2*c*x])/Sqrt[2*c^2*d - 2*a*c*f + b*(b - Sqrt[b^2 - 4*a*c])*f
] + ((b*B - 2*A*c + B*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c
] + 2*c*x])/Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f]
 - ((-(b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*Log[2*c*Sqrt[b^2 - 4*a
*c]*d + b^2*f*x - 4*a*c*f*x - b*Sqrt[b^2 - 4*a*c]*f*x + Sqrt[2]*S
qrt[b^2 - 4*a*c]*Sqrt[2*c^2*d - 2*a*c*f + b*(b - Sqrt[b^2 - 4*a*c
])*f]*Sqrt[d + f*x^2]])/Sqrt[2*c^2*d - 2*a*c*f + b*(b - Sqrt[b^2
- 4*a*c])*f] - ((b*B - 2*A*c + B*Sqrt[b^2 - 4*a*c])*Log[2*c*Sqrt[
b^2 - 4*a*c]*d - b^2*f*x + 4*a*c*f*x - b*Sqrt[b^2 - 4*a*c]*f*x +
Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^
2 - 4*a*c])*f]*Sqrt[d + f*x^2]])/Sqrt[2*c^2*d - 2*a*c*f + b*(b +
Sqrt[b^2 - 4*a*c])*f])/(Sqrt[2]*Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.067, size = 1771, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x)
```

```
[Out] 2/(-4*a*c+b^2)^(1/2)/(-2*(-b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2
*c^2*d)/c^2)^(1/2)*ln((-(-b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*
c^2*d)/c^2-f*(b+(-4*a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)+1/2*(-2*(-b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2
)^(1/2)*(4*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*f-4*f*(b+(-4*a*c+b^
2)^(1/2))/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)-2*(-b*f*(-4*a*c+b^2)
^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2))/(x+1/2*(b+(-4*a*c+b^2)^
(1/2))/c))*A-1/c/(-2*(-b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2
*d)/c^2)^(1/2)*ln((-(-b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*
d)/c^2-f*(b+(-4*a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c
)+1/2*(-2*(-b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1
/2)*(4*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*f-4*f*(b+(-4*a*c+b^2)^(
1/2))/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)-2*(-b*f*(-4*a*c+b^2)^(1/
2)+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2))/(x+1/2*(b+(-4*a*c+b^2)^(1/2
))/c))*B-1/(-4*a*c+b^2)^(1/2)/c/(-2*(-b*f*(-4*a*c+b^2)^(1/2)+2*a*
c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*ln((-(-b*f*(-4*a*c+b^2)^(1/2)+2*a*c
*f-b^2*f-2*c^2*d)/c^2-f*(b+(-4*a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*a*
c+b^2)^(1/2))/c)+1/2*(-2*(-b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2
*c^2*d)/c^2)^(1/2)*(4*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*f-4*f*(b
+(-4*a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)-2*(-b*f*(
-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2))/(x+1/2*(b+(-
4*a*c+b^2)^(1/2))/c))*b*B-2/(-4*a*c+b^2)^(1/2)/(-2*(b*f*(-4*a*c+b
^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*ln((-(b*f*(-4*a*c+b^2
)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b-(-4*a*c+b^2)^(1/2))/c*(x-
1/2/c*(-b+(-4*a*c+b^2)^(1/2)))+1/2*(-2*(b*f*(-4*a*c+b^2)^(1/2)+2*
a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*(4*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2
)))^2*f-4*f*(b-(-4*a*c+b^2)^(1/2))/c*(x-1/2/c*(-b+(-4*a*c+b^2)^(1
/2)))-2*(b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)
)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))*A-1/c/(-2*(b*f*(-4*a*c+b^2)^
(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*ln((-(b*f*(-4*a*c+b^2)^(1
/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b-(-4*a*c+b^2)^(1/2))/c*(x-1/2/
c*(-b+(-4*a*c+b^2)^(1/2)))+1/2*(-2*(b*f*(-4*a*c+b^2)^(1/2)+2*a*c*
f-b^2*f-2*c^2*d)/c^2)^(1/2)*(4*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))^
2*f-4*f*(b-(-4*a*c+b^2)^(1/2))/c*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))
)-2*(b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2))/(x
-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))*B+1/(-4*a*c+b^2)^(1/2)/c/(-2*(b*
f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*ln((-(b*f*
(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b-(-4*a*c+b^2)^(
1/2))/c*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))+1/2*(-2*(b*f*(-4*a*c+b^
2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*(4*(x-1/2/c*(-b+(-4*a*
c+b^2)^(1/2)))^2*f-4*f*(b-(-4*a*c+b^2)^(1/2))/c*(x-1/2/c*(-b+(-4*
a*c+b^2)^(1/2)))-2*(b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)
/c^2)^(1/2))/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))*b*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + b*x + a)*sqrt(f*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 56.0292, size = 12119, normalized size = 40.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + b*x + a)*sqrt(f*x^2 + d)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d +
 (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f + ((b^2*c^2 - 4*
```

```
a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3
*c)*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2
*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*
A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^
4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*
c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2
*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*
a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3
*c)*f^2))*log((2*(B^4*a*b^2 - A*B^3*b^3 - 2*A^3*B*b*c^2 - (2*A*B^
3*a*b - 3*A^2*B^2*b^2)*c)*d^2 + 2*(2*A*B^3*a^2*b - 3*A^2*B^2*a*b^
2 + A^3*B*b^3 + (2*A^3*B*a*b - A^4*b^2)*c)*d*f + sqrt(2)*((B^3*b^
4 - 8*A^2*B*a*c^3 + 2*(6*A*B^2*a*b + A^2*B*b^2)*c^2 - (4*B^3*a*b^
2 + 3*A*B^2*b^3)*c)*d^2 + (3*A*B^2*a*b^3 - A^2*B*b^4 + 4*(4*A^2*B
*a^2 - A^3*a*b)*c^2 - (12*A*B^2*a^2*b - A^3*b^3)*c)*d*f + (2*A^2*
B*a^2*b^2 - A^3*a*b^3 - 4*(2*A^2*B*a^3 - A^3*a^2*b)*c)*f^2 - ((B*
b^4*c^2 + 4*(2*B*a^2 + A*a*b)*c^4 - (6*B*a*b^2 + A*b^3)*c^3)*d^3
+ (B*b^6 - 4*(6*B*a^3 + A*a^2*b)*c^3 + (22*B*a^2*b^2 + 5*A*a*b^3)
*c^2 - (8*B*a*b^4 + A*b^5)*c)*d^2*f + (3*B*a^2*b^4 - A*a*b^5 + 4*
(6*B*a^4 - A*a^3*b)*c^2 - (18*B*a^3*b^2 - 5*A*a^2*b^3)*c)*d*f^2 +
 (2*B*a^4*b^2 - A*a^3*b^3 - 4*(2*B*a^5 - A*a^4*b)*c)*f^3)*sqrt(((
B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2
*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*
A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 -
 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c
^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*
d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))*sqrt(f*x^2 + d)*sqrt(((B^2*b^2
 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b +
A^2*b^2 - 2*A^2*a*c)*f + ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^
2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)*sqrt(((B^4*b^2 -
4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 -
 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b
+ A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c
^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^
3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a
^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^
2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)) - 4*((B^4*a^2*b
- A*B^3*a*b^2 - 2*A^3*B*a*c^2 - (2*A*B^3*a^2 - 3*A^2*B^2*a*b)*c)*
d*f + (2*A*B^3*a^3 - 3*A^2*B^2*a^2*b + A^3*B*a*b^2 + (2*A^3*B*a^2
 - A^4*a*b)*c)*f^2)*x + 2*((4*A^2*a*c^4 + (4*B^2*a^2 - 4*A*B*a*b
- A^2*b^2)*c^3 - (B^2*a*b^2 - A*B*b^3)*c^2)*d^3 - (B^2*a*b^4 - A*
B*b^5 + 8*A^2*a^2*c^3 + 2*(4*B^2*a^3 - 4*A*B*a^2*b - 3*A^2*a*b^2)
*c^2 - (6*B^2*a^2*b^2 - 6*A*B*a*b^3 - A^2*b^4)*c)*d^2*f - (B^2*a^
3*b^2 - A*B*a^2*b^3 - 4*A^2*a^3*c^2 - (4*B^2*a^4 - 4*A*B*a^3*b -
A^2*a^2*b^2)*c)*d*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c
^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b
)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4
 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (
b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b
^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/
x) - 1/4*sqrt(2)*sqrt(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c
)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f + ((b^2*c^2
 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 -
4*a^3*c)*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 +
 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f
+ (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^
5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a
*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^
3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2
 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 -
4*a^3*c)*f^2))*log((2*(B^4*a*b^2 - A*B^3*b^3 - 2*A^3*B*b*c^2 - (2
*A*B^3*a*b - 3*A^2*B^2*b^2)*c)*d^2 + 2*(2*A*B^3*a^2*b - 3*A^2*B^2
*a*b^2 + A^3*B*b^3 + (2*A^3*B*a*b - A^4*b^2)*c)*d*f - sqrt(2)*((B
^3*b^4 - 8*A^2*B*a*c^3 + 2*(6*A*B^2*a*b + A^2*B*b^2)*c^2 - (4*B^3
*a*b^2 + 3*A*B^2*b^3)*c)*d^2 + (3*A*B^2*a*b^3 - A^2*B*b^4 + 4*(4*
A^2*B*a^2 - A^3*a*b)*c^2 - (12*A*B^2*a^2*b - A^3*b^3)*c)*d*f + (2
*A^2*B*a^2*b^2 - A^3*a*b^3 - 4*(2*A^2*B*a^3 - A^3*a^2*b)*c)*f^2 -
 ((B*b^4*c^2 + 4*(2*B*a^2 + A*a*b)*c^4 - (6*B*a*b^2 + A*b^3)*c^3)
*d^3 + (B*b^6 - 4*(6*B*a^3 + A*a^2*b)*c^3 + (22*B*a^2*b^2 + 5*A*a
*b^3)*c^2 - (8*B*a*b^4 + A*b^5)*c)*d^2*f + (3*B*a^2*b^4 - A*a*b^5
 + 4*(6*B*a^4 - A*a^3*b)*c^2 - (18*B*a^3*b^2 - 5*A*a^2*b^3)*c)*d*
f^2 + (2*B*a^4*b^2 - A*a^3*b^3 - 4*(2*B*a^5 - A*a^4*b)*c)*f^3)*sq
rt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b
```

```
- A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2
 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*
c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*
b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*
c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))*sqrt(f*x^2 + d)*sqrt(((B^
2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a
*b + A^2*b^2 - 2*A^2*a*c)*f + ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6
*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)*sqrt(((B^4*b
^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*
b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B
*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*
b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 -
24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3
 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6
*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)) - 4*((B^4*a
^2*b - A*B^3*a*b^2 - 2*A^3*B*a*c^2 - (2*A*B^3*a^2 - 3*A^2*B^2*a*b
)*c)*d*f + (2*A*B^3*a^3 - 3*A^2*B^2*a^2*b + A^3*B*a*b^2 + (2*A^3*
B*a^2 - A^4*a*b)*c)*f^2)*x + 2*((4*A^2*a*c^4 + (4*B^2*a^2 - 4*A*B
*a*b - A^2*b^2)*c^3 - (B^2*a*b^2 - A*B*b^3)*c^2)*d^3 - (B^2*a*b^4
 - A*B*b^5 + 8*A^2*a^2*c^3 + 2*(4*B^2*a^3 - 4*A*B*a^2*b - 3*A^2*a
*b^2)*c^2 - (6*B^2*a^2*b^2 - 6*A*B*a*b^3 - A^2*b^4)*c)*d^2*f - (B
^2*a^3*b^2 - A*B*a^2*b^3 - 4*A^2*a^3*c^2 - (4*B^2*a^4 - 4*A*B*a^3
*b - A^2*a^2*b^2)*c)*d*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*
B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^
3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^
2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*
f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(
a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^
4)))/x) + 1/4*sqrt(2)*sqrt(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B
*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f - ((b^
2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b
^2 - 4*a^3*c)*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*
d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)
*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4
*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6
- 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 -
 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^
2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b
^2 - 4*a^3*c)*f^2))*log((2*(B^4*a*b^2 - A*B^3*b^3 - 2*A^3*B*b*c^2
 - (2*A*B^3*a*b - 3*A^2*B^2*b^2)*c)*d^2 + 2*(2*A*B^3*a^2*b - 3*A^
2*B^2*a*b^2 + A^3*B*b^3 + (2*A^3*B*a*b - A^4*b^2)*c)*d*f + sqrt(2
)*((B^3*b^4 - 8*A^2*B*a*c^3 + 2*(6*A*B^2*a*b + A^2*B*b^2)*c^2 - (
4*B^3*a*b^2 + 3*A*B^2*b^3)*c)*d^2 + (3*A*B^2*a*b^3 - A^2*B*b^4 +
4*(4*A^2*B*a^2 - A^3*a*b)*c^2 - (12*A*B^2*a^2*b - A^3*b^3)*c)*d*f
 + (2*A^2*B*a^2*b^2 - A^3*a*b^3 - 4*(2*A^2*B*a^3 - A^3*a^2*b)*c)*
f^2 + ((B*b^4*c^2 + 4*(2*B*a^2 + A*a*b)*c^4 - (6*B*a*b^2 + A*b^3)
*c^3)*d^3 + (B*b^6 - 4*(6*B*a^3 + A*a^2*b)*c^3 + (22*B*a^2*b^2 +
5*A*a*b^3)*c^2 - (8*B*a*b^4 + A*b^5)*c)*d^2*f + (3*B*a^2*b^4 - A*
a*b^5 + 4*(6*B*a^4 - A*a^3*b)*c^2 - (18*B*a^3*b^2 - 5*A*a^2*b^3)*
c)*d*f^2 + (2*B*a^4*b^2 - A*a^3*b^3 - 4*(2*B*a^5 - A*a^4*b)*c)*f^
3)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3
*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^
2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*
(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22
*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8
*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))*sqrt(f*x^2 + d)*sqrt
(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*
A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f - ((b^2*c^2 - 4*a*c^3)*d^2 + (b^
4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)*sqrt(((
B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2
*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*
A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 -
 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c
^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*
d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^
4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)) - 4*((
B^4*a^2*b - A*B^3*a*b^2 - 2*A^3*B*a*c^2 - (2*A*B^3*a^2 - 3*A^2*B^
2*a*b)*c)*d*f + (2*A*B^3*a^3 - 3*A^2*B^2*a^2*b + A^3*B*a*b^2 + (2
*A^3*B*a^2 - A^4*a*b)*c)*f^2)*x - 2*((4*A^2*a*c^4 + (4*B^2*a^2 -
4*A*B*a*b - A^2*b^2)*c^3 - (B^2*a*b^2 - A*B*b^3)*c^2)*d^3 - (B^2*
a*b^4 - A*B*b^5 + 8*A^2*a^2*c^3 + 2*(4*B^2*a^3 - 4*A*B*a^2*b - 3*
A^2*a*b^2)*c^2 - (6*B^2*a^2*b^2 - 6*A*B*a*b^3 - A^2*b^4)*c)*d^2*f
```

```
 - (B^2*a^3*b^2 - A*B*a^2*b^3 - 4*A^2*a^3*c^2 - (4*B^2*a^4 - 4*A*
B*a^3*b - A^2*a^2*b^2)*c)*d*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4
*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a
 - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)
/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)
*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2
+ 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*
c)*f^4)))/x) - 1/4*sqrt(2)*sqrt(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a
+ A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f -
 ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (
a^2*b^2 - 4*a^3*c)*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*
c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*
b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^
4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f +
(b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*
b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))
/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (
a^2*b^2 - 4*a^3*c)*f^2))*log((2*(B^4*a*b^2 - A*B^3*b^3 - 2*A^3*B*
b*c^2 - (2*A*B^3*a*b - 3*A^2*B^2*b^2)*c)*d^2 + 2*(2*A*B^3*a^2*b -
 3*A^2*B^2*a*b^2 + A^3*B*b^3 + (2*A^3*B*a*b - A^4*b^2)*c)*d*f - s
qrt(2)*((B^3*b^4 - 8*A^2*B*a*c^3 + 2*(6*A*B^2*a*b + A^2*B*b^2)*c^
2 - (4*B^3*a*b^2 + 3*A*B^2*b^3)*c)*d^2 + (3*A*B^2*a*b^3 - A^2*B*b
^4 + 4*(4*A^2*B*a^2 - A^3*a*b)*c^2 - (12*A*B^2*a^2*b - A^3*b^3)*c
)*d*f + (2*A^2*B*a^2*b^2 - A^3*a*b^3 - 4*(2*A^2*B*a^3 - A^3*a^2*b
)*c)*f^2 + ((B*b^4*c^2 + 4*(2*B*a^2 + A*a*b)*c^4 - (6*B*a*b^2 + A
*b^3)*c^3)*d^3 + (B*b^6 - 4*(6*B*a^3 + A*a^2*b)*c^3 + (22*B*a^2*b
^2 + 5*A*a*b^3)*c^2 - (8*B*a*b^4 + A*b^5)*c)*d^2*f + (3*B*a^2*b^4
 - A*a*b^5 + 4*(6*B*a^4 - A*a^3*b)*c^2 - (18*B*a^3*b^2 - 5*A*a^2*
b^3)*c)*d*f^2 + (2*B*a^4*b^2 - A*a^3*b^3 - 4*(2*B*a^5 - A*a^4*b)*
c)*f^3)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*
A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A
^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4
 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c
 + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*
c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))*sqrt(f*x^2 + d)
*sqrt(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2
 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f - ((b^2*c^2 - 4*a*c^3)*d^2
+ (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)*sq
rt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b
- A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2
 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*
c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*
b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*
c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*a*c^3)*d^2
+ (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)) -
 4*((B^4*a^2*b - A*B^3*a*b^2 - 2*A^3*B*a*c^2 - (2*A*B^3*a^2 - 3*A
^2*B^2*a*b)*c)*d*f + (2*A*B^3*a^3 - 3*A^2*B^2*a^2*b + A^3*B*a*b^2
 + (2*A^3*B*a^2 - A^4*a*b)*c)*f^2)*x - 2*((4*A^2*a*c^4 + (4*B^2*a
^2 - 4*A*B*a*b - A^2*b^2)*c^3 - (B^2*a*b^2 - A*B*b^3)*c^2)*d^3 -
(B^2*a*b^4 - A*B*b^5 + 8*A^2*a^2*c^3 + 2*(4*B^2*a^3 - 4*A*B*a^2*b
 - 3*A^2*a*b^2)*c^2 - (6*B^2*a^2*b^2 - 6*A*B*a*b^3 - A^2*b^4)*c)*
d^2*f - (B^2*a^3*b^2 - A*B*a^2*b^3 - 4*A^2*a^3*c^2 - (4*B^2*a^4 -
 4*A*B*a^3*b - A^2*a^2*b^2)*c)*d*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*
c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*
B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)
*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2
*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2
*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4
*a^5*c)*f^4)))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx}{\sqrt{d + fx^2}\,(a + bx + cx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+d)**(1/2),x)
```

```
[Out] Integral((A + B*x)/(sqrt(d + f*x**2)*(a + b*x + c*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx + A}{(cx^2 + bx + a)\sqrt{fx^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + b*x + a)*sqrt(f*x^2 + d)),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)/((c*x^2 + b*x + a)*sqrt(f*x^2 + d)), x)
```

3.24 $\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx$

Optimal. Leaf size=101

$$\frac{A\tan^{-1}\left(\frac{x\sqrt{cd-af}}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}}-\frac{B\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}}$$

```
[Out] (A*ArcTan[(Sqrt[c*d - a*f]*x)/(Sqrt[a]*Sqrt[d + f*x^2])])/(Sqrt[a
]*Sqrt[c*d - a*f]) - (B*ArcTanh[(Sqrt[c]*Sqrt[d + f*x^2])/Sqrt[c*
d - a*f]])/(Sqrt[c]*Sqrt[c*d - a*f])
```

Rubi [A] time = 0.318022, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{A\tan^{-1}\left(\frac{x\sqrt{cd-af}}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}}-\frac{B\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/((a + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] (A*ArcTan[(Sqrt[c*d - a*f]*x)/(Sqrt[a]*Sqrt[d + f*x^2])])/(Sqrt[a
]*Sqrt[c*d - a*f]) - (B*ArcTanh[(Sqrt[c]*Sqrt[d + f*x^2])/Sqrt[c*
d - a*f]])/(Sqrt[c]*Sqrt[c*d - a*f])
```

Rubi in Sympy [A] time = 28.9055, size = 155, normalized size = 1.53

$$-\frac{\left(A\sqrt{c}-B\sqrt{-a}\right)\operatorname{atan}\left(\frac{\sqrt{c}d-fx\sqrt{-a}}{\sqrt{d+fx^2}\sqrt{af-cd}}\right)}{2\sqrt{c}\sqrt{-a}\sqrt{af-cd}}+\frac{\left(A\sqrt{c}+B\sqrt{-a}\right)\operatorname{atan}\left(\frac{\sqrt{c}d+fx\sqrt{-a}}{\sqrt{d+fx^2}\sqrt{af-cd}}\right)}{2\sqrt{c}\sqrt{-a}\sqrt{af-cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/(c*x**2+a)/(f*x**2+d)**(1/2),x)
```

```
[Out] -(A*sqrt(c) - B*sqrt(-a))*atan((sqrt(c)*d - f*x*sqrt(-a))/(sqrt(d
 + f*x**2)*sqrt(a*f - c*d)))/(2*sqrt(c)*sqrt(-a)*sqrt(a*f - c*d))
 + (A*sqrt(c) + B*sqrt(-a))*atan((sqrt(c)*d + f*x*sqrt(-a))/(sqrt
(d + f*x**2)*sqrt(a*f - c*d)))/(2*sqrt(c)*sqrt(-a)*sqrt(a*f - c*d
))
```

Mathematica [C] time = 0.773739, size = 282, normalized size = 2.79

$$\frac{\left(-\sqrt{a}B+iA\sqrt{c}\right)\log\left(\frac{2i\sqrt{a}\sqrt{c}\left(\sqrt{d+fx^2}\sqrt{cd-af}+i\sqrt{a}fx+\sqrt{c}d\right)}{\left(\sqrt{a}+i\sqrt{c}x\right)\left(\sqrt{a}B-iA\sqrt{c}\right)\sqrt{cd-af}}\right)-\left(\sqrt{a}B+iA\sqrt{c}\right)\log\left(\frac{2\sqrt{a}\sqrt{c}\left(\sqrt{d+fx^2}\sqrt{cd-af}-i\sqrt{a}fx+\sqrt{c}d\right)}{\left(\sqrt{c}x+i\sqrt{a}\right)\left(\sqrt{a}B+iA\sqrt{c}\right)\sqrt{cd-af}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{cd-af}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/((a + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] (-((Sqrt[a]*B + I*A*Sqrt[c])*Log[(2*Sqrt[a]*Sqrt[c]*(Sqrt[c]*d -
I*Sqrt[a]*f*x + Sqrt[c*d - a*f]*Sqrt[d + f*x^2]))/((Sqrt[a]*B + I
*A*Sqrt[c])*Sqrt[c*d - a*f]*(I*Sqrt[a] + Sqrt[c]*x))]) + (-(Sqrt[
a]*B) + I*A*Sqrt[c])*Log[((2*I)*Sqrt[a]*Sqrt[c]*(Sqrt[c]*d + I*Sq
rt[a]*f*x + Sqrt[c*d - a*f]*Sqrt[d + f*x^2]))/((Sqrt[a]*B - I*A*S
qrt[c])*Sqrt[c*d - a*f]*(Sqrt[a] + I*Sqrt[c]*x))])/(2*Sqrt[a]*Sqr
t[c]*Sqrt[c*d - a*f])
```

Maple [B] time = 0.044, size = 608, normalized size = 6.

$$\frac{A}{2}\ln\left(1\left(-2\frac{fa-cd}{c}-2\frac{f\sqrt{-ac}}{c}\left(x+\frac{\sqrt{-ac}}{c}\right)+2\sqrt{-\frac{fa-cd}{c}}\sqrt{\left(x+\frac{\sqrt{-ac}}{c}\right)^{2}f-2\frac{f\sqrt{-ac}}{c}\left(x+\frac{\sqrt{-ac}}{c}\right)-\frac{fa-cd}{c}}\right)\left(x\right.\right.$$

$$-\frac{B}{2c}\ln\left(1\left(-2\frac{fa-cd}{c}-2\frac{f\sqrt{-ac}}{c}\left(x+\frac{\sqrt{-ac}}{c}\right)+2\sqrt{-\frac{fa-cd}{c}}\sqrt{\left(x+\frac{\sqrt{-ac}}{c}\right)^{2}f-2\frac{f\sqrt{-ac}}{c}\left(x+\frac{\sqrt{-ac}}{c}\right)-\frac{fa-cd}{c}}\right)\right.$$

$$-\frac{A}{2}\ln\left(1\left(-2\frac{fa-cd}{c}+2\frac{f\sqrt{-ac}}{c}\left(x-\frac{\sqrt{-ac}}{c}\right)+2\sqrt{-\frac{fa-cd}{c}}\sqrt{\left(x-\frac{\sqrt{-ac}}{c}\right)^{2}f+2\frac{f\sqrt{-ac}}{c}\left(x-\frac{\sqrt{-ac}}{c}\right)-\frac{fa-cd}{c}}\right)\right.$$

$$-\frac{B}{2c}\ln\left(1\left(-2\frac{fa-cd}{c}+2\frac{f\sqrt{-ac}}{c}\left(x-\frac{\sqrt{-ac}}{c}\right)+2\sqrt{-\frac{fa-cd}{c}}\sqrt{\left(x-\frac{\sqrt{-ac}}{c}\right)^{2}f+2\frac{f\sqrt{-ac}}{c}\left(x-\frac{\sqrt{-ac}}{c}\right)-\frac{fa-cd}{c}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x)
```

```
[Out] 1/2/(-a*c)^(1/2)/(-(a*f-c*d)/c)^(1/2)*ln((-2*(a*f-c*d)/c-2*f*(-a*
c)^(1/2)/c*(x+(-a*c)^(1/2)/c)+2*(-(a*f-c*d)/c)^(1/2)*((x+(-a*c)^(
1/2)/c)^2*f-2*f*(-a*c)^(1/2)/c*(x+(-a*c)^(1/2)/c)-(a*f-c*d)/c)^(1
/2))/(x+(-a*c)^(1/2)/c))*A-1/2/c/(-(a*f-c*d)/c)^(1/2)*ln((-2*(a*f
-c*d)/c-2*f*(-a*c)^(1/2)/c*(x+(-a*c)^(1/2)/c)+2*(-(a*f-c*d)/c)^(1
/2)*((x+(-a*c)^(1/2)/c)^2*f-2*f*(-a*c)^(1/2)/c*(x+(-a*c)^(1/2)/c)
-(a*f-c*d)/c)^(1/2))/(x+(-a*c)^(1/2)/c))*B-1/2/(-a*c)^(1/2)/(-(a*
f-c*d)/c)^(1/2)*ln((-2*(a*f-c*d)/c+2*f*(-a*c)^(1/2)/c*(x-(-a*c)^(
1/2)/c)+2*(-(a*f-c*d)/c)^(1/2)*((x-(-a*c)^(1/2)/c)^2*f+2*f*(-a*c)
^(1/2)/c*(x-(-a*c)^(1/2)/c)-(a*f-c*d)/c)^(1/2))/(x-(-a*c)^(1/2)/c
))*A-1/2/c/(-(a*f-c*d)/c)^(1/2)*ln((-2*(a*f-c*d)/c+2*f*(-a*c)^(1/
2)/c*(x-(-a*c)^(1/2)/c)+2*(-(a*f-c*d)/c)^(1/2)*((x-(-a*c)^(1/2)/c
)^2*f+2*f*(-a*c)^(1/2)/c*(x-(-a*c)^(1/2)/c)-(a*f-c*d)/c)^(1/2))/(
x-(-a*c)^(1/2)/c))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/((c*x^2 + a)*sqrt(f*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.399553, size = 2045, normalized size = 20.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + a)*sqrt(f*x^2 + d)),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a
*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f))*log(
((A*B^3*a + A^3*B*c)*f*x + (A^2*B*c^2*d - A^2*B*a*c*f + (B*a*c^3*
d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2
*a^2*c^2*d*f + a^3*c*f^2)))*sqrt(f*x^2 + d)*sqrt((B^2*a - A^2*c +
 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f +
 a^3*c*f^2)))/(a*c^2*d - a^2*c*f)) + sqrt(-A^2*B^2/(a*c^3*d^2 - 2
*a^2*c^2*d*f + a^3*c*f^2))*((B^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*
c + A^2*a*c^2)*d*f))/x) + 1/4*sqrt((B^2*a - A^2*c + 2*(a*c^2*d -
a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/
(a*c^2*d - a^2*c*f))*log(((A*B^3*a + A^3*B*c)*f*x - (A^2*B*c^2*d
- A^2*B*a*c*f + (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*sqr
t(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))*sqrt(f*x^2 +
 d)*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*
c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f)) + sqr
t(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*((B^2*a*c^2 +
 A^2*c^3)*d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f))/x) - 1/4*sqrt((B^2*
a - A^2*c - 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^
2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f))*log(((A*B^3*a + A^3
*B*c)*f*x + (A^2*B*c^2*d - A^2*B*a*c*f - (B*a*c^3*d^2 - 2*B*a^2*c
^2*d*f + B*a^3*c*f^2)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f +
a^3*c*f^2)))*sqrt(f*x^2 + d)*sqrt((B^2*a - A^2*c - 2*(a*c^2*d - a
^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(
a*c^2*d - a^2*c*f)) - sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f +
a^3*c*f^2))*((B^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A^2*a*c^2)*
d*f))/x) + 1/4*sqrt((B^2*a - A^2*c - 2*(a*c^2*d - a^2*c*f)*sqrt(-
A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*
c*f))*log(((A*B^3*a + A^3*B*c)*f*x - (A^2*B*c^2*d - A^2*B*a*c*f -
 (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*sqrt(-A^2*B^2/(a*c
^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))*sqrt(f*x^2 + d)*sqrt((B^2*a
 - A^2*c - 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2
*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f)) - sqrt(-A^2*B^2/(a*c
^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*((B^2*a*c^2 + A^2*c^3)*d^2 -
 (B^2*a^2*c + A^2*a*c^2)*d*f))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+a)/(f*x**2+d)**(1/2),x)
```

```
[Out] Integral((A + B*x)/((a + c*x**2)*sqrt(d + f*x**2)), x)
```

GIAC/XCAS [A] time = 1.28648, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((c*x^2 + a)*sqrt(f*x^2 + d)),x, algorithm="giac")
```

```
[Out] Done
```

3.25 $\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}}\,dx$

Optimal. Leaf size=139

$$\frac{1}{2}\sqrt{\sqrt{10}-\frac{13}{5}}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{1}{2}\sqrt{\frac{13}{5}+\sqrt{10}}\tanh^{-1}\left(\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)$$

```
[Out] (Sqrt[-13/5 + Sqrt[10]]*ArcTan[(3*(4 - Sqrt[10]) + (1 + 4*Sqrt[10
])*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2 + (Sqrt[13
/5 + Sqrt[10]]*ArcTanh[(3*(4 + Sqrt[10]) + (1 - 4*Sqrt[10])*x)/(2
*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2
```

Rubi [A] time = 0.484639, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{1}{2}\sqrt{\frac{1}{5}\left(5\sqrt{10}-13\right)}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(13+5\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x - 2*x^2]),x]
```

```
[Out] (Sqrt[(-13 + 5*Sqrt[10])/5]*ArcTan[(3*(4 - Sqrt[10]) + (1 + 4*Sqr
t[10])*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2 + (Sqr
t[(13 + 5*Sqrt[10])/5]*ArcTanh[(3*(4 + Sqrt[10]) + (1 - 4*Sqrt[10
])*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2
```

Rubi in Sympy [A] time = 39.8429, size = 144, normalized size = 1.04

$$-\frac{\sqrt{10}\left(-2\sqrt{10}+16\right)\operatorname{atan}\left(\frac{x\left(-8\sqrt{10}-2\right)-24+6\sqrt{10}}{4\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)}{40\sqrt{1+\sqrt{10}}}-\frac{\sqrt{10}\left(2\sqrt{10}+16\right)\operatorname{atanh}\left(\frac{x\left(-2+8\sqrt{10}\right)-24-6\sqrt{10}}{4\sqrt{-1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)}{40\sqrt{-1+\sqrt{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(1/2),x)
```

```
[Out] -sqrt(10)*(-2*sqrt(10) + 16)*atan((x*(-8*sqrt(10) - 2) - 24 + 6*s
qrt(10))/(4*sqrt(1 + sqrt(10))*sqrt(-2*x**2 + 3*x + 1)))/(40*sqrt
(1 + sqrt(10))) - sqrt(10)*(2*sqrt(10) + 16)*atanh((x*(-2 + 8*sqr
t(10)) - 24 - 6*sqrt(10))/(4*sqrt(-1 + sqrt(10))*sqrt(-2*x**2 + 3
*x + 1)))/(40*sqrt(-1 + sqrt(10)))
```

Mathematica [A] time = 0.674782, size = 161, normalized size = 1.16

$$\frac{-\sqrt{1+\sqrt{10}}\left(8+\sqrt{10}\right)\left(\log\left(-3x+\sqrt{10}+2\right)-\log\left(2\sqrt{10\left(\sqrt{10}-1\right)}\sqrt{-2x^2+3x+1}+\sqrt{10}x-40x+12\sqrt{10}+30\right)\right)-\left(\sqrt{\right.}{6\sqrt{10}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x - 2*x^2]),x]
```

```
[Out] (-((-8 + Sqrt[10])*Sqrt[-1 + Sqrt[10]]*ArcTan[(12 - 3*Sqrt[10] +
x + 4*Sqrt[10]*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])
- Sqrt[1 + Sqrt[10]]*(8 + Sqrt[10])*(Log[2 + Sqrt[10] - 3*x] - Lo
g[30 + 12*Sqrt[10] - 40*x + Sqrt[10]*x + 2*Sqrt[10*(-1 + Sqrt[10]
)]*Sqrt[1 + 3*x - 2*x^2]]))/(6*Sqrt[10])
```

Maple [B] time = 0.079, size = 324, normalized size = 2.3

$$\frac{2\sqrt{10}}{5\sqrt{1+\sqrt{10}}}\arctan\left(\frac{9}{2\sqrt{1+\sqrt{10}}}\left(-\frac{2}{9}-\frac{2\sqrt{10}}{9}+\left(\frac{1}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)\right)\frac{1}{\sqrt{-18\left(x-2/3+1/3\sqrt{10}\right)^{2}+9\left(1/3+4\right.}}\right.$$

$$-\frac{1}{2\sqrt{1+\sqrt{10}}}\arctan\left(\frac{9}{2\sqrt{1+\sqrt{10}}}\left(-\frac{2}{9}-\frac{2\sqrt{10}}{9}+\left(\frac{1}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)\right)\frac{}{\sqrt{-18\left(x-2/3+1/3\sqrt{10}\right)^{2}+9\left(1/3+\right.}}\right.$$

$$+\frac{2\sqrt{10}}{5\sqrt{-1+\sqrt{10}}}Artanh\left(\frac{9}{2\sqrt{-1+\sqrt{10}}}\left(-\frac{2}{9}+\frac{2\sqrt{10}}{9}+\left(\frac{1}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)\right)\frac{}{\sqrt{-18\left(x-2/3-1/3\sqrt{10}\right)^{2}+9\left(1/\right.}}\right.$$

$$+\frac{1}{2\sqrt{-1+\sqrt{10}}}Artanh\left(\frac{9}{2\sqrt{-1+\sqrt{10}}}\left(-\frac{2}{9}+\frac{2\sqrt{10}}{9}+\left(\frac{1}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)\right)\frac{}{\sqrt{-18\left(x-2/3-1/3\sqrt{10}\right)^{2}+9\left(1/\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x)
```

```
[Out] 2/5*10^(1/2)/(1+10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/
3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(1+10^(1/2))^(1/2)/(-18*(x-
2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1-1
0^(1/2))^(1/2))-1/2/(1+10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1
/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(1+10^(1/2))^(1/2)/(
-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/
2))-1-10^(1/2))^(1/2))+2/5*10^(1/2)/(-1+10^(1/2))^(1/2)*arctanh(9
/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(-
1+10^(1/2))^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/3*10^(1/2)
)*(x-2/3-1/3*10^(1/2))-1+10^(1/2))^(1/2))+1/2/(-1+10^(1/2))^(1/2)
*arctanh(9/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^
(1/2)))/(-1+10^(1/2))^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/
3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1+10^(1/2))^(1/2))
```

Maxima [A] time = 0.800441, size = 487, normalized size = 3.5

$$-\frac{1}{20}\sqrt{10}\left(\frac{\sqrt{10}\arcsin\left(\frac{8\sqrt{17}\sqrt{10}x}{17\left|6x+2\sqrt{10}-4\right|}+\frac{2\sqrt{17}x}{17\left|6x+2\sqrt{10}-4\right|}-\frac{6\sqrt{17}\sqrt{10}}{17\left|6x+2\sqrt{10}-4\right|}+\frac{24\sqrt{17}}{17\left|6x+2\sqrt{10}-4\right|}\right)}{\sqrt{\sqrt{10}+1}}-\frac{\sqrt{10}\log\left(-\frac{2}{9}\sqrt{10}+\frac{2\sqrt{-2x^{2}+3x+1}\sqrt{}}{3\left|6x-2\sqrt{10}-4\right|}\right.}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + 2)/((3*x^2 - 4*x - 2)*sqrt(-2*x^2 + 3*x + 1)),x, algorithm="maxima
```

```
[Out] -1/20*sqrt(10)*(sqrt(10)*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x
+ 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6
/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/
abs(6*x + 2*sqrt(10) - 4))/sqrt(sqrt(10) + 1) - sqrt(10)*log(-2/9
*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x
 - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9
/abs(6*x - 2*sqrt(10) - 4) + 1/18)/sqrt(sqrt(10) - 1) - 8*arcsin(
8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17
)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x +
2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/sqrt(
sqrt(10) + 1) - 8*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*
sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6
*x - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/sqrt
(sqrt(10) - 1))
```

Fricas [A] time = 0.300685, size = 466, normalized size = 3.35

$$2\sqrt{\frac{1}{10}}\sqrt{\sqrt{2}\left(10\sqrt{5}-13\sqrt{2}\right)}\arctan\left(-\frac{\sqrt{\frac{1}{10}}\left(7\sqrt{5}x+10\sqrt{2}x\right)\sqrt{\sqrt{2}\left(10\sqrt{5}-13\sqrt{2}\right)}}{9\left(\sqrt{2}(x+1)+\sqrt{5}x-x\sqrt{\frac{\sqrt{2}\left(\sqrt{2}(3x^2+5x+2)+\sqrt{5}(3x^2+2x)-2\sqrt{-2x^2+3x+1}\left(\sqrt{2}(x+1)+\sqrt{5}x\right)\right)}{x^2}}-\sqrt{\ldots}\right.}\right.$$

$$-\frac{1}{2}\sqrt{\frac{1}{10}}\sqrt{\sqrt{2}\left(10\sqrt{5}+13\sqrt{2}\right)}\log\left(-\frac{\sqrt{\frac{1}{10}}\left(7\sqrt{5}x-10\sqrt{2}x\right)\sqrt{\sqrt{2}\left(10\sqrt{5}+13\sqrt{2}\right)}+9\sqrt{2}(x+1)-9\sqrt{5}x-9\sqrt{2}\sqrt{-2x^2+\ldots}}{x}\right.$$

$$+\frac{1}{2}\sqrt{\frac{1}{10}}\sqrt{\sqrt{2}\left(10\sqrt{5}+13\sqrt{2}\right)}\log\left(\frac{\sqrt{\frac{1}{10}}\left(7\sqrt{5}x-10\sqrt{2}x\right)\sqrt{\sqrt{2}\left(10\sqrt{5}+13\sqrt{2}\right)}-9\sqrt{2}(x+1)+9\sqrt{5}x+9\sqrt{2}\sqrt{-2x^2+3\ldots}}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + 2)/((3*x^2 - 4*x - 2)*sqrt(-2*x^2 + 3*x + 1)),x, algorithm="fricas
```

```
[Out] 2*sqrt(1/10)*sqrt(sqrt(2)*(10*sqrt(5) - 13*sqrt(2)))*arctan(-1/9*
sqrt(1/10)*(7*sqrt(5)*x + 10*sqrt(2)*x)*sqrt(sqrt(2)*(10*sqrt(5)
- 13*sqrt(2)))/(sqrt(2)*(x + 1) + sqrt(5)*x - x*sqrt(sqrt(2)*(sqr
t(2)*(3*x^2 + 5*x + 2) + sqrt(5)*(3*x^2 + 2*x) - 2*sqrt(-2*x^2 +
3*x + 1)*(sqrt(2)*(x + 1) + sqrt(5)*x))/x^2) - sqrt(2)*sqrt(-2*x^
2 + 3*x + 1))) - 1/2*sqrt(1/10)*sqrt(sqrt(2)*(10*sqrt(5) + 13*sqr
t(2)))*log(-(sqrt(1/10)*(7*sqrt(5)*x - 10*sqrt(2)*x)*sqrt(sqrt(2)
*(10*sqrt(5) + 13*sqrt(2))) + 9*sqrt(2)*(x + 1) - 9*sqrt(5)*x - 9
*sqrt(2)*sqrt(-2*x^2 + 3*x + 1))/x) + 1/2*sqrt(1/10)*sqrt(sqrt(2)
*(10*sqrt(5) + 13*sqrt(2)))*log((sqrt(1/10)*(7*sqrt(5)*x - 10*sqr
t(2)*x)*sqrt(sqrt(2)*(10*sqrt(5) + 13*sqrt(2))) - 9*sqrt(2)*(x +
1) + 9*sqrt(5)*x + 9*sqrt(2)*sqrt(-2*x^2 + 3*x + 1))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x}{3x^2\sqrt{-2x^2+3x+1}-4x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}}\,dx-\int\frac{2}{3x^2\sqrt{-2x^2+3x+1}-4x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(1/2),x)
```

```
[Out] -Integral(x/(3*x**2*sqrt(-2*x**2 + 3*x + 1) - 4*x*sqrt(-2*x**2 +
3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x) - Integral(2/(3*x**2*sq
rt(-2*x**2 + 3*x + 1) - 4*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x
**2 + 3*x + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x+2}{(3x^2-4x-2)\sqrt{-2x^2+3x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + 2)/((3*x^2 - 4*x - 2)*sqrt(-2*x^2 + 3*x + 1)),x, algorithm="giac")
```

```
[Out] integrate(-(x + 2)/((3*x^2 - 4*x - 2)*sqrt(-2*x^2 + 3*x + 1)), x)
```

3.26 $\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{2(14x+15)}{17\sqrt{-2x^2+3x+1}}-\frac{9}{2}\sqrt{\frac{1}{5}\left(\sqrt{10}-3\right)}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{9}{2}\sqrt{\frac{1}{5}\left(3+\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)$$

```
[Out] (-2*(15 + 14*x))/(17*Sqrt[1 + 3*x - 2*x^2]) - (9*Sqrt[(-3 + Sqrt[
10])/5]*ArcTan[(3*(4 - Sqrt[10]) + (1 + 4*Sqrt[10])*x)/(2*Sqrt[1
+ Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2 + (9*Sqrt[(3 + Sqrt[10])/5
]*ArcTanh[(3*(4 + Sqrt[10]) + (1 - 4*Sqrt[10])*x)/(2*Sqrt[-1 + Sq
rt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2
```

Rubi [A] time = 0.581049, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{2(14x+15)}{17\sqrt{-2x^2+3x+1}}-\frac{9}{2}\sqrt{\frac{1}{5}\left(\sqrt{10}-3\right)}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{9}{2}\sqrt{\frac{1}{5}\left(3+\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(3/2)),x]
```

```
[Out] (-2*(15 + 14*x))/(17*Sqrt[1 + 3*x - 2*x^2]) - (9*Sqrt[(-3 + Sqrt[
10])/5]*ArcTan[(3*(4 - Sqrt[10]) + (1 + 4*Sqrt[10])*x)/(2*Sqrt[1
+ Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2 + (9*Sqrt[(3 + Sqrt[10])/5
]*ArcTanh[(3*(4 + Sqrt[10]) + (1 - 4*Sqrt[10])*x)/(2*Sqrt[-1 + Sq
rt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2
```

Rubi in Sympy [A] time = 50.5332, size = 168, normalized size = 1.01

$$-\frac{2\,(14x+15)}{17\sqrt{-2x^2+3x+1}}-\frac{9\sqrt{10}\left(-2\sqrt{10}+4\right)\operatorname{atan}\left(\frac{x\left(-8\sqrt{10}-2\right)-24+6\sqrt{10}}{4\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)}{40\sqrt{1+\sqrt{10}}}-\frac{9\sqrt{10}\left(4+2\sqrt{10}\right)\operatorname{atanh}\left(\frac{x\left(-2+8\sqrt{10}\right)-24-6\sqrt{10}}{4\sqrt{-1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)}{40\sqrt{-1+\sqrt{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(3/2),x)
```

```
[Out] -2*(14*x + 15)/(17*sqrt(-2*x**2 + 3*x + 1)) - 9*sqrt(10)*(-2*sqrt
(10) + 4)*atan((x*(-8*sqrt(10) - 2) - 24 + 6*sqrt(10))/(4*sqrt(1
+ sqrt(10))*sqrt(-2*x**2 + 3*x + 1)))/(40*sqrt(1 + sqrt(10))) - 9
*sqrt(10)*(4 + 2*sqrt(10))*atanh((x*(-2 + 8*sqrt(10)) - 24 - 6*sq
```

```
rt(10))/(4*sqrt(-1 + sqrt(10))*sqrt(-2*x**2 + 3*x + 1)))/(40*sqrt
(-1 + sqrt(10)))
```

Mathematica [A] time = 1.52075, size = 263, normalized size = 1.58

$$\frac{1}{170}\left(-\frac{280x}{\sqrt{-2x^2+3x+1}}-\frac{300}{\sqrt{-2x^2+3x+1}}+51\sqrt{10\left(1+\sqrt{10}\right)}\log\left(2\sqrt{10\left(\sqrt{10}-1\right)}\sqrt{-2x^2+3x+1}+\sqrt{10}x-40x+12\sqrt{10}+30\right)+255\sqrt{1+\sqrt{10}}\log\left(2\sqrt{10\left(\sqrt{10}-1\right)}\sqrt{-2x^2+3x+1}+\sqrt{10}x-40x+12\sqrt{10}+30\right)+\frac{153\left(\sqrt{10}-5\right)\tan^{-1}\left(\frac{4\sqrt{10}x+x-3\sqrt{10}+12}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)}{\sqrt{1+\sqrt{10}}}-51\sqrt{1+\sqrt{10}}\left(5+\sqrt{10}\right)\log\left(-3x+\sqrt{10}+2\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(3/2)),x]
```

```
[Out] (-300/Sqrt[1 + 3*x - 2*x^2] - (280*x)/Sqrt[1 + 3*x - 2*x^2] + (15
3*(-5 + Sqrt[10])*ArcTan[(12 - 3*Sqrt[10] + x + 4*Sqrt[10]*x)/(2*
Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/Sqrt[1 + Sqrt[10]] -
51*Sqrt[1 + Sqrt[10]]*(5 + Sqrt[10])*Log[2 + Sqrt[10] - 3*x] + 25
5*Sqrt[1 + Sqrt[10]]*Log[30 + 12*Sqrt[10] - 40*x + Sqrt[10]*x + 2
*Sqrt[10*(-1 + Sqrt[10])]*Sqrt[1 + 3*x - 2*x^2]] + 51*Sqrt[10*(1
+ Sqrt[10])]*Log[30 + 12*Sqrt[10] - 40*x + Sqrt[10]*x + 2*Sqrt[10
*(-1 + Sqrt[10])]*Sqrt[1 + 3*x - 2*x^2]])/170
```

Maple [B] time = 0.033, size = 760, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x)
```

```
[Out] 26/255*10^(1/2)/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1
/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)+32/
765/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1
/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)*10^(1/2)*x-62/1
53/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/
2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)*x+7/51/(-1/9-1/9
*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1
/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)+2/5*10^(1/2)/(-1/9-1/9*10^(1
/2))/(1+10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10
^(1/2))*(x-2/3+1/3*10^(1/2)))/(1+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*
10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1-10^(1/2))
^(1/2))-1/2/(-1/9-1/9*10^(1/2))/(1+10^(1/2))^(1/2)*arctan(9/2*(-2
/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(1+10^(1
/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x-2/
3+1/3*10^(1/2))-1-10^(1/2))^(1/2))-26/255*10^(1/2)/(-1/9+1/9*10^(
1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10
^(1/2))-1/9+1/9*10^(1/2))^(1/2)-32/765/(-1/9+1/9*10^(1/2))/(-2*(x
-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+
1/9*10^(1/2))^(1/2)*10^(1/2)*x-62/153/(-1/9+1/9*10^(1/2))/(-2*(x-
2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1
```

```
/9*10^(1/2))^(1/2)*x+7/51/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10^(
1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))
^(1/2)+2/5*10^(1/2)/(-1/9+1/9*10^(1/2))/(-1+10^(1/2))^(1/2)*arcta
nh(9/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))
)/(-1+10^(1/2))^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/3*10^(
1/2))*(x-2/3-1/3*10^(1/2))-1+10^(1/2))^(1/2))+1/2/(-1/9+1/9*10^(1
/2))/(-1+10^(1/2))^(1/2)*arctanh(9/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*
10^(1/2))*(x-2/3-1/3*10^(1/2)))/(-1+10^(1/2))^(1/2)/(-18*(x-2/3-1
/3*10^(1/2))^2+9*(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1+10^(1/
2))^(1/2))
```

Maxima [A] time = 0.806922, size = 915, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + 2)/((3*x^2 - 4*x - 2)*(-2*x^2 + 3*x + 1)^(3/2)),x, algorithm="maxi
```

```
[Out] 1/340*sqrt(10)*(124*sqrt(10)*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) +
 sqrt(-2*x^2 + 3*x + 1)) - 124*sqrt(10)*x/(sqrt(10)*sqrt(-2*x^2 +
 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) + 153*sqrt(10)*arcsin(8/17*sq
rt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs
(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(
10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/(sqrt(10)*sq
rt(sqrt(10) + 1) + sqrt(sqrt(10) + 1)) - 128*x/(sqrt(10)*sqrt(-2*
x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) - 128*x/(sqrt(10)*sqrt(-
2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 1224*arcsin(8/17*sqr
t(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(
6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(1
0) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/(sqrt(10)*sqr
t(sqrt(10) + 1) + sqrt(sqrt(10) + 1)) + 153*sqrt(10)*log(-2/9*sqr
t(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2
*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs
(6*x - 2*sqrt(10) - 4) + 1/18)/(sqrt(10) - 1)^(3/2) - 42*sqrt(10)
/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) + 42*
sqrt(10)/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1
)) + 1224*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqr
t(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*s
qrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/(sqrt(10) -
1)^(3/2) - 312/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3
*x + 1)) - 312/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3
*x + 1)))
```

Fricas [A] time = 0.29416, size = 586, normalized size = 3.53

$$-\frac{36\sqrt{\frac{1}{2}}\left(4x^2+\sqrt{-2x^2+3x+1}(3x+2)-6x-2\right)\sqrt{-\sqrt{10}\left(3\sqrt{10}-10\right)}\arctan\left(-\frac{\sqrt{\frac{1}{2}}\left(\sqrt{10}x+3x\right)\sqrt{-\cdots}}{\sqrt{10}(x+1)-x\sqrt{\frac{\sqrt{10}\left(15x^2+\sqrt{10}\left(3x^2+5x+2\right)-2\sqrt{-2x^2+3x+\cdots}\right.}{x^2}}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + 2)/((3*x^2 - 4*x - 2)*(-2*x^2 + 3*x + 1)^(3/2)),x, algorithm="fric
```

```
[Out] -1/10*(36*sqrt(1/2)*(4*x^2 + sqrt(-2*x^2 + 3*x + 1)*(3*x + 2) - 6
*x - 2)*sqrt(-sqrt(10)*(3*sqrt(10) - 10))*arctan(-sqrt(1/2)*(sqrt
(10)*x + 3*x)*sqrt(-sqrt(10)*(3*sqrt(10) - 10))/(sqrt(10)*(x + 1)
 - x*sqrt(sqrt(10)*(15*x^2 + sqrt(10)*(3*x^2 + 5*x + 2) - 2*sqrt(
-2*x^2 + 3*x + 1)*(sqrt(10)*(x + 1) + 5*x) + 10*x)/x^2) - sqrt(10
)*sqrt(-2*x^2 + 3*x + 1) + 5*x)) + 9*sqrt(1/2)*(4*x^2 + sqrt(-2*x
```

```
^2 + 3*x + 1)*(3*x + 2) - 6*x - 2)*sqrt(sqrt(10)*(3*sqrt(10) + 10
))*log(-9*(sqrt(1/2)*(sqrt(10)*x - 3*x)*sqrt(sqrt(10)*(3*sqrt(10)
 + 10)) + sqrt(10)*(x + 1) - sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - 5*
x)/x) - 9*sqrt(1/2)*(4*x^2 + sqrt(-2*x^2 + 3*x + 1)*(3*x + 2) - 6
*x - 2)*sqrt(sqrt(10)*(3*sqrt(10) + 10))*log(9*(sqrt(1/2)*(sqrt(1
0)*x - 3*x)*sqrt(sqrt(10)*(3*sqrt(10) + 10)) - sqrt(10)*(x + 1) +
 sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + 5*x)/x) + 120*x^2 + 20*sqrt(-2
*x^2 + 3*x + 1)*x - 20*x)/(4*x^2 + sqrt(-2*x^2 + 3*x + 1)*(3*x +
2) - 6*x - 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x+2}{(3x^2-4x-2)(-2x^2+3x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + 2)/((3*x^2 - 4*x - 2)*(-2*x^2 + 3*x + 1)^(3/2)),x, algorithm="giac
```

```
[Out] integrate(-(x + 2)/((3*x^2 - 4*x - 2)*(-2*x^2 + 3*x + 1)^(3/2)),
x)
```

3.27 $\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{2(14x+15)}{51\left(-2x^2+3x+1\right)^{3/2}}-\frac{2(4814x+291)}{867\sqrt{-2x^2+3x+1}}+\frac{9}{2}\sqrt{\frac{1}{5}\left(17\sqrt{10}-53\right)}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{9}{2}\sqrt{\frac{1}{5}\left(53+17\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)$$

```
[Out] (-2*(15 + 14*x))/(51*(1 + 3*x - 2*x^2)^(3/2)) - (2*(291 + 4814*x)
)/(867*Sqrt[1 + 3*x - 2*x^2]) + (9*Sqrt[(-53 + 17*Sqrt[10])/5]*Ar
cTan[(3*(4 - Sqrt[10]) + (1 + 4*Sqrt[10])*x)/(2*Sqrt[1 + Sqrt[10]
]*Sqrt[1 + 3*x - 2*x^2])])/2 + (9*Sqrt[(53 + 17*Sqrt[10])/5]*ArcT
anh[(3*(4 + Sqrt[10]) + (1 - 4*Sqrt[10])*x)/(2*Sqrt[-1 + Sqrt[10]
]*Sqrt[1 + 3*x - 2*x^2])])/2
```

Rubi [A] time = 0.708845, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{2(14x+15)}{51\left(-2x^2+3x+1\right)^{3/2}}-\frac{2(4814x+291)}{867\sqrt{-2x^2+3x+1}}+\frac{9}{2}\sqrt{\frac{1}{5}\left(17\sqrt{10}-53\right)}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{9}{2}\sqrt{\frac{1}{5}\left(53+17\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(5/2)),x]
```

```
[Out] (-2*(15 + 14*x))/(51*(1 + 3*x - 2*x^2)^(3/2)) - (2*(291 + 4814*x)
)/(867*Sqrt[1 + 3*x - 2*x^2]) + (9*Sqrt[(-53 + 17*Sqrt[10])/5]*Ar
cTan[(3*(4 - Sqrt[10]) + (1 + 4*Sqrt[10])*x)/(2*Sqrt[1 + Sqrt[10]
]*Sqrt[1 + 3*x - 2*x^2])])/2 + (9*Sqrt[(53 + 17*Sqrt[10])/5]*ArcT
anh[(3*(4 + Sqrt[10]) + (1 - 4*Sqrt[10])*x)/(2*Sqrt[-1 + Sqrt[10]
]*Sqrt[1 + 3*x - 2*x^2])])/2
```

Rubi in Sympy [A] time = 84.1818, size = 189, normalized size = 0.98

$$-\frac{2\left(14x+15\right)}{51\left(-2x^2+3x+1\right)^{\frac{3}{2}}}-\frac{4\left(2407x+\frac{291}{2}\right)}{867\sqrt{-2x^2+3x+1}}-\frac{\sqrt{10}\left(-\frac{23409\sqrt{10}}{2}+46818\right)\operatorname{atan}\left(\frac{x\left(-8\sqrt{10}-2\right)-24+6\sqrt{10}}{4\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)}{8670\sqrt{1+\sqrt{10}}}-\frac{\sqrt{10}\left(\frac{23409\sqrt{10}}{2}+46818\right)\operatorname{atanh}\left(\frac{x\left(-2+8\sqrt{10}\right)-24-6\sqrt{10}}{4\sqrt{-1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)}{8670\sqrt{-1+\sqrt{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(5/2),x)
```

```
[Out] -2*(14*x + 15)/(51*(-2*x**2 + 3*x + 1)**(3/2)) - 4*(2407*x + 291/
2)/(867*sqrt(-2*x**2 + 3*x + 1)) - sqrt(10)*(-23409*sqrt(10)/2 +
46818)*atan((x*(-8*sqrt(10) - 2) - 24 + 6*sqrt(10))/(4*sqrt(1 + s
qrt(10))*sqrt(-2*x**2 + 3*x + 1)))/(8670*sqrt(1 + sqrt(10))) - sq
rt(10)*(23409*sqrt(10)/2 + 46818)*atanh((x*(-2 + 8*sqrt(10)) - 24
 - 6*sqrt(10))/(4*sqrt(-1 + sqrt(10))*sqrt(-2*x**2 + 3*x + 1)))/(
8670*sqrt(-1 + sqrt(10)))
```

Mathematica [C]   time = 0.936274, size = 304, normalized size = 1.58

$$\frac{27i\left(\sqrt{10}-4\right)\log\left(9x^2+6\left(\sqrt{10}-2\right)x-4\sqrt{10}+14\right)}{4\sqrt{10\left(1+\sqrt{10}\right)}}+\frac{27\left(4+\sqrt{10}\right)\log\left(2\sqrt{10\left(\sqrt{10}-1\right)}\sqrt{-2x^2+3x+1}+\sqrt{10}x-40x+12\sqrt{10}+30\right)}{2\sqrt{10\left(\sqrt{10}-1\right)}}-\frac{27\left(\sqrt{10}-4\right)\tan^{-1}\left(\frac{4\sqrt{10}x+x-3\sqrt{10}+12}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)}{2\sqrt{10\left(1+\sqrt{10}\right)}}-\frac{2\left(-9628x^3+13860x^2+5925x+546\right)}{867\left(-2x^2+3x+1\right)^{3/2}}-\frac{27\left(4+\sqrt{10}\right)\log\left(-3x+\sqrt{10}+2\right)}{2\sqrt{10\left(\sqrt{10}-1\right)}}-\frac{27i\left(\sqrt{10}-4\right)\log\left(\left(3x+\sqrt{10}-2\right)^2\right)}{4\sqrt{10\left(1+\sqrt{10}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(5/2)),x]
```

```
[Out] (-2*(546 + 5925*x + 13860*x^2 - 9628*x^3))/(867*(1 + 3*x - 2*x^2)
^(3/2)) - (27*(-4 + Sqrt[10])*ArcTan[(12 - 3*Sqrt[10] + x + 4*Sqr
t[10]*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/(2*Sqrt[1
0*(1 + Sqrt[10])]) - (27*(4 + Sqrt[10])*Log[2 + Sqrt[10] - 3*x])/
(2*Sqrt[10*(-1 + Sqrt[10])]) - (((27*I)/4)*(-4 + Sqrt[10])*Log[(-
2 + Sqrt[10] + 3*x)^2])/Sqrt[10*(1 + Sqrt[10])] + (((27*I)/4)*(-4
 + Sqrt[10])*Log[14 - 4*Sqrt[10] + 6*(-2 + Sqrt[10])*x + 9*x^2])/
Sqrt[10*(1 + Sqrt[10])] + (27*(4 + Sqrt[10])*Log[30 + 12*Sqrt[10]
 - 40*x + Sqrt[10]*x + 2*Sqrt[10*(-1 + Sqrt[10])]*Sqrt[1 + 3*x -
2*x^2]])/(2*Sqrt[10*(-1 + Sqrt[10])])
```

Maple [B]   time = 0.027, size = 1560, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x)
```

```
[Out] 32/765/(-1/9-1/9*10^(1/2))^2/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*
10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)*10^(1/2)*x
+32/2295/(-1/9-1/9*10^(1/2))*10^(1/2)/(-2*(x-2/3+1/3*10^(1/2))^2+
(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(3/2)*x
+2/5*10^(1/2)/(-1/9+1/9*10^(1/2))^2/(-1+10^(1/2))^(1/2)*arctanh(9
/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(-
```

```
1+10^(1/2))^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/3*10^(1/2)
)*(x-2/3-1/3*10^(1/2))-1+10^(1/2))^(1/2))-32/765/(-1/9+1/9*10^(1/
2))^2/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10
^(1/2))-1/9+1/9*10^(1/2))^(1/2)*10^(1/2)*x-32/2295/(-1/9+1/9*10^(
1/2))*10^(1/2)/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2
/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(3/2)*x+2/5*10^(1/2)/(-1/9-1/9
*10^(1/2))^2/(1+10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/
3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(1+10^(1/2))^(1/2)/(-18*(x-
2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1-1
0^(1/2))^(1/2))-512/39015/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10^(
1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))
^(1/2)*10^(1/2)*x+512/39015/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10
^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2
))^(1/2)*10^(1/2)*x+7/51/(-1/9+1/9*10^(1/2))^2/(-2*(x-2/3-1/3*10^
(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2)
)^(1/2)+7/153/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3
-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(3/2)+7/51/
(-1/9-1/9*10^(1/2))^2/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2
))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)+7/153/(-1/9-1/9*1
0^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3
*10^(1/2))-1/9-1/9*10^(1/2))^(3/2)+248/2601/(-1/9-1/9*10^(1/2))/(
-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))
-1/9-1/9*10^(1/2))^(1/2)+248/2601/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-
1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*1
0^(1/2))^(1/2)-62/459/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2)
)^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(3/
2)*x+26/765/(-1/9-1/9*10^(1/2))*10^(1/2)/(-2*(x-2/3+1/3*10^(1/2))
^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(3/2
)+26/255/(-1/9-1/9*10^(1/2))^2/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/
3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)*10^(1/2)
-1/2/(-1/9-1/9*10^(1/2))^2/(1+10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/
9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(1+10^(1/2))^
(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x-2/3+1/3
*10^(1/2))-1-10^(1/2))^(1/2))-62/153/(-1/9-1/9*10^(1/2))^2/(-2*(x
-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-
1/9*10^(1/2))^(1/2)*x-26/255/(-1/9+1/9*10^(1/2))^2/(-2*(x-2/3-1/3
*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(
1/2))^(1/2)*10^(1/2)+1/2/(-1/9+1/9*10^(1/2))^2/(-1+10^(1/2))^(1/2
)*arctanh(9/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10
^(1/2)))/(-1+10^(1/2))^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4
/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1+10^(1/2))^(1/2))-26/765/(-1/9
+1/9*10^(1/2))*10^(1/2)/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1
/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(3/2)-62/459/(-1/9+1/
9*10^(1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-
1/3*10^(1/2))-1/9+1/9*10^(1/2))^(3/2)*x-62/153/(-1/9+1/9*10^(1/2)
)^2/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(
1/2))-1/9+1/9*10^(1/2))^(1/2)*x+128/13005*10^(1/2)/(-1/9+1/9*10^(
1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10
^(1/2))-1/9+1/9*10^(1/2))^(1/2)-992/7803/(-1/9+1/9*10^(1/2))/(-2*
(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/
9+1/9*10^(1/2))^(1/2)*x-128/13005*10^(1/2)/(-1/9-1/9*10^(1/2))/(-
2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-
1/9-1/9*10^(1/2))^(1/2)-992/7803/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1
/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10
^(1/2))^(1/2)*x
```

Maxima [A] time = 0.852525, size = 1723, normalized size = 8.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + 2)/((3*x^2 - 4*x - 2)*(-2*x^2 + 3*x + 1)^(5/2)),x, algorithm="maxi
```

```
[Out] 1/17340*sqrt(10)*(2108*sqrt(10)*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3
/2) + (-2*x^2 + 3*x + 1)^(3/2)) - 2108*sqrt(10)*x/(sqrt(10)*(-2*x
^2 + 3*x + 1)^(3/2) - (-2*x^2 + 3*x + 1)^(3/2)) - 56916*sqrt(10)*
x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + 11*sqrt(-2*x^2 + 3*x + 1))
```

```
 + 56916*sqrt(10)*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - 11*sqrt(
-2*x^2 + 3*x + 1)) + 1984*sqrt(10)*x/(sqrt(10)*sqrt(-2*x^2 + 3*x
+ 1) + sqrt(-2*x^2 + 3*x + 1)) - 1984*sqrt(10)*x/(sqrt(10)*sqrt(-
2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 70227*sqrt(10)*arcsi
n(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(
17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x
+ 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/(2*
sqrt(10)*sqrt(sqrt(10) + 1) + 11*sqrt(sqrt(10) + 1)) - 2176*x/(sq
rt(10)*(-2*x^2 + 3*x + 1)^(3/2) + (-2*x^2 + 3*x + 1)^(3/2)) - 217
6*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) - (-2*x^2 + 3*x + 1)^(3/2)
) + 58752*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + 11*sqrt(-2*x^2 +
 3*x + 1)) + 58752*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - 11*sqrt
(-2*x^2 + 3*x + 1)) - 2048*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + s
qrt(-2*x^2 + 3*x + 1)) - 2048*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1)
- sqrt(-2*x^2 + 3*x + 1)) + 561816*arcsin(8/17*sqrt(17)*sqrt(10)*
x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10
) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17
*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/(2*sqrt(10)*sqrt(sqrt(10) +
1) + 11*sqrt(sqrt(10) + 1)) - 714*sqrt(10)/(sqrt(10)*(-2*x^2 + 3*
x + 1)^(3/2) + (-2*x^2 + 3*x + 1)^(3/2)) + 714*sqrt(10)/(sqrt(10)
*(-2*x^2 + 3*x + 1)^(3/2) - (-2*x^2 + 3*x + 1)^(3/2)) + 19278*sqr
t(10)/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + 11*sqrt(-2*x^2 + 3*x +
 1)) - 19278*sqrt(10)/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - 11*sqr
t(-2*x^2 + 3*x + 1)) - 1488*sqrt(10)/(sqrt(10)*sqrt(-2*x^2 + 3*x
+ 1) + sqrt(-2*x^2 + 3*x + 1)) + 1488*sqrt(10)/(sqrt(10)*sqrt(-2*
x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 5304/(sqrt(10)*(-2*x^2
 + 3*x + 1)^(3/2) + (-2*x^2 + 3*x + 1)^(3/2)) - 5304/(sqrt(10)*(-
2*x^2 + 3*x + 1)^(3/2) - (-2*x^2 + 3*x + 1)^(3/2)) + 143208/(2*sq
rt(10)*sqrt(-2*x^2 + 3*x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) + 1432
08/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)
) + 1536/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1
)) + 1536/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x +
1)) + 70227*sqrt(10)*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x +
1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/ab
s(6*x - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/(
sqrt(10) - 1)^(5/2) + 561816*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2
+ 3*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqr
t(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) +
 1/18)/(sqrt(10) - 1)^(5/2))
```

Fricas [A] time = 0.299087, size = 872, normalized size = 4.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + 2)/((3*x^2 - 4*x - 2)*(-2*x^2 + 3*x + 1)^(5/2)),x, algorithm="fric
```

```
[Out] 1/30*(24240*x^6 - 116400*x^5 + 26460*x^4 + 149600*x^3 + 108*sqrt(
1/2)*(152*x^6 - 72*x^5 - 834*x^4 + 324*x^3 + 774*x^2 - (90*x^5 -
363*x^4 + 9*x^3 + 482*x^2 + 240*x + 32)*sqrt(-2*x^2 + 3*x + 1) +
288*x + 32)*sqrt(-sqrt(10)*(53*sqrt(10) - 170))*arctan(-1/9*sqrt(
1/2)*(4*sqrt(10)*x + 13*x)*sqrt(-sqrt(10)*(53*sqrt(10) - 170))/(s
qrt(10)*(x + 1) - x*sqrt(sqrt(10)*(15*x^2 + sqrt(10)*(3*x^2 + 5*x
 + 2) - 2*sqrt(-2*x^2 + 3*x + 1)*(sqrt(10)*(x + 1) + 5*x) + 10*x)
/x^2) - sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + 5*x)) + 27*sqrt(1/2)*(1
52*x^6 - 72*x^5 - 834*x^4 + 324*x^3 + 774*x^2 - (90*x^5 - 363*x^4
 + 9*x^3 + 482*x^2 + 240*x + 32)*sqrt(-2*x^2 + 3*x + 1) + 288*x +
 32)*sqrt(sqrt(10)*(53*sqrt(10) + 170))*log(-9*(sqrt(1/2)*(4*sqrt
(10)*x - 13*x)*sqrt(sqrt(10)*(53*sqrt(10) + 170)) + 9*sqrt(10)*(x
 + 1) - 9*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - 45*x)/x) - 27*sqrt(1/
2)*(152*x^6 - 72*x^5 - 834*x^4 + 324*x^3 + 774*x^2 - (90*x^5 - 36
3*x^4 + 9*x^3 + 482*x^2 + 240*x + 32)*sqrt(-2*x^2 + 3*x + 1) + 28
8*x + 32)*sqrt(sqrt(10)*(53*sqrt(10) + 170))*log(9*(sqrt(1/2)*(4*
sqrt(10)*x - 13*x)*sqrt(sqrt(10)*(53*sqrt(10) + 170)) - 9*sqrt(10
)*(x + 1) + 9*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + 45*x)/x) + 64800*
x^2 + 40*(718*x^5 + 345*x^4 - 2150*x^3 - 1332*x^2 - 192*x)*sqrt(-
2*x^2 + 3*x + 1) + 7680*x)/(152*x^6 - 72*x^5 - 834*x^4 + 324*x^3
```

```
+ 774*x^2 - (90*x^5 - 363*x^4 + 9*x^3 + 482*x^2 + 240*x + 32)*sqr
t(-2*x^2 + 3*x + 1) + 288*x + 32)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x+2}{(3x^2-4x-2)(-2x^2+3x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + 2)/((3*x^2 - 4*x - 2)*(-2*x^2 + 3*x + 1)^(5/2)),x, algorithm="giac
```

```
[Out] integrate(-(x + 2)/((3*x^2 - 4*x - 2)*(-2*x^2 + 3*x + 1)^(5/2)),
x)
```

3.28 $\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}} \, dx$

Optimal. Leaf size=151

$$\frac{1}{2}\sqrt{1-\frac{7\sqrt{\frac{2}{5}}}{5}}\tanh^{-1}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)-\frac{1}{2}\sqrt{1+\frac{7\sqrt{\frac{2}{5}}}{5}}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

```
[Out] -(Sqrt[1 + (7*Sqrt[2/5])/5]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*S
qrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/2
+ (Sqrt[1 - (7*Sqrt[2/5])/5]*ArcTanh[(3*(4 + Sqrt[10]) + (17 + 4*
Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/2
```

Rubi [A] time = 0.601403, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{1}{10}\sqrt{25-7\sqrt{10}}\tanh^{-1}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)-\frac{1}{10}\sqrt{25+7\sqrt{10}}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x + 2*x^2]),x]
```

```
[Out] -(Sqrt[25 + 7*Sqrt[10]]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[
10])*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/10 + (
Sqrt[25 - 7*Sqrt[10]]*ArcTanh[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10
])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/10
```

Rubi in Sympy [A] time = 40.304, size = 150, normalized size = 0.99

$$\frac{\sqrt{10}\left(-2\sqrt{10}+16\right)\operatorname{atanh}\left(\frac{x\left(-34+8\sqrt{10}\right)-24+6\sqrt{10}}{4\sqrt{-17\sqrt{10}+55}\sqrt{2x^2+3x+1}}\right)}{40\sqrt{-17\sqrt{10}+55}}-\frac{\sqrt{10}\left(2\sqrt{10}+16\right)\operatorname{atanh}\left(\frac{x\left(-34-8\sqrt{10}\right)-24-6\sqrt{10}}{4\sqrt{17\sqrt{10}+55}\sqrt{2x^2+3x+1}}\right)}{40\sqrt{17\sqrt{10}+55}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(1/2),x)
```

```
[Out] sqrt(10)*(-2*sqrt(10) + 16)*atanh((x*(-34 + 8*sqrt(10)) - 24 + 6*
sqrt(10))/(4*sqrt(-17*sqrt(10) + 55)*sqrt(2*x**2 + 3*x + 1)))/(40
*sqrt(-17*sqrt(10) + 55)) - sqrt(10)*(2*sqrt(10) + 16)*atanh((x*(
-34 - 8*sqrt(10)) - 24 - 6*sqrt(10))/(4*sqrt(17*sqrt(10) + 55)*sq
rt(2*x**2 + 3*x + 1)))/(40*sqrt(17*sqrt(10) + 55))
```

Mathematica [A] time = 1.253, size = 220, normalized size = 1.46

$$\frac{\frac{\left(\sqrt{10}-8\right)\log\left(-2\sqrt{550-170\sqrt{10}}\sqrt{2x^2+3x+1}-17\sqrt{10}x+40x-12\sqrt{10}+30\right)}{\sqrt{55-17\sqrt{10}}}+\frac{\left(8+\sqrt{10}\right)\log\left(2\sqrt{550+170\sqrt{10}}\sqrt{2x^2+3x+1}+17\sqrt{10}x+40x+12\sqrt{10}+30\right)}{\sqrt{55+17\sqrt{10}}}-\frac{\left(\sqrt{10}-8\right)\log\left(-\cdots\right)}{\sqrt{55-1\cdots}}}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x + 2*x^2]),x]
```

```
[Out] (-(((-8 + Sqrt[10])*Log[2 - Sqrt[10] - 3*x])/Sqrt[55 - 17*Sqrt[10
]]) - ((8 + Sqrt[10])*Log[2 + Sqrt[10] - 3*x])/Sqrt[55 + 17*Sqrt[
10]] + ((-8 + Sqrt[10])*Log[30 - 12*Sqrt[10] + 40*x - 17*Sqrt[10]
*x - 2*Sqrt[550 - 170*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2]])/Sqrt[55 -
 17*Sqrt[10]] + ((8 + Sqrt[10])*Log[30 + 12*Sqrt[10] + 40*x + 17*
Sqrt[10]*x + 2*Sqrt[550 + 170*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2]])/S
qrt[55 + 17*Sqrt[10]])/(2*Sqrt[10])
```

Maple [A] time = 0.06, size = 186, normalized size = 1.2

$$\frac{\left(-8+\sqrt{10}\right)\sqrt{10}}{20\sqrt{55-17\sqrt{10}}}\mathit{Artanh}\left(\frac{9}{2\sqrt{55-17\sqrt{10}}}\left(\frac{110}{9}-\frac{34\sqrt{10}}{9}+\left(\frac{17}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)\right)\frac{1}{\sqrt{18\left(x-2/3+1/3\sqrt{10}\right)^{2}+\cdots}}\right.$$
$$+\frac{\left(8+\sqrt{10}\right)\sqrt{10}}{20\sqrt{55+17\sqrt{10}}}\mathit{Artanh}\left(\frac{9}{2\sqrt{55+17\sqrt{10}}}\left(\frac{110}{9}+\frac{34\sqrt{10}}{9}+\left(\frac{17}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)\right)\frac{1}{\sqrt{18\left(x-2/3-1/3\sqrt{10}\right)^{2}\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x)
```

```
[Out] 1/20*(-8+10^(1/2))*10^(1/2)/(55-17*10^(1/2))^(1/2)*arctanh(9/2*(1
10/9-34/9*10^(1/2)+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(55-
17*10^(1/2))^(1/2)/(18*(x-2/3+1/3*10^(1/2))^2+9*(17/3-4/3*10^(1/2
))*(x-2/3+1/3*10^(1/2))+55-17*10^(1/2))^(1/2))+1/20*(8+10^(1/2))*
10^(1/2)/(55+17*10^(1/2))^(1/2)*arctanh(9/2*(110/9+34/9*10^(1/2)+
(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(55+17*10^(1/2))^(1/2)/
(18*(x-2/3-1/3*10^(1/2))^2+9*(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1
/2))+55+17*10^(1/2))^(1/2))
```

Maxima [A] time = 0.799835, size = 490, normalized size = 3.25

$$\frac{1}{60}\sqrt{10}\left(\frac{3\sqrt{10}\log\left(\frac{2}{9}\sqrt{10}+\frac{2\sqrt{2x^{2}+3x+1}\sqrt{17\sqrt{10}+55}}{3\left|6x-2\sqrt{10}-4\right|}+\frac{34\sqrt{10}}{9\left|6x-2\sqrt{10}-4\right|}+\frac{110}{9\left|6x-2\sqrt{10}-4\right|}+\frac{17}{18}\right)}{\sqrt{17\sqrt{10}+55}}+\frac{\sqrt{10}\log\left(-\frac{2}{9}\sqrt{10}+\frac{2\sqrt{2x^{2}+3x+1}\sqrt{-\cdots}}{\left|6x+2\sqrt{10}-\cdots\right.}\cdots\right.}{\sqrt{-\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + 2)/((3*x^2 - 4*x - 2)*sqrt(2*x^2 + 3*x + 1)),x, algorithm="maxima"
```

```
[Out] 1/60*sqrt(10)*(3*sqrt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x
 + 1)*sqrt(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqr
t(10)/abs(6*x - 2*sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4)
 + 17/18)/sqrt(17*sqrt(10) + 55) + sqrt(10)*log(-2/9*sqrt(10) + 2
*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(10) + 55/9)/abs(6*x + 2*sq
rt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110/9/abs
(6*x + 2*sqrt(10) - 4) + 17/18)/sqrt(-17/9*sqrt(10) + 55/9) + 24*
log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*sqrt(10) + 5
5)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(10)
 - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/sqrt(17*sqrt(10)
 + 55) - 8*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9
```

```
*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6
*x + 2*sqrt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/s
qrt(-17/9*sqrt(10) + 55/9))
```

Fricas [A] time = 0.285419, size = 431, normalized size = 2.85

$$-\frac{1}{10}\sqrt{\frac{1}{2}}\sqrt{\sqrt{10}\left(5\sqrt{10}+14\right)}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(2\sqrt{10}x-5x\right)\sqrt{\sqrt{10}\left(5\sqrt{10}+14\right)}+3\sqrt{10}(x+1)-3\sqrt{10}\sqrt{2x^2+3x+1}+15x}{x}\right)$$
$$+\frac{1}{10}\sqrt{\frac{1}{2}}\sqrt{\sqrt{10}\left(5\sqrt{10}+14\right)}\log\left(\frac{\sqrt{\frac{1}{2}}\left(2\sqrt{10}x-5x\right)\sqrt{\sqrt{10}\left(5\sqrt{10}+14\right)}-3\sqrt{10}(x+1)+3\sqrt{10}\sqrt{2x^2+3x+1}-15x}{x}\right)$$
$$-\frac{1}{10}\sqrt{\frac{1}{2}}\sqrt{\sqrt{10}\left(5\sqrt{10}-14\right)}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(2\sqrt{10}x+5x\right)\sqrt{\sqrt{10}\left(5\sqrt{10}-14\right)}+3\sqrt{10}(x+1)-3\sqrt{10}\sqrt{2x^2+3x+1}-15x}{x}\right)$$
$$+\frac{1}{10}\sqrt{\frac{1}{2}}\sqrt{\sqrt{10}\left(5\sqrt{10}-14\right)}\log\left(\frac{\sqrt{\frac{1}{2}}\left(2\sqrt{10}x+5x\right)\sqrt{\sqrt{10}\left(5\sqrt{10}-14\right)}-3\sqrt{10}(x+1)+3\sqrt{10}\sqrt{2x^2+3x+1}+15x}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + 2)/((3*x^2 - 4*x - 2)*sqrt(2*x^2 + 3*x + 1)),x, algorithm="fricas"

[Out] -1/10*sqrt(1/2)*sqrt(sqrt(10)*(5*sqrt(10) + 14))*log(-(sqrt(1/2)*
(2*sqrt(10)*x - 5*x)*sqrt(sqrt(10)*(5*sqrt(10) + 14)) + 3*sqrt(10
)*(x + 1) - 3*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 15*x)/x) + 1/10*sq
rt(1/2)*sqrt(sqrt(10)*(5*sqrt(10) + 14))*log((sqrt(1/2)*(2*sqrt(1
0)*x - 5*x)*sqrt(sqrt(10)*(5*sqrt(10) + 14)) - 3*sqrt(10)*(x + 1)
 + 3*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 15*x)/x) - 1/10*sqrt(1/2)*s
qrt(sqrt(10)*(5*sqrt(10) - 14))*log(-(sqrt(1/2)*(2*sqrt(10)*x + 5
*x)*sqrt(sqrt(10)*(5*sqrt(10) - 14)) + 3*sqrt(10)*(x + 1) - 3*sqr
t(10)*sqrt(2*x^2 + 3*x + 1) - 15*x)/x) + 1/10*sqrt(1/2)*sqrt(sqrt
(10)*(5*sqrt(10) - 14))*log((sqrt(1/2)*(2*sqrt(10)*x + 5*x)*sqrt(
sqrt(10)*(5*sqrt(10) - 14)) - 3*sqrt(10)*(x + 1) + 3*sqrt(10)*sqr
t(2*x^2 + 3*x + 1) + 15*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x}{3x^2\sqrt{2x^2+3x+1}-4x\sqrt{2x^2+3x+1}-2\sqrt{2x^2+3x+1}}\,dx$$
$$-\int\frac{2}{3x^2\sqrt{2x^2+3x+1}-4x\sqrt{2x^2+3x+1}-2\sqrt{2x^2+3x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(1/2),x)

[Out] -Integral(x/(3*x**2*sqrt(2*x**2 + 3*x + 1) - 4*x*sqrt(2*x**2 + 3*
x + 1) - 2*sqrt(2*x**2 + 3*x + 1)), x) - Integral(2/(3*x**2*sqrt(
2*x**2 + 3*x + 1) - 4*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 +
3*x + 1)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + 2)/((3*x^2 - 4*x - 2)*sqrt(2*x^2 + 3*x + 1)),x, algorithm="giac")

[Out] Exception raised: RuntimeError
```

3.29 $\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}}\,dx$

Optimal. Leaf size=174

$$\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}}-\frac{1}{10}\sqrt{\frac{3}{5}\left(2065+653\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)+\frac{1}{10}\sqrt{\frac{3}{5}\left(2065-653\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

```
[Out] (2*(21 + 22*x))/(5*Sqrt[1 + 3*x + 2*x^2]) - (Sqrt[(3*(2065 + 653*
Sqrt[10]))/5]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10])*x)/(2
*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/10 + (Sqrt[(3*(2
065 - 653*Sqrt[10]))/5]*ArcTanh[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[
10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/10
```

Rubi [A] time = 0.724863, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}}-\frac{1}{10}\sqrt{\frac{3}{5}\left(2065+653\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)+\frac{1}{10}\sqrt{\frac{3}{5}\left(2065-653\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(3/2)),x]
```

```
[Out] (2*(21 + 22*x))/(5*Sqrt[1 + 3*x + 2*x^2]) - (Sqrt[(3*(2065 + 653*
Sqrt[10]))/5]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10])*x)/(2
*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/10 + (Sqrt[(3*(2
065 - 653*Sqrt[10]))/5]*ArcTanh[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[
10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/10
```

Rubi in Sympy [A] time = 53.0049, size = 170, normalized size = 0.98

$$\frac{2\,(66x+63)}{15\sqrt{2x^2+3x+1}}+\frac{\sqrt{10}\left(81\sqrt{10}+270\right)\operatorname{atanh}\left(\frac{x\left(-34+8\sqrt{10}\right)-24+6\sqrt{10}}{4\sqrt{-17\sqrt{10}+55}\sqrt{2x^2+3x+1}}\right)}{300\sqrt{-17\sqrt{10}+55}}-\frac{\sqrt{10}\left(-81\sqrt{10}+270\right)\operatorname{atanh}\left(\frac{x\left(-34-8\sqrt{10}\right)-24-6\sqrt{10}}{4\sqrt{17\sqrt{10}+55}\sqrt{2x^2+3x+1}}\right)}{300\sqrt{17\sqrt{10}+55}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(3/2),x)
```

```
[Out] 2*(66*x + 63)/(15*sqrt(2*x**2 + 3*x + 1)) + sqrt(10)*(81*sqrt(10)
 + 270)*atanh((x*(-34 + 8*sqrt(10)) - 24 + 6*sqrt(10))/(4*sqrt(-1
7*sqrt(10) + 55)*sqrt(2*x**2 + 3*x + 1)))/(300*sqrt(-17*sqrt(10)
+ 55)) - sqrt(10)*(-81*sqrt(10) + 270)*atanh((x*(-34 - 8*sqrt(10)
```

```
) - 24 - 6*sqrt(10))/(4*sqrt(17*sqrt(10) + 55)*sqrt(2*x**2 + 3*x
+ 1)))/(300*sqrt(17*sqrt(10) + 55))
```

Mathematica [A] time = 1.7923, size = 252, normalized size = 1.45

$$\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}} - \frac{9\left(10+3\sqrt{10}\right)\log\left(-2\sqrt{550-170\sqrt{10}}\sqrt{2x^2+3x+1}-17\sqrt{10}x+40x-12\sqrt{10}+30\right)}{10\sqrt{550-170\sqrt{10}}} - \frac{9\left(3\sqrt{10}-10\right)\log\left(2\sqrt{550+170\sqrt{10}}\sqrt{2x^2+3x+1}+17\sqrt{10}x+40x+12\sqrt{10}+30\right)}{10\sqrt{550+170\sqrt{10}}} + \frac{9\left(10+3\sqrt{10}\right)\log\left(-3x-\sqrt{10}+2\right)}{10\sqrt{550-170\sqrt{10}}} + \frac{9\left(3\sqrt{10}-10\right)\log\left(-3x+\sqrt{10}+2\right)}{10\sqrt{550+170\sqrt{10}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(3/2)),x]
```

```
[Out] (2*(21 + 22*x))/(5*Sqrt[1 + 3*x + 2*x^2]) + (9*(10 + 3*Sqrt[10])*
Log[2 - Sqrt[10] - 3*x])/(10*Sqrt[550 - 170*Sqrt[10]]) + (9*(-10
+ 3*Sqrt[10])*Log[2 + Sqrt[10] - 3*x])/(10*Sqrt[550 + 170*Sqrt[10
]]) - (9*(10 + 3*Sqrt[10])*Log[30 - 12*Sqrt[10] + 40*x - 17*Sqrt[
10]*x - 2*Sqrt[550 - 170*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2]])/(10*Sq
rt[550 - 170*Sqrt[10]]) - (9*(-10 + 3*Sqrt[10])*Log[30 + 12*Sqrt[
10] + 40*x + 17*Sqrt[10]*x + 2*Sqrt[550 + 170*Sqrt[10]]*Sqrt[1 +
3*x + 2*x^2]])/(10*Sqrt[550 + 170*Sqrt[10]])
```

Maple [B] time = 0.026, size = 466, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x)
```

```
[Out] -1/20*(-8+10^(1/2))*10^(1/2)*(1/3/(55/9-17/9*10^(1/2))/(2*(x-2/3+
1/3*10^(1/2))^2+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55/9-17/
9*10^(1/2))^(1/2)-1/3*(17/3-4/3*10^(1/2))/(55/9-17/9*10^(1/2))*(3
+4*x)/(440/9-136/9*10^(1/2)-(17/3-4/3*10^(1/2))^2)/(2*(x-2/3+1/3*
10^(1/2))^2+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55/9-17/9*10
^(1/2))^(1/2)-1/(55/9-17/9*10^(1/2))/(55-17*10^(1/2))^(1/2)*arcta
nh(9/2*(110/9-34/9*10^(1/2)+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/
2)))/(55-17*10^(1/2))^(1/2)/(18*(x-2/3+1/3*10^(1/2))^2+9*(17/3-4/
3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55-17*10^(1/2))^(1/2)))-1/20*(8+
10^(1/2))*10^(1/2)*(1/3/(55/9+17/9*10^(1/2))/(2*(x-2/3-1/3*10^(1/
2))^2+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*10^(1/2)
)^(1/2)-1/3*(17/3+4/3*10^(1/2))/(55/9+17/9*10^(1/2))*(3+4*x)/(440
/9+136/9*10^(1/2)-(17/3+4/3*10^(1/2))^2)/(2*(x-2/3-1/3*10^(1/2))^
2+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*10^(1/2))^(1
/2)-1/(55/9+17/9*10^(1/2))/(55+17*10^(1/2))^(1/2)*arctanh(9/2*(11
0/9+34/9*10^(1/2)+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(55+1
7*10^(1/2))^(1/2)/(18*(x-2/3-1/3*10^(1/2))^2+9*(17/3+4/3*10^(1/2)
)*(x-2/3-1/3*10^(1/2))+55+17*10^(1/2))^(1/2)))
```

Maxima [A] time = 0.804822, size = 902, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + 2)/((3*x^2 - 4*x - 2)*(2*x^2 + 3*x + 1)^(3/2)),x, algorithm="maxim
```

```
[Out] -1/60*sqrt(10)*(588*sqrt(10)*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1)
 + 55*sqrt(2*x^2 + 3*x + 1)) - 588*sqrt(10)*x/(17*sqrt(10)*sqrt(2
*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)) + 2112*x/(17*sqrt(10)
*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) + 2112*x/(17*s
qrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)) - 27*sq
rt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*sqrt(
10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*s
qrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(
10) + 55)^(3/2) - sqrt(10)*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x
 + 1)*sqrt(-17/9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/
9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10)
 - 4) + 17/18)/(-17/9*sqrt(10) + 55/9)^(3/2) + 450*sqrt(10)/(17*s
qrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) - 450*s
qrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x
+ 1)) - 216*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*
sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x
- 2*sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*
sqrt(10) + 55)^(3/2) + 8*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x +
 1)*sqrt(-17/9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*
sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10) -
 4) + 17/18)/(-17/9*sqrt(10) + 55/9)^(3/2) + 1656/(17*sqrt(10)*sq
rt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) + 1656/(17*sqrt(1
0)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)))
```

Fricas [A] time = 0.288493, size = 763, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + 2)/((3*x^2 - 4*x - 2)*(2*x^2 + 3*x + 1)^(3/2)),x, algorithm="frica
```

```
[Out] 1/10*(72*x^2 + (sqrt(3/10)*sqrt(2*x^2 + 3*x + 1)*sqrt(sqrt(10)*(4
13*sqrt(10) + 1306))*(3*x + 2) - 2*sqrt(3/10)*(2*x^2 + 3*x + 1)*s
qrt(sqrt(10)*(413*sqrt(10) + 1306)))*log(-(5*sqrt(3/10)*(13*sqrt(
10)*x - 41*x)*sqrt(sqrt(10)*(413*sqrt(10) + 1306)) + 9*sqrt(10)*(
x + 1) - 9*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 45*x)/x) - (sqrt(3/10
)*sqrt(2*x^2 + 3*x + 1)*sqrt(sqrt(10)*(413*sqrt(10) + 1306))*(3*x
 + 2) - 2*sqrt(3/10)*(2*x^2 + 3*x + 1)*sqrt(sqrt(10)*(413*sqrt(10
) + 1306)))*log((5*sqrt(3/10)*(13*sqrt(10)*x - 41*x)*sqrt(sqrt(10
)*(413*sqrt(10) + 1306)) - 9*sqrt(10)*(x + 1) + 9*sqrt(10)*sqrt(2
*x^2 + 3*x + 1) - 45*x)/x) + (sqrt(3/10)*sqrt(2*x^2 + 3*x + 1)*sq
rt(sqrt(10)*(413*sqrt(10) - 1306))*(3*x + 2) - 2*sqrt(3/10)*(2*x^
2 + 3*x + 1)*sqrt(sqrt(10)*(413*sqrt(10) - 1306)))*log(-(5*sqrt(3
/10)*(13*sqrt(10)*x + 41*x)*sqrt(sqrt(10)*(413*sqrt(10) - 1306))
+ 9*sqrt(10)*(x + 1) - 9*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 45*x)/x
) - (sqrt(3/10)*sqrt(2*x^2 + 3*x + 1)*sqrt(sqrt(10)*(413*sqrt(10)
 - 1306))*(3*x + 2) - 2*sqrt(3/10)*(2*x^2 + 3*x + 1)*sqrt(sqrt(10
)*(413*sqrt(10) - 1306)))*log((5*sqrt(3/10)*(13*sqrt(10)*x + 41*x
)*sqrt(sqrt(10)*(413*sqrt(10) - 1306)) - 9*sqrt(10)*(x + 1) + 9*s
qrt(10)*sqrt(2*x^2 + 3*x + 1) + 45*x)/x) - 76*sqrt(2*x^2 + 3*x +
1)*x + 76*x)/(4*x^2 - sqrt(2*x^2 + 3*x + 1)*(3*x + 2) + 6*x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + 2)/((3*x^2 - 4*x - 2)*(2*x^2 + 3*x + 1)^(3/2)),x, algorithm="giac'
```

```
[Out] Exception raised: RuntimeError
```

3.30 $\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=197

$$\frac{2(22x+21)}{15\left(2x^2+3x+1\right)^{3/2}}+\frac{2(230x+273)}{15\sqrt{2x^2+3x+1}}-\frac{1}{50}\sqrt{\frac{1}{3}\left(4885115+1544809\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)+\frac{1}{50}\sqrt{\frac{1}{3}\left(4885115-1544809\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

```
[Out] (2*(21 + 22*x))/(15*(1 + 3*x + 2*x^2)^(3/2)) + (2*(273 + 230*x))/
(15*Sqrt[1 + 3*x + 2*x^2]) - (Sqrt[(4885115 + 1544809*Sqrt[10])/3
]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10])*x)/(2*Sqrt[55 - 1
7*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/50 + (Sqrt[(4885115 - 154480
9*Sqrt[10])/3]*ArcTanh[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(
2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/50
```

Rubi [A] time = 0.816461, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{2(22x+21)}{15\left(2x^2+3x+1\right)^{3/2}}+\frac{2(230x+273)}{15\sqrt{2x^2+3x+1}}-\frac{1}{50}\sqrt{\frac{1}{3}\left(4885115+1544809\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)+\frac{1}{50}\sqrt{\frac{1}{3}\left(4885115-1544809\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(5/2)),x]
```


```
[Out] (2*(21 + 22*x))/(15*(1 + 3*x + 2*x^2)^(3/2)) + (2*(273 + 230*x))/
(15*Sqrt[1 + 3*x + 2*x^2]) - (Sqrt[(4885115 + 1544809*Sqrt[10])/3
]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10])*x)/(2*Sqrt[55 - 1
7*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/50 + (Sqrt[(4885115 - 154480
9*Sqrt[10])/3]*ArcTanh[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(
2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/50
```

Rubi in Sympy [A] time = 79.2848, size = 194, normalized size = 0.98

$$\frac{2(66x+63)}{45\left(2x^2+3x+1\right)^{\frac{3}{2}}}+\frac{4\left(5175x+\frac{12285}{2}\right)}{675\sqrt{2x^2+3x+1}}+\frac{\sqrt{10}\left(\frac{27135\sqrt{10}}{2}+42930\right)\operatorname{atanh}\left(\frac{x\left(-34+8\sqrt{10}\right)-24+6\sqrt{10}}{4\sqrt{-17\sqrt{10}+55}\sqrt{2x^2+3x+1}}\right)}{6750\sqrt{-17\sqrt{10}+55}}-\frac{\sqrt{10}\left(-\frac{27135\sqrt{10}}{2}+42930\right)\operatorname{atanh}\left(\frac{x\left(-34-8\sqrt{10}\right)-24-6\sqrt{10}}{4\sqrt{17\sqrt{10}+55}\sqrt{2x^2+3x+1}}\right)}{6750\sqrt{17\sqrt{10}+55}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(5/2),x)
```

```
[Out] 2*(66*x + 63)/(45*(2*x**2 + 3*x + 1)**(3/2)) + 4*(5175*x + 12285/
2)/(675*sqrt(2*x**2 + 3*x + 1)) + sqrt(10)*(27135*sqrt(10)/2 + 42
930)*atanh((x*(-34 + 8*sqrt(10)) - 24 + 6*sqrt(10))/(4*sqrt(-17*s
qrt(10) + 55)*sqrt(2*x**2 + 3*x + 1)))/(6750*sqrt(-17*sqrt(10) +
55)) - sqrt(10)*(-27135*sqrt(10)/2 + 42930)*atanh((x*(-34 - 8*sqr
t(10)) - 24 - 6*sqrt(10))/(4*sqrt(17*sqrt(10) + 55)*sqrt(2*x**2 +
 3*x + 1)))/(6750*sqrt(17*sqrt(10) + 55))
```

Mathematica [A] time = 3.36487, size = 283, normalized size = 1.44

$$\frac{1}{300}\left(\frac{40(22x+21)}{(2x^2+3x+1)^{3/2}}+\frac{40(230x+273)}{\sqrt{2x^2+3x+1}}\right.$$
$$-9\sqrt{\frac{10}{55-17\sqrt{10}}}\left(212+67\sqrt{10}\right)\log\left(-2\sqrt{550-170\sqrt{10}}\sqrt{2x^2+3x+1}-17\sqrt{10}x+40x-12\sqrt{10}+30\right)$$
$$-9\sqrt{\frac{10}{55+17\sqrt{10}}}\left(67\sqrt{10}-212\right)\log\left(2\sqrt{550+170\sqrt{10}}\sqrt{2x^2+3x+1}+17\sqrt{10}x+40x+12\sqrt{10}+30\right)$$
$$\left.+9\sqrt{\frac{10}{55-17\sqrt{10}}}\left(212+67\sqrt{10}\right)\log\left(-3x-\sqrt{10}+2\right)+9\sqrt{\frac{10}{55+17\sqrt{10}}}\left(67\sqrt{10}-212\right)\log\left(-3x+\sqrt{10}+2\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(5/2)),x]
```

```
[Out] ((40*(21 + 22*x))/(1 + 3*x + 2*x^2)^(3/2) + (40*(273 + 230*x))/Sq
rt[1 + 3*x + 2*x^2] + 9*Sqrt[10/(55 - 17*Sqrt[10])]*(212 + 67*Sqr
t[10])*Log[2 - Sqrt[10] - 3*x] + 9*Sqrt[10/(55 + 17*Sqrt[10])]*(-
212 + 67*Sqrt[10])*Log[2 + Sqrt[10] - 3*x] - 9*Sqrt[10/(55 - 17*S
qrt[10])]*(212 + 67*Sqrt[10])*Log[30 - 12*Sqrt[10] + 40*x - 17*Sq
rt[10]*x - 2*Sqrt[550 - 170*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2]] - 9*
Sqrt[10/(55 + 17*Sqrt[10])]*(-212 + 67*Sqrt[10])*Log[30 + 12*Sqrt
[10] + 40*x + 17*Sqrt[10]*x + 2*Sqrt[550 + 170*Sqrt[10]]*Sqrt[1 +
 3*x + 2*x^2]])/300
```

Maple [B] time = 0.023, size = 878, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x)
```

```
[Out] -1/20*(-8+10^(1/2))*10^(1/2)*(1/9/(55/9-17/9*10^(1/2))/(2*(x-2/3+
1/3*10^(1/2))^2+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55/9-17/
9*10^(1/2))^(3/2)-1/6*(17/3-4/3*10^(1/2))/(55/9-17/9*10^(1/2))*(2
/3*(3+4*x)/(440/9-136/9*10^(1/2)-(17/3-4/3*10^(1/2))^2)/(2*(x-2/3
+1/3*10^(1/2))^2+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55/9-17
/9*10^(1/2))^(3/2)+32/3/(440/9-136/9*10^(1/2)-(17/3-4/3*10^(1/2))
^2)^2*(3+4*x)/(2*(x-2/3+1/3*10^(1/2))^2+(17/3-4/3*10^(1/2))*(x-2/
3+1/3*10^(1/2))+55/9-17/9*10^(1/2))^(1/2))+1/3/(55/9-17/9*10^(1/2
))*(1/(55/9-17/9*10^(1/2))/(2*(x-2/3+1/3*10^(1/2))^2+(17/3-4/3*10
^(1/2))*(x-2/3+1/3*10^(1/2))+55/9-17/9*10^(1/2))^(1/2)-(17/3-4/3*
10^(1/2))/(55/9-17/9*10^(1/2))*(3+4*x)/(440/9-136/9*10^(1/2)-(17/
3-4/3*10^(1/2))^2)/(2*(x-2/3+1/3*10^(1/2))^2+(17/3-4/3*10^(1/2))*
(x-2/3+1/3*10^(1/2))+55/9-17/9*10^(1/2))^(1/2)-3/(55/9-17/9*10^(1
/2))/(55-17*10^(1/2))^(1/2)*arctanh(9/2*(110/9-34/9*10^(1/2)+(17/
3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(55-17*10^(1/2))^(1/2)/(18*
```

```
(x-2/3+1/3*10^(1/2))^2+9*(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))
+55-17*10^(1/2))^(1/2))))-1/20*(8+10^(1/2))*10^(1/2)*(1/9/(55/9+1
7/9*10^(1/2))/(2*(x-2/3-1/3*10^(1/2))^2+(17/3+4/3*10^(1/2))*(x-2/
3-1/3*10^(1/2))+55/9+17/9*10^(1/2))^(3/2)-1/6*(17/3+4/3*10^(1/2))
/(55/9+17/9*10^(1/2))*(2/3*(3+4*x)/(440/9+136/9*10^(1/2)-(17/3+4/
3*10^(1/2))^2)/(2*(x-2/3-1/3*10^(1/2))^2+(17/3+4/3*10^(1/2))*(x-2
/3-1/3*10^(1/2))+55/9+17/9*10^(1/2))^(3/2)+32/3/(440/9+136/9*10^(
1/2)-(17/3+4/3*10^(1/2))^2)^2*(3+4*x)/(2*(x-2/3-1/3*10^(1/2))^2+(
17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*10^(1/2))^(1/2)
)+1/3/(55/9+17/9*10^(1/2))*(1/(55/9+17/9*10^(1/2))/(2*(x-2/3-1/3*
10^(1/2))^2+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*10
^(1/2))^(1/2)-(17/3+4/3*10^(1/2))/(55/9+17/9*10^(1/2))*(3+4*x)/(4
40/9+136/9*10^(1/2)-(17/3+4/3*10^(1/2))^2)/(2*(x-2/3-1/3*10^(1/2)
)^2+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*10^(1/2))^
(1/2)-3/(55/9+17/9*10^(1/2))/(55+17*10^(1/2))^(1/2)*arctanh(9/2*(
110/9+34/9*10^(1/2)+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(55
+17*10^(1/2))^(1/2)/(18*(x-2/3-1/3*10^(1/2))^2+9*(17/3+4/3*10^(1/
2))*(x-2/3-1/3*10^(1/2))+55+17*10^(1/2))^(1/2))))
```

Maxima [A] time = 0.837167, size = 1723, normalized size = 8.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + 2)/((3*x^2 - 4*x - 2)*(2*x^2 + 3*x + 1)^(5/2)),x, algorithm="maxim
```

```
[Out] -1/300*sqrt(10)*(980*sqrt(10)*x/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3
/2) + 55*(2*x^2 + 3*x + 1)^(3/2)) - 980*sqrt(10)*x/(17*sqrt(10)*(
2*x^2 + 3*x + 1)^(3/2) - 55*(2*x^2 + 3*x + 1)^(3/2)) + 5292*sqrt(
10)*x/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 1183*sqrt(2*x^2 + 3*x
 + 1)) - 5292*sqrt(10)*x/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 11
83*sqrt(2*x^2 + 3*x + 1)) - 15680*sqrt(10)*x/(17*sqrt(10)*sqrt(2*
x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) + 15680*sqrt(10)*x/(17
*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)) + 352
0*x/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 55*(2*x^2 + 3*x + 1)^(
3/2)) + 3520*x/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) - 55*(2*x^2 +
 3*x + 1)^(3/2)) + 19008*x/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) +
1183*sqrt(2*x^2 + 3*x + 1)) + 19008*x/(374*sqrt(10)*sqrt(2*x^2 +
3*x + 1) - 1183*sqrt(2*x^2 + 3*x + 1)) - 56320*x/(17*sqrt(10)*sqr
t(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) - 56320*x/(17*sqrt
(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)) + 750*sqrt
(10)/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 55*(2*x^2 + 3*x + 1)^
(3/2)) - 750*sqrt(10)/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) - 55*(
2*x^2 + 3*x + 1)^(3/2)) + 4050*sqrt(10)/(374*sqrt(10)*sqrt(2*x^2
+ 3*x + 1) + 1183*sqrt(2*x^2 + 3*x + 1)) - 4050*sqrt(10)/(374*sqr
t(10)*sqrt(2*x^2 + 3*x + 1) - 1183*sqrt(2*x^2 + 3*x + 1)) - 11760
*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*
x + 1)) + 11760*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*
sqrt(2*x^2 + 3*x + 1)) + 2760/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2
) + 55*(2*x^2 + 3*x + 1)^(3/2)) + 2760/(17*sqrt(10)*(2*x^2 + 3*x
+ 1)^(3/2) - 55*(2*x^2 + 3*x + 1)^(3/2)) + 14904/(374*sqrt(10)*sq
rt(2*x^2 + 3*x + 1) + 1183*sqrt(2*x^2 + 3*x + 1)) + 14904/(374*sq
rt(10)*sqrt(2*x^2 + 3*x + 1) - 1183*sqrt(2*x^2 + 3*x + 1)) - 4224
0/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1))
- 42240/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x
+ 1)) - 1215*sqrt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1
)*sqrt(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10
)/abs(6*x - 2*sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 1
7/18)/(17*sqrt(10) + 55)^(5/2) - 5*sqrt(10)*log(-2/9*sqrt(10) + 2
*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(10) + 55/9)/abs(6*x + 2*sq
rt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110/9/abs
(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) + 55/9)^(5/2) - 9
720*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*sqrt(10)
 + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt
(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10)
 + 55)^(5/2) + 40*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqr
t(-17/9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10
```

```
)/abs(6*x + 2*sqrt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 1
7/18)/(-17/9*sqrt(10) + 55/9)^(5/2))
```

Fricas [A] time = 0.289555, size = 1088, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + 2)/((3*x^2 - 4*x - 2)*(2*x^2 + 3*x + 1)^(5/2)),x, algorithm="frica
```

```
[Out] 1/150*(735600*x^6 + 2817840*x^5 + 4240620*x^4 + 3131360*x^3 + 113
4240*x^2 + 3*(sqrt(1/6)*(198*x^5 + 717*x^4 + 1017*x^3 + 706*x^2 +
 240*x + 32)*sqrt(2*x^2 + 3*x + 1)*sqrt(sqrt(10)*(977023*sqrt(10)
 + 3089618)) - 2*sqrt(1/6)*(140*x^6 + 612*x^5 + 1095*x^4 + 1026*x
^3 + 531*x^2 + 144*x + 16)*sqrt(sqrt(10)*(977023*sqrt(10) + 30896
18)))*log(-(sqrt(1/6)*(1412*sqrt(10)*x - 4465*x)*sqrt(sqrt(10)*(9
77023*sqrt(10) + 3089618)) + 81*sqrt(10)*(x + 1) - 81*sqrt(10)*sq
rt(2*x^2 + 3*x + 1) + 405*x)/x) - 3*(sqrt(1/6)*(198*x^5 + 717*x^4
 + 1017*x^3 + 706*x^2 + 240*x + 32)*sqrt(2*x^2 + 3*x + 1)*sqrt(sq
rt(10)*(977023*sqrt(10) + 3089618)) - 2*sqrt(1/6)*(140*x^6 + 612*
x^5 + 1095*x^4 + 1026*x^3 + 531*x^2 + 144*x + 16)*sqrt(sqrt(10)*(
977023*sqrt(10) + 3089618)))*log((sqrt(1/6)*(1412*sqrt(10)*x - 44
65*x)*sqrt(sqrt(10)*(977023*sqrt(10) + 3089618)) - 81*sqrt(10)*(x
 + 1) + 81*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 405*x)/x) + 3*(sqrt(1
/6)*(198*x^5 + 717*x^4 + 1017*x^3 + 706*x^2 + 240*x + 32)*sqrt(2*
x^2 + 3*x + 1)*sqrt(sqrt(10)*(977023*sqrt(10) - 3089618)) - 2*sqr
t(1/6)*(140*x^6 + 612*x^5 + 1095*x^4 + 1026*x^3 + 531*x^2 + 144*x
 + 16)*sqrt(sqrt(10)*(977023*sqrt(10) - 3089618)))*log(-(sqrt(1/6
)*(1412*sqrt(10)*x + 4465*x)*sqrt(sqrt(10)*(977023*sqrt(10) - 308
9618)) + 81*sqrt(10)*(x + 1) - 81*sqrt(10)*sqrt(2*x^2 + 3*x + 1)
- 405*x)/x) - 3*(sqrt(1/6)*(198*x^5 + 717*x^4 + 1017*x^3 + 706*x^
2 + 240*x + 32)*sqrt(2*x^2 + 3*x + 1)*sqrt(sqrt(10)*(977023*sqrt(
10) - 3089618)) - 2*sqrt(1/6)*(140*x^6 + 612*x^5 + 1095*x^4 + 102
6*x^3 + 531*x^2 + 144*x + 16)*sqrt(sqrt(10)*(977023*sqrt(10) - 30
89618)))*log((sqrt(1/6)*(1412*sqrt(10)*x + 4465*x)*sqrt(sqrt(10)*
(977023*sqrt(10) - 3089618)) - 81*sqrt(10)*(x + 1) + 81*sqrt(10)*
sqrt(2*x^2 + 3*x + 1) + 405*x)/x) - 40*(13006*x^5 + 40059*x^4 + 4
5326*x^3 + 22308*x^2 + 4032*x)*sqrt(2*x^2 + 3*x + 1) + 161280*x)/
(280*x^6 + 1224*x^5 + 2190*x^4 + 2052*x^3 + 1062*x^2 - (198*x^5 +
 717*x^4 + 1017*x^3 + 706*x^2 + 240*x + 32)*sqrt(2*x^2 + 3*x + 1)
 + 288*x + 32)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + 2)/((3*x^2 - 4*x - 2)*(2*x^2 + 3*x + 1)^(5/2)),x, algorithm="giac'

[Out] Exception raised: RuntimeError
```

3.31 $\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx$

Optimal. Leaf size=15

$$-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

```
[Out] -ArcTanh[Sqrt[5 + 2*x + x^2]]
```

Rubi [A] time = 0.0524999, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]

[Out] -ArcTanh[Sqrt[5 + 2*x + x^2]]
```

Rubi in Sympy [A] time = 13.311, size = 14, normalized size = 0.93

$$-\operatorname{atanh}\left(\sqrt{x^2+2x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x)/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)

[Out] -atanh(sqrt(x**2 + 2*x + 5))
```

Mathematica [B] time = 0.0178435, size = 41, normalized size = 2.73

$$\frac{1}{2}\log\left(1-\sqrt{x^2+2x+5}\right)-\frac{1}{2}\log\left(\sqrt{x^2+2x+5}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]

[Out] Log[1 - Sqrt[5 + 2*x + x^2]]/2 - Log[1 + Sqrt[5 + 2*x + x^2]]/2
```

Maple [A] time = 0.011, size = 14, normalized size = 0.9

$$-\mathit{Artanh}\left(\sqrt{x^2+2\,x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x)

[Out] -arctanh((x^2+2*x+5)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^2+2\,x+5}(x^2+2\,x+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 1)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)
```

Fricas [A] time = 0.272681, size = 66, normalized size = 4.4

$$\frac{1}{2}\,\log\left(x^2-\sqrt{x^2+2\,x+5}(x+2)+3\,x+6\right)-\frac{1}{2}\,\log\left(x^2-\sqrt{x^2+2\,x+5}x+x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 1)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 - sqrt(x^2 + 2*x + 5)*(x + 2) + 3*x + 6) - 1/2*log(x^
2 - sqrt(x^2 + 2*x + 5)*x + x + 4)
```

Sympy [A] time = 2.39663, size = 36, normalized size = 2.4

$$\frac{\log\left(-1+\frac{1}{\sqrt{x^2+2x+5}}\right)}{2}-\frac{\log\left(1+\frac{1}{\sqrt{x^2+2x+5}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)
```

```
[Out] log(-1 + 1/sqrt(x**2 + 2*x + 5))/2 - log(1 + 1/sqrt(x**2 + 2*x +
5))/2
```

GIAC/XCAS [A] time = 0.276584, size = 78, normalized size = 5.2

$$\frac{1}{2}\ln\left(\left(x-\sqrt{x^2+2\,x+5}\right)^2+4\,x-4\,\sqrt{x^2+2\,x+5}+7\right)-\frac{1}{2}\ln\left(\left(x-\sqrt{x^2+2\,x+5}\right)^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 1)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)),x, algorithm="giac")
```

```
[Out] 1/2*ln((x - sqrt(x^2 + 2*x + 5))^2 + 4*x - 4*sqrt(x^2 + 2*x + 5)
+ 7) - 1/2*ln((x - sqrt(x^2 + 2*x + 5))^2 + 3)
```

3.32 $\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx$

Optimal. Leaf size=44

$$\sqrt{3}\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

```
[Out] Sqrt[3]*ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])] - ArcTanh[S
qrt[5 + 2*x + x^2]]
```

Rubi [A] time = 0.145462, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\sqrt{3}\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])] - ArcTanh[S
qrt[5 + 2*x + x^2]]
```

Rubi in Sympy [A] time = 54.5304, size = 42, normalized size = 0.95

$$\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2x+2)}{6\sqrt{x^2+2x+5}}\right)-\mathrm{atanh}\left(\sqrt{x^2+2x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4+x)/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)
```

```
[Out] sqrt(3)*atan(sqrt(3)*(2*x + 2)/(6*sqrt(x**2 + 2*x + 5))) - atanh(
sqrt(x**2 + 2*x + 5))
```

Mathematica [B] time = 0.152868, size = 109, normalized size = 2.48

$$\frac{1}{2}\left(\log\left(\left(x^2+2x+4\right)^2\right)-\log\left(\left(x^2+2x+4\right)\left(x^2+2\sqrt{x^2+2x+5}+2x+6\right)\right)\right.$$
$$\left.+2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(x^2+\left(\sqrt{x^2+2x+5}+2\right)x+\sqrt{x^2+2x+5}+4\right)}{2x^2+4x+11}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(Sqrt[3]*(4 + x^2 + Sqrt[5 + 2*x + x^2] + x*(2
+ Sqrt[5 + 2*x + x^2])))/(11 + 4*x + 2*x^2)] + Log[(4 + 2*x + x^2
)^2] - Log[(4 + 2*x + x^2)*(6 + 2*x + x^2 + 2*Sqrt[5 + 2*x + x^2]
)])/2
```

Maple [A] time = 0.013, size = 40, normalized size = 0.9

$$-Artanh\left(\sqrt{x^2+2x+5}\right)+\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(2x+2)}{6}\frac{1}{\sqrt{x^2+2x+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x)
```

```
[Out] -arctanh((x^2+2*x+5)^(1/2))+3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5
)^(1/2)*(2*x+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x+4}{\sqrt{x^2+2x+5}(x^2+2x+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 4)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)),x, algorithm="maxima")
```

```
[Out] integrate((x + 4)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)
```

Fricas [A] time = 0.276216, size = 134, normalized size = 3.05

$$-\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}+2\right)\right)+\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}\right)\right)$$
$$+\frac{1}{2}\log\left(x^2-\sqrt{x^2+2x+5}(x+2)+3x+6\right)-\frac{1}{2}\log\left(x^2-\sqrt{x^2+2x+5}x+x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 4)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) + sqr
t(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5))) + 1/2*log(x^2
 - sqrt(x^2 + 2*x + 5)*(x + 2) + 3*x + 6) - 1/2*log(x^2 - sqrt(x^
2 + 2*x + 5)*x + x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x+4}{(x^2+2x+4)\sqrt{x^2+2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4+x)/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)
```

```
[Out] Integral((x + 4)/((x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)
```

GIAC/XCAS [A] time = 0.276905, size = 146, normalized size = 3.32

$$-\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}+2\right)\right)+\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}\right)\right)$$
$$+\frac{1}{2}\ln\left(\left(x-\sqrt{x^2+2x+5}\right)^2+4x-4\sqrt{x^2+2x+5}+7\right)-\frac{1}{2}\ln\left(\left(x-\sqrt{x^2+2x+5}\right)^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 4)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) + sqr
t(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5))) + 1/2*ln((x -
 sqrt(x^2 + 2*x + 5))^2 + 4*x - 4*sqrt(x^2 + 2*x + 5) + 7) - 1/2*
ln((x - sqrt(x^2 + 2*x + 5))^2 + 3)
```

3.33 $\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx$

Optimal. Leaf size=24

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)$$

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[5 + x + x^2]/Sqrt[2]])
```

Rubi [A] time = 0.0567864, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x)/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]

[Out] -(Sqrt[2]*ArcTanh[Sqrt[5 + x + x^2]/Sqrt[2]])
```

Rubi in Sympy [A] time = 14.8139, size = 24, normalized size = 1.

$$-\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{x^2+x+5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+2*x)/(x**2+x+3)/(x**2+x+5)**(1/2),x)

[Out] -sqrt(2)*atanh(sqrt(2)*sqrt(x**2 + x + 5)/2)
```

Mathematica [A] time = 0.0192771, size = 24, normalized size = 1.

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x)/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]

[Out] -(Sqrt[2]*ArcTanh[Sqrt[5 + x + x^2]/Sqrt[2]])
```

Maple [A] time = 0.018, size = 20, normalized size = 0.8

$$-Artanh\left(\frac{\sqrt{2}}{2}\sqrt{x^2+x+5}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x)

[Out] -arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+1}{\sqrt{x^2+x+5}(x^2+x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 1)/(sqrt(x^2 + x + 5)*(x^2 + x + 3)),x, algorithm="maxima")

[Out] integrate((2*x + 1)/(sqrt(x^2 + x + 5)*(x^2 + x + 3)), x)
```

Fricas [A] time = 0.272334, size = 180, normalized size = 7.5

$$\frac{1}{2}\sqrt{2}\log\left(\frac{8x^4+16x^3+85x^2+8\sqrt{2}\left(2x^3+3x^2+11x+5\right)-2\left(4x^3+6x^2+\sqrt{2}\left(8x^2+8x+21\right)+30x+14\right)\sqrt{x^2+x+5}}{8x^4+16x^3+53x^2-4\left(2x^3+3x^2+7x+3\right)\sqrt{x^2+x+5}+45x+63}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 1)/(sqrt(x^2 + x + 5)*(x^2 + x + 3)),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*log((8*x^4 + 16*x^3 + 85*x^2 + 8*sqrt(2)*(2*x^3 + 3*x
^2 + 11*x + 5) - 2*(4*x^3 + 6*x^2 + sqrt(2)*(8*x^2 + 8*x + 21) +
30*x + 14)*sqrt(x^2 + x + 5) + 77*x + 147)/(8*x^4 + 16*x^3 + 53*x
^2 - 4*(2*x^3 + 3*x^2 + 7*x + 3)*sqrt(x^2 + x + 5) + 45*x + 63))
```

Sympy [A] time = 2.30833, size = 68, normalized size = 2.83

$$2\left(\begin{cases} -\frac{\sqrt{2}\operatorname{acoth}\left(\frac{\sqrt{2}}{\sqrt{x^2+x+5}}\right)}{2} & \text{for}\ \frac{1}{x^2+x+5} > \frac{1}{2} \\ -\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}}{\sqrt{x^2+x+5}}\right)}{2} & \text{for}\ \frac{1}{x^2+x+5} < \frac{1}{2} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+2*x)/(x**2+x+3)/(x**2+x+5)**(1/2),x)

[Out] 2*Piecewise((-sqrt(2)*acoth(sqrt(2)/sqrt(x**2 + x + 5))/2, 1/(x**
2 + x + 5) > 1/2), (-sqrt(2)*atanh(sqrt(2)/sqrt(x**2 + x + 5))/2,
 1/(x**2 + x + 5) < 1/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+1}{\sqrt{x^2+x+5}(x^2+x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 1)/(sqrt(x^2 + x + 5)*(x^2 + x + 3)),x, algorithm="giac")

[Out] integrate((2*x + 1)/(sqrt(x^2 + x + 5)*(x^2 + x + 3)), x)
```

3.34 $\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx$

Optimal. Leaf size=56

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{11}}(2x+1)}{\sqrt{x^2+x+5}}\right)}{\sqrt{22}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[(Sqrt[2/11]*(1 + 2*x))/Sqrt[5 + x + x^2]]/Sqrt[22]) - Ar
cTanh[Sqrt[5 + x + x^2]/Sqrt[2]]/Sqrt[2]
```

Rubi [A] time = 0.136249, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{11}}(2x+1)}{\sqrt{x^2+x+5}}\right)}{\sqrt{22}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```

```
[Out] -(ArcTan[(Sqrt[2/11]*(1 + 2*x))/Sqrt[5 + x + x^2]]/Sqrt[22]) - Ar
cTanh[Sqrt[5 + x + x^2]/Sqrt[2]]/Sqrt[2]
```

Rubi in Sympy [A] time = 34.2877, size = 56, normalized size = 1.

$$-\frac{\sqrt{22}\,\mathrm{atan}\left(\frac{\sqrt{22}(2x+1)}{11\sqrt{x^2+x+5}}\right)}{22}-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{x^2+x+5}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**2+x+3)/(x**2+x+5)**(1/2),x)
```

```
[Out] -sqrt(22)*atan(sqrt(22)*(2*x + 1)/(11*sqrt(x**2 + x + 5)))/22 - s
qrt(2)*atanh(sqrt(2)*sqrt(x**2 + x + 5)/2)/2
```

Mathematica [B] time = 0.212243, size = 126, normalized size = 2.25

$$\frac{\log\left(\left(x^2+x+3\right)^2\right)-\log\left(\left(x^2+x+3\right)\left(x^2+2\sqrt{2}\sqrt{x^2+x+5}+x+7\right)\right)+\log(16)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{11}(7x^2+7x-3)}{-19x^2+\left(24\sqrt{2}\sqrt{x^2+x+5}-19\right)x+12\sqrt{2}\sqrt{x^2+x+5}-57}\right)}{\sqrt{22}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```

```
[Out] -(ArcTan[(Sqrt[11]*(-3 + 7*x + 7*x^2))/(-57 - 19*x^2 + 12*Sqrt[2]
*Sqrt[5 + x + x^2] + x*(-19 + 24*Sqrt[2]*Sqrt[5 + x + x^2]))]/Sqr
```

```
t[22]) + (Log[16] + Log[(3 + x + x^2)^2] - Log[(3 + x + x^2)*(7 +
 x + x^2 + 2*Sqrt[2]*Sqrt[5 + x + x^2])])/(2*Sqrt[2])
```

Maple [A] time = 0.014, size = 45, normalized size = 0.8

$$-\frac{\sqrt{2}}{2} \mathit{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{x^2+x+5}\right)-\frac{\sqrt{22}}{22}\arctan\left(\frac{(1+2x)\sqrt{22}}{11}\frac{1}{\sqrt{x^2+x+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^2+x+3)/(x^2+x+5)^(1/2),x)
```

```
[Out] -1/2*arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)-1/22*arctan(1/1
1*(1+2*x)*22^(1/2)/(x^2+x+5)^(1/2))*22^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2+x+5}(x^2+x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^2 + x + 5)*(x^2 + x + 3)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^2 + x + 5)*(x^2 + x + 3)), x)
```

Fricas [A] time = 0.29276, size = 433, normalized size = 7.73

$$\frac{1}{132}\sqrt{11}\sqrt{3}\left(\sqrt{11}\sqrt{6}\log\left(132\sqrt{6}\sqrt{3}(2x+1)+792x^2-132\sqrt{x^2+x+5}\left(2\sqrt{6}\sqrt{3}+6x+3\right)+792x+3960\right)-\sqrt{11}\sqrt{6}\log\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^2 + x + 5)*(x^2 + x + 3)),x, algorithm="fricas")
```

```
[Out] 1/132*sqrt(11)*sqrt(3)*(sqrt(11)*sqrt(6)*log(132*sqrt(6)*sqrt(3)*
(2*x + 1) + 792*x^2 - 132*sqrt(x^2 + x + 5)*(2*sqrt(6)*sqrt(3) +
6*x + 3) + 792*x + 3960) - sqrt(11)*sqrt(6)*log(-132*sqrt(6)*sqrt
(3)*(2*x + 1) + 792*x^2 + 132*sqrt(x^2 + x + 5)*(2*sqrt(6)*sqrt(3
) - 6*x - 3) + 792*x + 3960) - 4*sqrt(6)*arctan(33*sqrt(11)*sqrt(
3)/(2*sqrt(33)*sqrt(11)*sqrt(3)*sqrt(sqrt(6)*sqrt(3)*(2*x + 1) +
6*x^2 - sqrt(x^2 + x + 5)*(2*sqrt(6)*sqrt(3) + 6*x + 3) + 6*x + 3
0) - 33*sqrt(3)*(2*x + 1) + 66*sqrt(3)*sqrt(x^2 + x + 5) - 66*sqr
t(6))) + 4*sqrt(6)*arctan(33*sqrt(11)*sqrt(3)/(sqrt(11)*sqrt(3)*s
qrt(-132*sqrt(6)*sqrt(3)*(2*x + 1) + 792*x^2 + 132*sqrt(x^2 + x +
 5)*(2*sqrt(6)*sqrt(3) - 6*x - 3) + 792*x + 3960) - 33*sqrt(3)*(2
*x + 1) + 66*sqrt(3)*sqrt(x^2 + x + 5) + 66*sqrt(6))))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^2+x+3)\sqrt{x^2+x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2+x+3)/(x**2+x+5)**(1/2),x)
```

```
[Out] Integral(x/((x**2 + x + 3)*sqrt(x**2 + x + 5)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2+x+5}\,(x^2+x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^2 + x + 5)*(x^2 + x + 3)),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^2 + x + 5)*(x^2 + x + 3)), x)
```

3.35 $\int \frac{A+Bx}{\sqrt{d+ex+fx^2}(ae+bex+bfx^2)^2}\,dx$

Optimal. Leaf size=249

$$\frac{(Be-2Af)\left(8aef-b\left(4df+e^2\right)\right)\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{2e^{3/2}f(bd-ae)^{3/2}(be-4af)^{3/2}}-\frac{\sqrt{d+ex+fx^2}(e(Ab-2aB)-bx(Be-2Af))}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex+fx^2}}{\sqrt{bd-ae}}\right)}{2\sqrt{b}f(bd-ae)^{3/2}}$$

```
[Out] -((((A*b - 2*a*B)*e - b*(B*e - 2*A*f)*x)*Sqrt[d + e*x + f*x^2])/(
e*(b*d - a*e)*(b*e - 4*a*f)*(a*e + b*e*x + b*f*x^2))) + ((B*e - 2
*A*f)*(8*a*e*f - b*(e^2 + 4*d*f))*ArcTanh[(Sqrt[b*d - a*e]*(e + 2
*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a*f]*Sqrt[d + e*x + f*x^2])])/(2*e^(
3/2)*(b*d - a*e)^(3/2)*f*(b*e - 4*a*f)^(3/2)) + (B*ArcTanh[(Sqrt[
b]*Sqrt[d + e*x + f*x^2])/Sqrt[b*d - a*e]])/(2*Sqrt[b]*(b*d - a*e
)^(3/2)*f)
```

Rubi [A] time = 1.94445, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139

$$\frac{(Be-2Af)\left(8aef-b\left(4df+e^2\right)\right)\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{2e^{3/2}f(bd-ae)^{3/2}(be-4af)^{3/2}}-\frac{\sqrt{d+ex+fx^2}(e(Ab-2aB)-bx(Be-2Af))}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex+fx^2}}{\sqrt{bd-ae}}\right)}{2\sqrt{b}f(bd-ae)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```


```
[Out] -((((A*b - 2*a*B)*e - b*(B*e - 2*A*f)*x)*Sqrt[d + e*x + f*x^2])/(
e*(b*d - a*e)*(b*e - 4*a*f)*(a*e + b*e*x + b*f*x^2))) + ((B*e - 2
*A*f)*(8*a*e*f - b*(e^2 + 4*d*f))*ArcTanh[(Sqrt[b*d - a*e]*(e + 2
*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a*f]*Sqrt[d + e*x + f*x^2])])/(2*e^(
3/2)*(b*d - a*e)^(3/2)*f*(b*e - 4*a*f)^(3/2)) + (B*ArcTanh[(Sqrt[
b]*Sqrt[d + e*x + f*x^2])/Sqrt[b*d - a*e]])/(2*Sqrt[b]*(b*d - a*e
)^(3/2)*f)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((B*x+A)/(b*f*x**2+b*e*x+a*e)**2/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [B] time = 1.85023, size = 767, normalized size = 3.08

$$-\frac{-(ae+bx(e+fx))\log\left(b(e+2fx)-\sqrt{b}\sqrt{e}\sqrt{be-4af}\right)\left(-8abef(Be-2Af)-b^{3/2}Be^{5/2}\sqrt{be-4af}+4a\sqrt{b}Be^{3/2}f\sqrt{be-}\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

```
[Out] -(4*b*Sqrt[e]*Sqrt[b*d - a*e]*f*Sqrt[b*e - 4*a*f]*Sqrt[d + x*(e +
 f*x)]*(-(B*e*(2*a + b*x)) + A*b*(e + 2*f*x)) - (-(b^(3/2)*B*e^(5
/2)*Sqrt[b*e - 4*a*f]) + 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f
] - 8*a*b*e*f*(B*e - 2*A*f) + b^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a
*e + b*x*(e + f*x))*Log[-(Sqrt[b]*Sqrt[e]*Sqrt[b*e - 4*a*f]) + b*
(e + 2*f*x)] + (b^(3/2)*B*e^(5/2)*Sqrt[b*e - 4*a*f] - 4*a*Sqrt[b]
*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e - 2*A*f) + b^2*(B
*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log[Sqrt[b]*Sqrt
[e]*Sqrt[b*e - 4*a*f] + b*(e + 2*f*x)] - (b^(3/2)*B*e^(5/2)*Sqrt[
b*e - 4*a*f] - 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*
e*f*(B*e - 2*A*f) + b^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(
e + f*x))*Log[Sqrt[b]*(e^(3/2)*Sqrt[b*e - 4*a*f] + Sqrt[b]*(e^2 -
 4*d*f) + 2*Sqrt[e]*f*Sqrt[b*e - 4*a*f]*x - 4*Sqrt[b*d - a*e]*f*S
qrt[d + x*(e + f*x)])] + (-(b^(3/2)*B*e^(5/2)*Sqrt[b*e - 4*a*f])
+ 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e - 2*
A*f) + b^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log
[Sqrt[b]*(e^(3/2)*Sqrt[b*e - 4*a*f] - Sqrt[b]*(e^2 - 4*d*f) + 2*S
qrt[e]*f*Sqrt[b*e - 4*a*f]*x + 4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e
+ f*x)])])/(4*b*e^(3/2)*(b*d - a*e)^(3/2)*f*(b*e - 4*a*f)^(3/2)*(
a*e + b*x*(e + f*x)))
```

Maple [B] time = 0.028, size = 3606, normalized size = 14.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] -1/e/(4*a*f-b*e)/(a*e-b*d)/(x+1/2*e/f+1/2/b/f*(-b*e*(4*a*f-b*e))^
(1/2))*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a
*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(
a*e-b*d))^(1/2)*A+1/2/f/(4*a*f-b*e)/(a*e-b*d)/(x+1/2*e/f+1/2/b/f*
(-b*e*(4*a*f-b*e))^(1/2))*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/
b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e)
)^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2)*B+1/2/f/e/(4*a*f-b*e)/b/(a*e-b
*d)/(x+1/2*e/f+1/2/b/f*(-b*e*(4*a*f-b*e))^(1/2))*((x+1/2*(b*e+(-b
*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2
*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2)*B*(-b*e
*(4*a*f-b*e))^(1/2)-1/2/e/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^(1/2)/
(a*e-b*d)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-
b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*
(a*e-b*d))^(1/2)*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-
(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/
b/f)-1/b*(a*e-b*d))^(1/2))/(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/
b/f))*A+1/4/f/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^(1/2)/(a*e-b*d)/(-
1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^(1/2)/
b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(
1/2)*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f
-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*
e-b*d))^(1/2))/(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*B-1/4/
f/b/(a*e-b*d)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)-(-b*e*(4*
a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-
1/b*(a*e-b*d))^(1/2)*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^
2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/
2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/
2))/b/f))*B-1/e/(4*a*f-b*e)/(a*e-b*d)/(x+1/2*e/f-1/2/b/f*(-b*e*(4
*a*f-b*e))^(1/2))*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*
f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2
))/b/f)-1/b*(a*e-b*d))^(1/2)*A+1/2/f/(4*a*f-b*e)/(a*e-b*d)/(x+1/2
*e/f-1/2/b/f*(-b*e*(4*a*f-b*e))^(1/2))*((x-1/2*(-b*e+(-b*e*(4*a*f
-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-
```

```
b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2)*B-1/2/f/e/(4*a*
f-b*e)/b/(a*e-b*d)/(x+1/2*e/f-1/2/b/f*(-b*e*(4*a*f-b*e))^(1/2))*(
(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e)
)^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*
d))^(1/2)*B*(-b*e*(4*a*f-b*e))^(1/2)+1/2/e/(4*a*f-b*e)/b*(-b*e*(4
*a*f-b*e))^(1/2)/(a*e-b*d)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b
*d)+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1
/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e
))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*
(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x-1/2*(-b*e+(-b*e
*(4*a*f-b*e))^(1/2))/b/f))*A-1/4/f/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e
))^(1/2)/(a*e-b*d)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)+(-b*
e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f
)+2*(-1/b*(a*e-b*d))^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2)
)/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b
*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-
b*e))^(1/2))/b/f))*B-1/4/f/b/(a*e-b*d)/(-1/b*(a*e-b*d))^(1/2)*ln(
(-2/b*(a*e-b*d)+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*
a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x-1/2*(-b*e+(-b*
e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*
(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x-1/2
*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*B-2/(-b*e*(4*a*f-b*e))^(1/
2)/e/(4*a*f-b*e)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)+(-b*e*
(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+
2*(-1/b*(a*e-b*d))^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/
b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e
))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*
e))^(1/2))/b/f))*A*f+1/(-b*e*(4*a*f-b*e))^(1/2)/(4*a*f-b*e)/(-1/b
*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)+(-b*e*(4*a*f-b*e))^(1/2)/b*(
x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/
2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-
b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*
e-b*d))^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*B+2/(
-b*e*(4*a*f-b*e))^(1/2)/e/(4*a*f-b*e)/(-1/b*(a*e-b*d))^(1/2)*ln((
-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*
f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x+1/2*(b*e+(-b*e*(
4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*
e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x+1/2*(b*
e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*A*f-1/(-b*e*(4*a*f-b*e))^(1/2)/
(4*a*f-b*e)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)-(-b*e*(4*a*
f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/
b*(a*e-b*d))^(1/2)*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*
f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2)
)/b/f)-1/b*(a*e-b*d))^(1/2))/(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2)
)/b/f))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx + A}{(bfx^2 + bex + ae)^2 \sqrt{fx^2 + ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((b*f*x^2 + b*e*x + a*e)^2*sqrt(f*x^2 + e*x + d)),x, algorithm
```

```
[Out] integrate((B*x + A)/((b*f*x^2 + b*e*x + a*e)^2*sqrt(f*x^2 + e*x +
 d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((b*f*x^2 + b*e*x + a*e)^2*sqrt(f*x^2 + e*x + d)),x, algorithm
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*f*x**2+b*e*x+a*e)**2/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x + A)/((b*f*x^2 + b*e*x + a*e)^2*sqrt(f*x^2 + e*x + d)),x, algorithm
```

```
[Out] Exception raised: TypeError
```

3.36 $\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2}\,dx$

Optimal. Leaf size=48

$$-\frac{2(-2ah+x(2cg-bh)+bg)}{d^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*(b*g - 2*a*h + (2*c*g - b*h)*x))/((b^2 - 4*a*c)*d^2*Sqrt[a +
b*x + c*x^2])
```

Rubi [A] time = 0.148237, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{2(-2ah+x(2cg-bh)+bg)}{d^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^2,x]
```

```
[Out] (-2*(b*g - 2*a*h + (2*c*g - b*h)*x))/((b^2 - 4*a*c)*d^2*Sqrt[a +
b*x + c*x^2])
```

Rubi in Sympy [A] time = 33.2541, size = 46, normalized size = 0.96

$$\frac{4ah-2bg+x\left(2bh-4cg\right)}{d^2\left(-4ac+b^2\right)\sqrt{a+bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((h*x+g)*(c*x**2+b*x+a)**(1/2)/(c*d*x**2+b*d*x+a*d)**2,x)
```

```
[Out] (4*a*h - 2*b*g + x*(2*b*h - 4*c*g))/(d**2*(-4*a*c + b**2)*sqrt(a
+ b*x + c*x**2))
```

Mathematica [A] time = 0.0680517, size = 46, normalized size = 0.96

$$\frac{4ah-2bg+2bhx-4cgx}{d^2\left(b^2-4ac\right)\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^2,x]
```

```
[Out] (-2*b*g + 4*a*h - 4*c*g*x + 2*b*h*x)/((b^2 - 4*a*c)*d^2*Sqrt[a +
x*(b + c*x)])
```

Maple [A] time = 0.007, size = 48, normalized size = 1.

$$-2\,\frac{bhx-2\,cgx+2\,ah-bg}{\sqrt{cx^2+bx+a}d^2\left(4\,ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x)
```

```
[Out] -2/(c*x^2+b*x+a)^(1/2)*(b*h*x-2*c*g*x+2*a*h-b*g)/d^2/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+bx+a}(hx+g)}{(cdx^2+bdx+ad)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)^2,x, algorithm='
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)
^2, x)
```

Fricas [A] time = 0.456367, size = 115, normalized size = 2.4

$$-\frac{2\sqrt{cx^2+bx+a}(bg-2\,ah+(2\,cg-bh)x)}{(b^2c-4\,ac^2)d^2x^2+(b^3-4\,abc)d^2x+(ab^2-4\,a^2c)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)^2,x, algorithm='
```

```
[Out] -2*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h)*x)/((b^2*c
- 4*a*c^2)*d^2*x^2 + (b^3 - 4*a*b*c)*d^2*x + (a*b^2 - 4*a^2*c)*d^
2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x+g)*(c*x**2+b*x+a)**(1/2)/(c*d*x**2+b*d*x+a*d)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275177, size = 109, normalized size = 2.27

$$-\frac{2\left(\frac{(2\,cd^2g-bd^2h)x}{b^2d^4-4\,acd^4}+\frac{bd^2g-2\,ad^2h}{b^2d^4-4\,acd^4}\right)}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)^2,x, algorithm='
```

```
[Out] -2*((2*c*d^2*g - b*d^2*h)*x/(b^2*d^4 - 4*a*c*d^4) + (b*d^2*g - 2*
a*d^2*h)/(b^2*d^4 - 4*a*c*d^4))/sqrt(c*x^2 + b*x + a)
```

3.37 $\int \frac{3+2x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

Optimal. Leaf size=17

$$\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

```
[Out] ArcTanh[x/Sqrt[-3 - 4*x - x^2]]
```

Rubi [A] time = 0.0665603, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 2*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] ArcTanh[x/Sqrt[-3 - 4*x - x^2]]
```

Rubi in Sympy [A] time = 18.129, size = 15, normalized size = 0.88

$$\operatorname{atanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3+2*x)/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)

[Out] atanh(x/sqrt(-x**2 - 4*x - 3))
```

Mathematica [C] time = 6.28269, size = 1057, normalized size = 62.18

$$-\frac{i\left(-i+\sqrt{2}\right)\tan^{-1}\left(\frac{6i\sqrt{2}x^4+8x^4+6i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x^3+20i\sqrt{2}x^3+52x^3+24i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x^2+11i\sqrt{2}x^2+112x^2+33i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x-}{8\sqrt{2}x^4+6ix^4+40\sqrt{2}x^3+16ix^3+58\sqrt{2}x^2+19ix^2+32\sqrt{2}x+28ix+6\sqrt{2}+21i}\right.}{2\sqrt{1-2i\sqrt{2}}}$$

$$+\frac{\left(i+\sqrt{2}\right)\tanh^{-1}\left(\frac{6\sqrt{2}x^4+8ix^4+6\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}x^3+20\sqrt{2}x^3+52ix^3+24\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}x^2+11\sqrt{2}x^2+112ix^2+33\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}x-16\sqrt{2}x}{8\sqrt{2}x^4-6ix^4+40\sqrt{2}x^3-16ix^3+58\sqrt{2}x^2-19ix^2+32\sqrt{2}x-28ix+6\sqrt{2}-21i}\right.}{2\sqrt{1+2i\sqrt{2}}}$$

$$+\frac{\left(i+\sqrt{2}\right)\log\left(\left(-2ix+\sqrt{2}-2i\right)^2\left(2ix+\sqrt{2}+2i\right)^2\right)}{4\sqrt{1+2i\sqrt{2}}}$$

$$+\frac{\left(-i+\sqrt{2}\right)\log\left(\left(-2ix+\sqrt{2}-2i\right)^2\left(2ix+\sqrt{2}+2i\right)^2\right)}{4\sqrt{1-2i\sqrt{2}}}$$

$$-\frac{\left(-i+\sqrt{2}\right)\log\left((2x^2+4x+3)\left(2i\sqrt{2}x^2+2x^2-2\sqrt{2\left(1-2i\sqrt{2}\right)}\sqrt{-x^2-4x-3}x+8i\sqrt{2}x+4x-2\sqrt{2\left(1-2i\sqrt{2}\right)}\sqrt{-x^2}\right.\right.}{4\sqrt{1-2i\sqrt{2}}}$$

$$-\frac{\left(i+\sqrt{2}\right)\log\left((2x^2+4x+3)\left(-2i\sqrt{2}x^2+2x^2-2\sqrt{2\left(1+2i\sqrt{2}\right)}\sqrt{-x^2-4x-3}x-8i\sqrt{2}x+4x-2\sqrt{2\left(1+2i\sqrt{2}\right)}\sqrt{-x^2}\right.\right.}{4\sqrt{1+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 2*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ((-I/2)*(-I + Sqrt[2])*ArcTan[(24 - (12*I)*Sqrt[2] + 92*x - (16*I
)*Sqrt[2]*x + 112*x^2 + (11*I)*Sqrt[2]*x^2 + 52*x^3 + (20*I)*Sqrt
[2]*x^3 + 8*x^4 + (6*I)*Sqrt[2]*x^4 + (18*I)*Sqrt[1 - (2*I)*Sqrt[
2]]*Sqrt[-3 - 4*x - x^2] + (33*I)*Sqrt[1 - (2*I)*Sqrt[2]]*x*Sqrt[
-3 - 4*x - x^2] + (24*I)*Sqrt[1 - (2*I)*Sqrt[2]]*x^2*Sqrt[-3 - 4*
x - x^2] + (6*I)*Sqrt[1 - (2*I)*Sqrt[2]]*x^3*Sqrt[-3 - 4*x - x^2]
)/(21*I + 6*Sqrt[2] + (28*I)*x + 32*Sqrt[2]*x + (19*I)*x^2 + 58*S
qrt[2]*x^2 + (16*I)*x^3 + 40*Sqrt[2]*x^3 + (6*I)*x^4 + 8*Sqrt[2]*
x^4)])/Sqrt[1 - (2*I)*Sqrt[2]] + ((I + Sqrt[2])*ArcTanh[(24*I - 1
2*Sqrt[2] + (92*I)*x - 16*Sqrt[2]*x + (112*I)*x^2 + 11*Sqrt[2]*x^
2 + (52*I)*x^3 + 20*Sqrt[2]*x^3 + (8*I)*x^4 + 6*Sqrt[2]*x^4 + 18*
Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] + 33*Sqrt[1 + (2*I)*
Sqrt[2]]*x*Sqrt[-3 - 4*x - x^2] + 24*Sqrt[1 + (2*I)*Sqrt[2]]*x^2*
Sqrt[-3 - 4*x - x^2] + 6*Sqrt[1 + (2*I)*Sqrt[2]]*x^3*Sqrt[-3 - 4*
x - x^2])/(-21*I + 6*Sqrt[2] - (28*I)*x + 32*Sqrt[2]*x - (19*I)*x
^2 + 58*Sqrt[2]*x^2 - (16*I)*x^3 + 40*Sqrt[2]*x^3 - (6*I)*x^4 + 8
*Sqrt[2]*x^4)])/(2*Sqrt[1 + (2*I)*Sqrt[2]]) + ((-I + Sqrt[2])*Log
[(-2*I + Sqrt[2] - (2*I)*x)^2*(2*I + Sqrt[2] + (2*I)*x)^2])/(4*Sq
rt[1 - (2*I)*Sqrt[2]]) + ((I + Sqrt[2])*Log[(-2*I + Sqrt[2] - (2*
I)*x)^2*(2*I + Sqrt[2] + (2*I)*x)^2])/(4*Sqrt[1 + (2*I)*Sqrt[2]])
 - ((-I + Sqrt[2])*Log[(3 + 4*x + 2*x^2)*(3 + (6*I)*Sqrt[2] + 4*x
 + (8*I)*Sqrt[2]*x + 2*x^2 + (2*I)*Sqrt[2]*x^2 - 2*Sqrt[2*(1 - (2
*I)*Sqrt[2])]*Sqrt[-3 - 4*x - x^2] - 2*Sqrt[2*(1 - (2*I)*Sqrt[2])
]*x*Sqrt[-3 - 4*x - x^2])])/(4*Sqrt[1 - (2*I)*Sqrt[2]]) - ((I + S
qrt[2])*Log[(3 + 4*x + 2*x^2)*(3 - (6*I)*Sqrt[2] + 4*x - (8*I)*Sq
rt[2]*x + 2*x^2 - (2*I)*Sqrt[2]*x^2 - 2*Sqrt[2*(1 + (2*I)*Sqrt[2]
)]*Sqrt[-3 - 4*x - x^2] - 2*Sqrt[2*(1 + (2*I)*Sqrt[2])]*x*Sqrt[-3
 - 4*x - x^2])])/(4*Sqrt[1 + (2*I)*Sqrt[2]])
```

Maple [B] time = 0.015, size = 94, normalized size = 5.5

$$-\frac{\sqrt{3}\sqrt{4}}{6}\sqrt{3\,\frac{x^2}{(-3/2-x)^2}-12}\,Artanh\left(3\,\frac{x}{-3/2-x}\,\frac{1}{\sqrt{3\,\frac{x^2}{(-3/2-x)^2}-12}}\right)\frac{1}{\sqrt{1\left(x^2\left(-\frac{3}{2}-x\right)^{-2}-4\right)\left(1+x\left(-\frac{3}{2}-x\right)^{-1}\right)^{-2}}}\left(1+x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] -1/6*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/
(1+x/(-3/2-x))*(3*x^2/(-3/2-x)^2-12)^(1/2)*arctanh(3*x/(-3/2-x)/(
3*x^2/(-3/2-x)^2-12)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x+3}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="maxima"
```

```
[Out] integrate((2*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Fricas [A] time = 0.288099, size = 76, normalized size = 4.47

$$-\frac{1}{4}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right)+\frac{1}{4}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="fricas")
```

```
[Out] -1/4*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/4*log((2*
sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+3}{\sqrt{-(x+1)(x+3)}(2x^2+4x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral((2*x + 3)/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x
)
```

GIAC/XCAS [A] time = 0.274404, size = 132, normalized size = 7.76

$$\frac{1}{2}\ln\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)-\frac{1}{2}\ln\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="giac")
```

```
[Out] 1/2*ln(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x
- 3) - 1)^2/(x + 2)^2 + 1) - 1/2*ln(2*(sqrt(-x^2 - 4*x - 3) - 1)/
(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

3.38 $\int \frac{3+4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

Optimal. Leaf size=86

$$\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

```
[Out] Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]] - Sqrt
[2]*ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]] + ArcTanh[
x/Sqrt[-3 - 4*x - x^2]]
```

Rubi [A] time = 0.437591, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281

$$\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 4*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]] - Sqrt
[2]*ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]] + ArcTanh[
x/Sqrt[-3 - 4*x - x^2]]
```

Rubi in Sympy [A] time = 61.2249, size = 83, normalized size = 0.97

$$-\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3\left(\frac{x}{3}+1\right)}{2\sqrt{-x^2-4x-3}}-\frac{1}{2}\right)\right)-\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3\left(\frac{x}{3}+1\right)}{2\sqrt{-x^2-4x-3}}+\frac{1}{2}\right)\right)+\mathrm{atanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3+4*x)/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] -sqrt(2)*atan(sqrt(2)*(3*(x/3 + 1)/(2*sqrt(-x**2 - 4*x - 3)) - 1/
2)) - sqrt(2)*atan(sqrt(2)*(3*(x/3 + 1)/(2*sqrt(-x**2 - 4*x - 3))
 + 1/2)) + atanh(x/sqrt(-x**2 - 4*x - 3))
```

Mathematica [C] time = 6.27008, size = 1079, normalized size = 12.55

$$-\frac{i\left(i+2\sqrt{2}\right)\tan^{-1}\left(\frac{6i\sqrt{2}x^4-16x^4+18i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x^3+68i\sqrt{2}x^3-68x^3+72i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x^2+185i\sqrt{2}x^2-44x^2+99i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x}{32\sqrt{2}x^4+66ix^4+208\sqrt{2}x^3+304ix^3+466\sqrt{2}x^2+493ix^2+440\sqrt{2}x+340ix+150\sqrt{2}+93i}\right.}{2\sqrt{1-2i\sqrt{2}}}$$

$$-\frac{i\left(-i+2\sqrt{2}\right)\tan^{-1}\left(\frac{6i\sqrt{2}x^4+16x^4+18i\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}x^3+68i\sqrt{2}x^3+68x^3+72i\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}x^2+185i\sqrt{2}x^2+44x^2+99i\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}}{32\sqrt{2}x^4-66ix^4+208\sqrt{2}x^3-304ix^3+466\sqrt{2}x^2-493ix^2+440\sqrt{2}x-340ix+150\sqrt{2}-9}\right.}{2\sqrt{1+2i\sqrt{2}}}$$

$$+\frac{\left(i+2\sqrt{2}\right)\log\left(\left(-2ix+\sqrt{2}-2i\right)^2\left(2ix+\sqrt{2}+2i\right)^2\right)}{4\sqrt{1-2i\sqrt{2}}}$$

$$+\frac{\left(-i+2\sqrt{2}\right)\log\left(\left(-2ix+\sqrt{2}-2i\right)^2\left(2ix+\sqrt{2}+2i\right)^2\right)}{4\sqrt{1+2i\sqrt{2}}}$$

$$-\frac{\left(i+2\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(2i\sqrt{2}x^2+2x^2-2\sqrt{2\left(1-2i\sqrt{2}\right)}\sqrt{-x^2-4x-3}x+8i\sqrt{2}x+4x-2\sqrt{2\left(1-2i\sqrt{2}\right)}\sqrt{-x^2}\right.\right.}{4\sqrt{1-2i\sqrt{2}}}$$

$$-\frac{\left(-i+2\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(-2i\sqrt{2}x^2+2x^2-2\sqrt{2\left(1+2i\sqrt{2}\right)}\sqrt{-x^2-4x-3}x-8i\sqrt{2}x+4x-2\sqrt{2\left(1+2i\sqrt{2}\right)}\sqrt{-}\right.\right.}{4\sqrt{1+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 4*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ((-I/2)*(I + 2*Sqrt[2])*ArcTan[(60 + (51*I)*Sqrt[2] + 68*x + (176
*I)*Sqrt[2]*x - 44*x^2 + (185*I)*Sqrt[2]*x^2 - 68*x^3 + (68*I)*Sq
rt[2]*x^3 - 16*x^4 + (6*I)*Sqrt[2]*x^4 + (54*I)*Sqrt[1 - (2*I)*Sq
rt[2]]*Sqrt[-3 - 4*x - x^2] + (99*I)*Sqrt[1 - (2*I)*Sqrt[2]]*x*Sq
rt[-3 - 4*x - x^2] + (72*I)*Sqrt[1 - (2*I)*Sqrt[2]]*x^2*Sqrt[-3 -
 4*x - x^2] + (18*I)*Sqrt[1 - (2*I)*Sqrt[2]]*x^3*Sqrt[-3 - 4*x -
x^2])/(93*I + 150*Sqrt[2] + (340*I)*x + 440*Sqrt[2]*x + (493*I)*x
^2 + 466*Sqrt[2]*x^2 + (304*I)*x^3 + 208*Sqrt[2]*x^3 + (66*I)*x^4
 + 32*Sqrt[2]*x^4)])/Sqrt[1 - (2*I)*Sqrt[2]] - ((I/2)*(-I + 2*Sqr
t[2])*ArcTan[(-60 + (51*I)*Sqrt[2] - 68*x + (176*I)*Sqrt[2]*x + 4
4*x^2 + (185*I)*Sqrt[2]*x^2 + 68*x^3 + (68*I)*Sqrt[2]*x^3 + 16*x^
4 + (6*I)*Sqrt[2]*x^4 + (54*I)*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 -
4*x - x^2] + (99*I)*Sqrt[1 + (2*I)*Sqrt[2]]*x*Sqrt[-3 - 4*x - x^2
] + (72*I)*Sqrt[1 + (2*I)*Sqrt[2]]*x^2*Sqrt[-3 - 4*x - x^2] + (18
*I)*Sqrt[1 + (2*I)*Sqrt[2]]*x^3*Sqrt[-3 - 4*x - x^2])/(-93*I + 15
0*Sqrt[2] - (340*I)*x + 440*Sqrt[2]*x - (493*I)*x^2 + 466*Sqrt[2]
*x^2 - (304*I)*x^3 + 208*Sqrt[2]*x^3 - (66*I)*x^4 + 32*Sqrt[2]*x^
4)])/Sqrt[1 + (2*I)*Sqrt[2]] + ((-I + 2*Sqrt[2])*Log[(-2*I + Sqrt
[2] - (2*I)*x)^2*(2*I + Sqrt[2] + (2*I)*x)^2])/(4*Sqrt[1 + (2*I)*
Sqrt[2]]) + ((I + 2*Sqrt[2])*Log[(-2*I + Sqrt[2] - (2*I)*x)^2*(2*
I + Sqrt[2] + (2*I)*x)^2])/(4*Sqrt[1 - (2*I)*Sqrt[2]]) - ((I + 2*
Sqrt[2])*Log[(3 + 4*x + 2*x^2)*(3 + (6*I)*Sqrt[2] + 4*x + (8*I)*S
qrt[2]*x + 2*x^2 + (2*I)*Sqrt[2]*x^2 - 2*Sqrt[2*(1 - (2*I)*Sqrt[2
])]*Sqrt[-3 - 4*x - x^2] - 2*Sqrt[2*(1 - (2*I)*Sqrt[2])]*x*Sqrt[-
3 - 4*x - x^2])])/(4*Sqrt[1 - (2*I)*Sqrt[2]]) - ((-I + 2*Sqrt[2])
*Log[(3 + 4*x + 2*x^2)*(3 - (6*I)*Sqrt[2] + 4*x - (8*I)*Sqrt[2]*x
 + 2*x^2 - (2*I)*Sqrt[2]*x^2 - 2*Sqrt[2*(1 + (2*I)*Sqrt[2])]*Sqrt
[-3 - 4*x - x^2] - 2*Sqrt[2*(1 + (2*I)*Sqrt[2])]*x*Sqrt[-3 - 4*x
- x^2])])/(4*Sqrt[1 + (2*I)*Sqrt[2]])
```

Maple [A] time = 0.011, size = 123, normalized size = 1.4

$$\frac{\sqrt{3}\sqrt{4}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\right)-Artanh\left(3\frac{x}{-3/2-x}\frac{1}{\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}}\right)\right)\frac{1}{\sqrt{1\left(x^2\left(-\frac{3}{2}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] 1/6*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1
/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(3*x/(-3/2-x)/(3*x
^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(
1/2)/(1+x/(-3/2-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4\,x+3}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Fricas [A] time = 0.29282, size = 167, normalized size = 1.94

$$\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x+3\,\sqrt{-x^2-4\,x-3}\right)}{2\,(2\,x+3)}\right)+\frac{1}{2}\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(x-3\,\sqrt{-x^2-4\,x-3}\right)}{2\,(2\,x+3)}\right)-\frac{1}{4}\,\log\left(-\frac{2\,\sqrt{-x^2-4\,x-3}x+4\,x+3}{x^2}\right)+\frac{1}{4}\,\log\left(\frac{2\,\sqrt{-x^2-4\,x-3}x-4\,x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 3*sqrt(-x^2 - 4*x - 3))/(2*x
+ 3)) + 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(x - 3*sqrt(-x^2 - 4*x -
3))/(2*x + 3)) - 1/4*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^
2) + 1/4*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x+3}{\sqrt{-(x+1)(x+3)}\,(2x^2+4x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3+4*x)/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral((4*x + 3)/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x
)
```

GIAC/XCAS [A] time = 0.273383, size = 220, normalized size = 2.56

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)+\frac{1}{2}\ln\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)-\frac{1}{2}\ln\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="giac")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2)
+ 1)) + sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x
 + 2) + 1)) + 1/2*ln(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sq
rt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/2*ln(2*(sqrt(-x^2 -
4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 +
3)
```

3.39 $\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^{3/2}}\,dx$

Optimal. Leaf size=136

$$\frac{h\sqrt{a+bx+cx^2}\log\left(a+bx+cx^2\right)}{2cd\sqrt{ad+bdx+cdx^2}}-\frac{\sqrt{a+bx+cx^2}(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{cd\sqrt{b^2-4ac}\sqrt{ad+bdx+cdx^2}}$$

```
[Out] -(((2*c*g - b*h)*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*d*Sqrt[a*d + b*d*x + c*d*x^2])
) + (h*Sqrt[a + b*x + c*x^2]*Log[a + b*x + c*x^2])/(2*c*d*Sqrt[a*
d + b*d*x + c*d*x^2])
```

Rubi [A] time = 0.299394, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132

$$\frac{h\sqrt{a+bx+cx^2}\log\left(a+bx+cx^2\right)}{2cd\sqrt{ad+bdx+cdx^2}}-\frac{\sqrt{a+bx+cx^2}(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{cd\sqrt{b^2-4ac}\sqrt{ad+bdx+cdx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^(3/2),x]
```

```
[Out] -(((2*c*g - b*h)*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*d*Sqrt[a*d + b*d*x + c*d*x^2])
) + (h*Sqrt[a + b*x + c*x^2]*Log[a + b*x + c*x^2])/(2*c*d*Sqrt[a*
d + b*d*x + c*d*x^2])
```

Rubi in Sympy [A] time = 42.4655, size = 129, normalized size = 0.95

$$\frac{h\sqrt{ad+bdx+cdx^2}\log\left(a+bx+cx^2\right)}{2cd^2\sqrt{a+bx+cx^2}}+\frac{(bh-2cg)\sqrt{ad+bdx+cdx^2}\,\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{cd^2\sqrt{-4ac+b^2}\sqrt{a+bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x+g)*(c*x**2+b*x+a)**(1/2)/(c*d*x**2+b*d*x+a*d)**(3/2),x)
```

```
[Out] h*sqrt(a*d + b*d*x + c*d*x**2)*log(a + b*x + c*x**2)/(2*c*d**2*sq
rt(a + b*x + c*x**2)) + (b*h - 2*c*g)*sqrt(a*d + b*d*x + c*d*x**2
)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(c*d**2*sqrt(-4*a*c + b*
*2)*sqrt(a + b*x + c*x**2))
```

Mathematica [A] time = 0.181597, size = 108, normalized size = 0.79

$$\frac{(a+x(b+cx))^{3/2}\left((4cg-2bh)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)+h\sqrt{4ac-b^2}\log(a+x(b+cx))\right)}{2c\sqrt{4ac-b^2}(d(a+x(b+cx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^(3/2),x]
```

```
[Out] ((a + x*(b + c*x))^(3/2)*((4*c*g - 2*b*h)*ArcTan[(b + 2*c*x)/Sqrt
[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*h*Log[a + x*(b + c*x)]))/(2*
```

```
c*Sqrt[-b^2 + 4*a*c]*(d*(a + x*(b + c*x)))^(3/2))
```

Maple [A] time = 0.025, size = 121, normalized size = 0.9

$$\frac{1}{2\,cd^2}\sqrt{d\,(cx^2+bx+a)}\left(h\ln\left(cx^2+bx+a\right)\sqrt{4\,ac-b^2}-2\,\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)bh+4\,\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)cg\right)\frac{1}{\sqrt{cx^2+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x)
```

```
[Out] 1/2/(c*x^2+b*x+a)^(1/2)*(d*(c*x^2+b*x+a))^(1/2)*(h*ln(c*x^2+b*x+a
)*(4*a*c-b^2)^(1/2)-2*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*h+4*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c*g)/d^2/c/(4*a*c-b^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)^(3/2),x, algorit
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)^(3/2),x, algorit
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(d\,(a+bx+cx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x**2+b*x+a)**(1/2)/(c*d*x**2+b*d*x+a*d)**(3/2),x)
```

```
[Out] Integral((g + h*x)*sqrt(a + b*x + c*x**2)/(d*(a + b*x + c*x**2))*
*(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+bx+a}(hx+g)}{(cdx^2+bdx+ad)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)^(3/2),x, algorit
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)
^(3/2), x)
```

3.40 $\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=212

$$-\frac{ac^2\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}(8bc-15adx)}{60d^2(a+bx)}-\frac{acx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)}+\frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{5d(a+bx)}$$

```
[Out] -(a*c*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(8*d*(a +
b*x)) + (b*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(
5*d*(a + b*x)) - ((8*b*c - 15*a*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2
]*(c + d*x^2)^(3/2))/(60*d^2*(a + b*x)) - (a*c^2*Sqrt[a^2 + 2*a*b
*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(3/2)*(a
 + b*x))
```

Rubi [A] time = 0.393442, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$

$$-\frac{ac^2\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}(8bc-15adx)}{60d^2(a+bx)}-\frac{acx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)}+\frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{5d(a+bx)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] -(a*c*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(8*d*(a +
b*x)) + (b*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(
5*d*(a + b*x)) - ((8*b*c - 15*a*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2
]*(c + d*x^2)^(3/2))/(60*d^2*(a + b*x)) - (a*c^2*Sqrt[a^2 + 2*a*b
*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(3/2)*(a
 + b*x))
```

Rubi in Sympy [A] time = 40.4385, size = 196, normalized size = 0.92

$$-\frac{ac^2\sqrt{a^2+2abx+b^2x^2}\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{\frac{3}{2}}(a+bx)}-\frac{acx\sqrt{c+dx^2}\sqrt{a^2+2abx+b^2x^2}}{8d(a+bx)}+\frac{bx^2\left(c+dx^2\right)^{\frac{3}{2}}\sqrt{a^2+2abx+b^2x^2}}{5d(a+bx)}-\frac{\left(c+dx^2\right)^{\frac{3}{2}}(-30adx+16bc)\sqrt{a^2+2abx+b^2x^2}}{120d^2(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] -a*c**2*sqrt(a**2 + 2*a*b*x + b**2*x**2)*atanh(sqrt(d)*x/sqrt(c +
 d*x**2))/(8*d**(3/2)*(a + b*x)) - a*c*x*sqrt(c + d*x**2)*sqrt(a*
*2 + 2*a*b*x + b**2*x**2)/(8*d*(a + b*x)) + b*x**2*(c + d*x**2)**
(3/2)*sqrt(a**2 + 2*a*b*x + b**2*x**2)/(5*d*(a + b*x)) - (c + d*x
**2)**(3/2)*(-30*a*d*x + 16*b*c)*sqrt(a**2 + 2*a*b*x + b**2*x**2)
/(120*d**2*(a + b*x))
```

Mathematica [A] time = 0.128794, size = 108, normalized size = 0.51

$$\frac{\sqrt{(a+bx)^2}\left(\sqrt{c+dx^2}\left(15adx\left(c+2dx^2\right)+8b\left(-2c^2+cdx^2+3d^2x^4\right)\right)-15ac^2\sqrt{d}\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)\right)}{120d^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(Sqrt[c + d*x^2]*(15*a*d*x*(c + 2*d*x^2) + 8*b
*(-2*c^2 + c*d*x^2 + 3*d^2*x^4)) - 15*a*c^2*Sqrt[d]*Log[d*x + Sqr
t[d]*Sqrt[c + d*x^2]]))/(120*d^2*(a + b*x))
```

Maple [C] time = 0.043, size = 105, normalized size = 0.5

$$\frac{csgn(bx+a)}{120}\left(24\,d^{5/2}\left(dx^2+c\right)^{3/2}x^2b+30\,ax\left(dx^2+c\right)^{3/2}d^{5/2}-16\,d^{3/2}\left(dx^2+c\right)^{3/2}bc-15\,acx\sqrt{dx^2+c}d^{5/2}-15\,ac^2\ln\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/120*csgn(b*x+a)*(24*d^(5/2)*(d*x^2+c)^(3/2)*x^2*b+30*a*x*(d*x^2
+c)^(3/2)*d^(5/2)-16*d^(3/2)*(d*x^2+c)^(3/2)*b*c-15*a*c*x*(d*x^2+
c)^(1/2)*d^(5/2)-15*a*c^2*ln(x*d^(1/2)+(d*x^2+c)^(1/2))*d^2)/d^(7
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.323893, size = 1, normalized size = 0.

$$\left[\frac{15\,ac^2d\log\left(2\,\sqrt{dx^2+c}dx-\left(2\,dx^2+c\right)\sqrt{d}\right)+2\left(24\,bd^2x^4+30\,ad^2x^3+8\,bcdx^2+15\,acdx-16\,bc^2\right)\sqrt{dx^2+c}\sqrt{d}}{240\,d^{\frac{5}{2}}},\right.$$
$$\left.-\frac{15\,ac^2d\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-\left(24\,bd^2x^4+30\,ad^2x^3+8\,bcdx^2+15\,acdx-16\,bc^2\right)\sqrt{dx^2+c}\sqrt{-d}}{120\,\sqrt{-d}d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)*x^2,x, algorithm="fricas")
```

```
[Out] [1/240*(15*a*c^2*d*log(2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt
(d)) + 2*(24*b*d^2*x^4 + 30*a*d^2*x^3 + 8*b*c*d*x^2 + 15*a*c*d*x
- 16*b*c^2)*sqrt(d*x^2 + c)*sqrt(d))/d^(5/2), -1/120*(15*a*c^2*d*
```

```
arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (24*b*d^2*x^4 + 30*a*d^2*x^3
 + 8*b*c*d*x^2 + 15*a*c*d*x - 16*b*c^2)*sqrt(d*x^2 + c)*sqrt(-d))
/(sqrt(-d)*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{c+dx^2}\sqrt{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)
```

GIAC/XCAS [A] time = 0.269735, size = 158, normalized size = 0.75

$$\frac{ac^2\ln\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)\operatorname{sign}(bx+a)}{8\,d^{\frac{3}{2}}} + \frac{1}{120}\sqrt{dx^2+c}\left(\left(2\left(3\,(4\,bx\operatorname{sign}(bx+a)+5\,a\operatorname{sign}(bx+a))x+\frac{4\,bc\operatorname{sign}(bx+a)}{d}\right)x+\frac{15\,ac\operatorname{sign}(bx+a)}{d}\right)x-\frac{16\,bc^2\operatorname{sign}(bx}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)*x^2,x, algorithm="giac")
```

```
[Out] 1/8*a*c^2*ln(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sign(b*x + a)/d^(
3/2) + 1/120*sqrt(d*x^2 + c)*((2*(3*(4*b*x*sign(b*x + a) + 5*a*si
gn(b*x + a))*x + 4*b*c*sign(b*x + a)/d)*x + 15*a*c*sign(b*x + a)/
d)*x - 16*b*c^2*sign(b*x + a)/d^2)
```

3.41 $\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=161

$$-\frac{bc^2\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}-\frac{bcx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)}+\frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{12d(a+bx)}$$

```
[Out] -(b*c*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(8*d*(a +
b*x)) + ((4*a + 3*b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^
(3/2))/(12*d*(a + b*x)) - (b*c^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Ar
cTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(3/2)*(a + b*x))
```

Rubi [A] time = 0.225531, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152

$$-\frac{bc^2\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}-\frac{bcx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)}+\frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{12d(a+bx)}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] -(b*c*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(8*d*(a +
b*x)) + ((4*a + 3*b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^
(3/2))/(12*d*(a + b*x)) - (b*c^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Ar
cTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(3/2)*(a + b*x))
```

Rubi in Sympy [A] time = 28.4304, size = 146, normalized size = 0.91

$$-\frac{bc^2\sqrt{a^2+2abx+b^2x^2}\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{\frac{3}{2}}(a+bx)}-\frac{bcx\sqrt{c+dx^2}\sqrt{a^2+2abx+b^2x^2}}{8d(a+bx)}+\frac{(8a+6bx)\left(c+dx^2\right)^{\frac{3}{2}}\sqrt{a^2+2abx+b^2x^2}}{24d(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] -b*c**2*sqrt(a**2 + 2*a*b*x + b**2*x**2)*atanh(sqrt(d)*x/sqrt(c +
 d*x**2))/(8*d**(3/2)*(a + b*x)) - b*c*x*sqrt(c + d*x**2)*sqrt(a*
*2 + 2*a*b*x + b**2*x**2)/(8*d*(a + b*x)) + (8*a + 6*b*x)*(c + d*
x**2)**(3/2)*sqrt(a**2 + 2*a*b*x + b**2*x**2)/(24*d*(a + b*x))
```

Mathematica [A] time = 0.095338, size = 96, normalized size = 0.6

$$\frac{\sqrt{(a+bx)^2}\left(\sqrt{d}\sqrt{c+dx^2}\left(8a\left(c+dx^2\right)+3bx\left(c+2dx^2\right)\right)-3bc^2\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)\right)}{24d^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(Sqrt[d]*Sqrt[c + d*x^2]*(8*a*(c + d*x^2) + 3*
b*x*(c + 2*d*x^2)) - 3*b*c^2*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]]))
/(24*d^(3/2)*(a + b*x))
```

Maple [C] time = 0.013, size = 84, normalized size = 0.5

$$\frac{csgn(bx+a)}{24}\left(6\,bx\left(dx^2+c\right)^{3/2}d^{3/2}+8\,a\left(dx^2+c\right)^{3/2}d^{3/2}-3\,bcx\sqrt{dx^2+c}d^{3/2}-3\,bc^2\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d\right)d^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/24*csgn(b*x+a)*(6*b*x*(d*x^2+c)^(3/2)*d^(3/2)+8*a*(d*x^2+c)^(3/
2)*d^(3/2)-3*b*c*x*(d*x^2+c)^(1/2)*d^(3/2)-3*b*c^2*ln(x*d^(1/2)+(
d*x^2+c)^(1/2))*d)/d^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.324643, size = 1, normalized size = 0.01

$$\left[\frac{3\,bc^2\log\left(2\,\sqrt{dx^2+c}dx-\left(2\,dx^2+c\right)\sqrt{d}\right)+2\left(6\,bdx^3+8\,adx^2+3\,bcx+8\,ac\right)\sqrt{dx^2+c}\sqrt{d}}{48\,d^{\frac{3}{2}}},\right.$$
$$\left.-\frac{3\,bc^2\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-\left(6\,bdx^3+8\,adx^2+3\,bcx+8\,ac\right)\sqrt{dx^2+c}\sqrt{-d}}{24\,\sqrt{-d}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)*x,x, algorithm="fricas")
```

```
[Out] [1/48*(3*b*c^2*log(2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d))
 + 2*(6*b*d*x^3 + 8*a*d*x^2 + 3*b*c*x + 8*a*c)*sqrt(d*x^2 + c)*sq
rt(d))/d^(3/2), -1/24*(3*b*c^2*arctan(sqrt(-d)*x/sqrt(d*x^2 + c))
 - (6*b*d*x^3 + 8*a*d*x^2 + 3*b*c*x + 8*a*c)*sqrt(d*x^2 + c)*sqrt
(-d))/(sqrt(-d)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{c+dx^2}\sqrt{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)
```

GIAC/XCAS [A] time = 0.271875, size = 132, normalized size = 0.82

$$\frac{bc^2\ln\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)\operatorname{sign}(bx+a)}{8\,d^{\frac{3}{2}}} + \frac{1}{24}\sqrt{dx^2+c}\left(\left(2\,(3\,bx\operatorname{sign}(bx+a)+4\,a\operatorname{sign}(bx+a))x+\frac{3\,bc\operatorname{sign}(bx+a)}{d}\right)x+\frac{8\,ac\operatorname{sign}(bx+a)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)*x,x, algorithm="giac")
```

```
[Out] 1/8*b*c^2*ln(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sign(b*x + a)/d^(
3/2) + 1/24*sqrt(d*x^2 + c)*((2*(3*b*x*sign(b*x + a) + 4*a*sign(b
*x + a))*x + 3*b*c*sign(b*x + a)/d)*x + 8*a*c*sign(b*x + a)/d)
```

3.42 $\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=148

$$\frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{3d(a+bx)}+\frac{ax\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)}+\frac{ac\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)}$$

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*(a + b*x))
 + (b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(3*d*(a +
b*x)) + (a*c*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sq
rt[c + d*x^2]])/(2*Sqrt[d]*(a + b*x))
```

Rubi [A] time = 0.149308, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$

$$\frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{3d(a+bx)}+\frac{ax\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)}+\frac{ac\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*(a + b*x))
 + (b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(3*d*(a +
b*x)) + (a*c*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sq
rt[c + d*x^2]])/(2*Sqrt[d]*(a + b*x))
```

Rubi in Sympy [A] time = 26.1897, size = 136, normalized size = 0.92

$$\frac{ac\sqrt{a^2+2abx+b^2x^2}\,\mathrm{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}\,(a+bx)}+\frac{ax\sqrt{c+dx^2}\sqrt{a^2+2abx+b^2x^2}}{2\,(a+bx)}+\frac{b\left(c+dx^2\right)^{\frac{3}{2}}\sqrt{a^2+2abx+b^2x^2}}{3d\,(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] a*c*sqrt(a**2 + 2*a*b*x + b**2*x**2)*atanh(sqrt(d)*x/sqrt(c + d*x
**2))/(2*sqrt(d)*(a + b*x)) + a*x*sqrt(c + d*x**2)*sqrt(a**2 + 2*
a*b*x + b**2*x**2)/(2*(a + b*x)) + b*(c + d*x**2)**(3/2)*sqrt(a**
2 + 2*a*b*x + b**2*x**2)/(3*d*(a + b*x))
```

Mathematica [A] time = 0.0746258, size = 85, normalized size = 0.57

$$\frac{\sqrt{(a+bx)^2}\left(\sqrt{c+dx^2}\left(3adx+2b\left(c+dx^2\right)\right)+3ac\sqrt{d}\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)\right)}{6d(a+bx)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(Sqrt[c + d*x^2]*(3*a*d*x + 2*b*(c + d*x^2)) +
 3*a*c*Sqrt[d]*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]]))/(6*d*(a + b*x
```

```
))
```

Maple [C] time = 0.013, size = 65, normalized size = 0.4

$$\frac{\operatorname{csgn}(bx+a)}{6}\left(2\,b\left(dx^{2}+c\right)^{3/2}\sqrt{d}+3\,ax\sqrt{dx^{2}+c}\,d^{3/2}+3\,ac\ln\left(x\sqrt{d}+\sqrt{dx^{2}+c}\right)d\right)d^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/6*csgn(b*x+a)*(2*b*(d*x^2+c)^(3/2)*d^(1/2)+3*a*x*(d*x^2+c)^(1/2
)*d^(3/2)+3*a*c*ln(x*d^(1/2)+(d*x^2+c)^(1/2))*d)/d^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.339508, size = 1, normalized size = 0.01

$$\left[\frac{3\,acd\log\left(-2\,\sqrt{dx^{2}+c}dx-\left(2\,dx^{2}+c\right)\sqrt{d}\right)+2\left(2\,bdx^{2}+3\,adx+2\,bc\right)\sqrt{dx^{2}+c}\sqrt{d}}{12\,d^{\frac{3}{2}}},\ \frac{3\,acd\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^{2}+c}}\right)+\left(2\,bdx^{2}+3\right.}{6\,\sqrt{-d}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*c*d*log(-2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)
) + 2*(2*b*d*x^2 + 3*a*d*x + 2*b*c)*sqrt(d*x^2 + c)*sqrt(d))/d^(3
/2), 1/6*(3*a*c*d*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (2*b*d*x^2
 + 3*a*d*x + 2*b*c)*sqrt(d*x^2 + c)*sqrt(-d))/(sqrt(-d)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c+dx^{2}}\sqrt{(a+bx)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)
```

GIAC/XCAS [A] time = 0.271617, size = 107, normalized size = 0.72

$$-\frac{ac\ln\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)\operatorname{sign}(bx+a)}{2\sqrt{d}}+\frac{1}{6}\sqrt{dx^2+c}\left((2\,bx\operatorname{sign}(bx+a)+3\,a\operatorname{sign}(bx+a))x+\frac{2\,bc\operatorname{sign}(bx+a)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2),x, algorithm="giac")
```

```
[Out] -1/2*a*c*ln(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sign(b*x + a)/sqrt
(d) + 1/6*sqrt(d*x^2 + c)*((2*b*x*sign(b*x + a) + 3*a*sign(b*x +
a))*x + 2*b*c*sign(b*x + a)/d)
```

3.43 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx$

Optimal. Leaf size=160

$$\frac{\sqrt{a^2+2abx+b^2x^2}(2a+bx)\sqrt{c+dx^2}}{2(a+bx)}+\frac{bc\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)}-\frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}$$

```
[Out] ((2*a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*(a
 + b*x)) + (b*c*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)
/Sqrt[c + d*x^2]])/(2*Sqrt[d]*(a + b*x)) - (a*Sqrt[c]*Sqrt[a^2 +
2*a*b*x + b^2*x^2]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(a + b*x)
```

Rubi [A] time = 0.377451, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$

$$\frac{\sqrt{a^2+2abx+b^2x^2}(2a+bx)\sqrt{c+dx^2}}{2(a+bx)}+\frac{bc\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)}-\frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x,x]
```

```
[Out] ((2*a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*(a
 + b*x)) + (b*c*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)
/Sqrt[c + d*x^2]])/(2*Sqrt[d]*(a + b*x)) - (a*Sqrt[c]*Sqrt[a^2 +
2*a*b*x + b^2*x^2]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(a + b*x)
```

Rubi in Sympy [A] time = 42.1211, size = 150, normalized size = 0.94

$$-\frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}+\frac{bc\sqrt{a^2+2abx+b^2x^2}\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}\,(a+bx)}+\frac{(4a+2bx)\sqrt{c+dx^2}\sqrt{a^2+2abx+b^2x^2}}{4\,(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x,x)
```

```
[Out] -a*sqrt(c)*sqrt(a**2 + 2*a*b*x + b**2*x**2)*atanh(sqrt(c + d*x**2
)/sqrt(c))/(a + b*x) + b*c*sqrt(a**2 + 2*a*b*x + b**2*x**2)*atanh
(sqrt(d)*x/sqrt(c + d*x**2))/(2*sqrt(d)*(a + b*x)) + (4*a + 2*b*x
)*sqrt(c + d*x**2)*sqrt(a**2 + 2*a*b*x + b**2*x**2)/(4*(a + b*x))
```

Mathematica [A] time = 0.106102, size = 139, normalized size = 0.87

$$\frac{\sqrt{(a+bx)^2}\left(2a\sqrt{d}\sqrt{c+dx^2}-2a\sqrt{c}\sqrt{d}\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)+2a\sqrt{c}\sqrt{d}\log(x)+b\sqrt{d}x\sqrt{c+dx^2}+bc\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right.\right.}{2\sqrt{d}(a+bx)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(2*a*Sqrt[d]*Sqrt[c + d*x^2] + b*Sqrt[d]*x*Sqr
t[c + d*x^2] + 2*a*Sqrt[c]*Sqrt[d]*Log[x] - 2*a*Sqrt[c]*Sqrt[d]*L
og[c + Sqrt[c]*Sqrt[c + d*x^2]] + b*c*Log[d*x + Sqrt[d]*Sqrt[c +
d*x^2]]))/(2*Sqrt[d]*(a + b*x))
```

Maple [C] time = 0.014, size = 94, normalized size = 0.6

$$-\frac{csgn(bx+a)}{2}\left(2\sqrt{c}\ln\left(2\,\frac{\sqrt{c}\sqrt{dx^2+c}+c}{x}\right)a\sqrt{d}-bx\sqrt{dx^2+c}\sqrt{d}-2\sqrt{dx^2+c}a\sqrt{d}-bc\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\right)\frac{1}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x)
```

```
[Out] -1/2*csgn(b*x+a)*(2*c^(1/2)*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*a
*d^(1/2)-b*x*(d*x^2+c)^(1/2)*d^(1/2)-2*(d*x^2+c)^(1/2)*a*d^(1/2)-
b*c*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))/d^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.341941, size = 1, normalized size = 0.01

$$\left[\frac{bc\log\left(-2\sqrt{dx^2+c}dx-\left(2dx^2+c\right)\sqrt{d}\right)+2a\sqrt{c}\sqrt{d}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\sqrt{dx^2+c}(bx+2a)\sqrt{d}}{4\sqrt{d}},\frac{bc\arctan\left(\frac{\sqrt{-d}}{\sqrt{dx^2}}\right.}{}\right.$$

$$-\frac{4a\sqrt{-c}\sqrt{d}\arctan\left(\frac{c}{\sqrt{dx^2+c}\sqrt{-c}}\right)-bc\log\left(-2\sqrt{dx^2+c}dx-\left(2dx^2+c\right)\sqrt{d}\right)-2\sqrt{dx^2+c}(bx+2a)\sqrt{d}}{4\sqrt{d}},\frac{bc\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*c*log(-2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)) + 2
*a*sqrt(c)*sqrt(d)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)
/x^2) + 2*sqrt(d*x^2 + c)*(b*x + 2*a)*sqrt(d))/sqrt(d), 1/2*(b*c*
arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + a*sqrt(c)*sqrt(-d)*log(-(d*x
^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + sqrt(d*x^2 + c)*(b*x
 + 2*a)*sqrt(-d))/sqrt(-d), -1/4*(4*a*sqrt(-c)*sqrt(d)*arctan(c/(
sqrt(d*x^2 + c)*sqrt(-c))) - b*c*log(-2*sqrt(d*x^2 + c)*d*x - (2*
d*x^2 + c)*sqrt(d)) - 2*sqrt(d*x^2 + c)*(b*x + 2*a)*sqrt(d))/sqrt
(d), 1/2*(b*c*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - 2*a*sqrt(-c)*s
```

```
qrt(-d)*arctan(c/(sqrt(d*x^2 + c)*sqrt(-c))) + sqrt(d*x^2 + c)*(b
*x + 2*a)*sqrt(-d))/sqrt(-d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}\sqrt{(a+bx)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x, x)
```

GIAC/XCAS [A] time = 0.274964, size = 138, normalized size = 0.86

$$\frac{2\,ac\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^2+c}}{\sqrt{-c}}\right)\operatorname{sign}(bx+a)}{\sqrt{-c}}-\frac{bc\ln\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)\operatorname{sign}(bx+a)}{2\sqrt{d}}+\frac{1}{2}\sqrt{dx^2+c}\left(bx\operatorname{sign}(bx+a)+2\,a\operatorname{sign}(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c*arctan(-(sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c))*sign(b*x +
a)/sqrt(-c) - 1/2*b*c*ln(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sign(
b*x + a)/sqrt(d) + 1/2*sqrt(d*x^2 + c)*(b*x*sign(b*x + a) + 2*a*s
ign(b*x + a))
```

3.44 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2}\,dx$

Optimal. Leaf size=156

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(a-bx)\sqrt{c+dx^2}}{x(a+bx)}+\frac{a\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx}-\frac{b\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}$$

```
[Out] -(((a - b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(x*(a
 + b*x))) + (a*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqr
t[d]*x)/Sqrt[c + d*x^2]])/(a + b*x) - (b*Sqrt[c]*Sqrt[a^2 + 2*a*b
*x + b^2*x^2]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(a + b*x)
```

Rubi [A] time = 0.371093, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(a-bx)\sqrt{c+dx^2}}{x(a+bx)}+\frac{a\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx}-\frac{b\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^2,x]
```

```
[Out] -(((a - b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(x*(a
 + b*x))) + (a*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqr
t[d]*x)/Sqrt[c + d*x^2]])/(a + b*x) - (b*Sqrt[c]*Sqrt[a^2 + 2*a*b
*x + b^2*x^2]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(a + b*x)
```

Rubi in Sympy [A] time = 38.2923, size = 146, normalized size = 0.94

$$\frac{a\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\,\mathrm{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx}-\frac{b\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\,\mathrm{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}-\frac{(2a-2bx)\sqrt{c+dx^2}\sqrt{a^2+2abx+b^2x^2}}{2x(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x**2,x)
```

```
[Out] a*sqrt(d)*sqrt(a**2 + 2*a*b*x + b**2*x**2)*atanh(sqrt(d)*x/sqrt(c
 + d*x**2))/(a + b*x) - b*sqrt(c)*sqrt(a**2 + 2*a*b*x + b**2*x**2
)*atanh(sqrt(c + d*x**2)/sqrt(c))/(a + b*x) - (2*a - 2*b*x)*sqrt(
c + d*x**2)*sqrt(a**2 + 2*a*b*x + b**2*x**2)/(2*x*(a + b*x))
```

Mathematica [A] time = 0.0794963, size = 120, normalized size = 0.77

$$\frac{\sqrt{(a+bx)^2}\left(-a\sqrt{c+dx^2}+a\sqrt{d}x\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+bx\sqrt{c+dx^2}-b\sqrt{c}x\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)+b\sqrt{c}x\log(x)\right)}{x(a+bx)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^2,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(-(a*Sqrt[c + d*x^2]) + b*x*Sqrt[c + d*x^2] +
b*Sqrt[c]*x*Log[x] - b*Sqrt[c]*x*Log[c + Sqrt[c]*Sqrt[c + d*x^2]]
 + a*Sqrt[d]*x*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]]))/(x*(a + b*x))
```

Maple [C] time = 0.017, size = 108, normalized size = 0.7

$$-\frac{csgn\left(bx+a\right)}{cx}\left(c^{\frac{3}{2}}\ln\left(2\,\frac{\sqrt{c}\sqrt{dx^2+c}+c}{x}\right)bx-a\sqrt{d}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)cx-adx^2\sqrt{dx^2+c}+a\left(dx^2+c\right)^{\frac{3}{2}}-\sqrt{dx^2+c}bc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x)
```

```
[Out] -csgn(b*x+a)*(c^(3/2)*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*b*x-a*d
^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))*c*x-a*d*x^2*(d*x^2+c)^(1/2)+
a*(d*x^2+c)^(3/2)-(d*x^2+c)^(1/2)*b*c*x)/c/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.335737, size = 1, normalized size = 0.01

$$\left[\frac{a\sqrt{d}x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+b\sqrt{c}x\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,\sqrt{dx^2+c}(bx-a)}{2\,x},\ \frac{2\,a\sqrt{-d}x\arctan\left(\frac{dx}{\sqrt{dx^2+c}\right.}\right.}{}\right.$$

$$-\frac{2\,b\sqrt{-c}x\arctan\left(\frac{c}{\sqrt{dx^2+c}\sqrt{-c}}\right)-a\sqrt{d}x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-2\,\sqrt{dx^2+c}(bx-a)}{2\,x},\ \frac{a\sqrt{-d}x\arctan\left(\frac{dx}{\sqrt{dx^2+c}\sqrt{-}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c)
 + b*sqrt(c)*x*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2
) + 2*sqrt(d*x^2 + c)*(b*x - a))/x, 1/2*(2*a*sqrt(-d)*x*arctan(d*
x/(sqrt(d*x^2 + c)*sqrt(-d))) + b*sqrt(c)*x*log(-(d*x^2 - 2*sqrt(
d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*sqrt(d*x^2 + c)*(b*x - a))/x,
-1/2*(2*b*sqrt(-c)*x*arctan(c/(sqrt(d*x^2 + c)*sqrt(-c))) - a*sqr
t(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*sqrt(d
*x^2 + c)*(b*x - a))/x, (a*sqrt(-d)*x*arctan(d*x/(sqrt(d*x^2 + c)
*sqrt(-d))) - b*sqrt(-c)*x*arctan(c/(sqrt(d*x^2 + c)*sqrt(-c))) +
 sqrt(d*x^2 + c)*(b*x - a))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^2}\sqrt{(a + bx)^2}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x**2, x)
```

GIAC/XCAS [A] time = 0.277115, size = 170, normalized size = 1.09

$$\frac{2\,bc\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^2+c}}{\sqrt{-c}}\right)\operatorname{sign}\left(bx+a\right)}{\sqrt{-c}} - a\sqrt{d}\ln\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)\operatorname{sign}\left(bx+a\right) + \sqrt{dx^2+c}b\operatorname{sign}\left(bx+a\right) + \frac{2\,ac\sqrt{d}\operatorname{sign}\left(bx+a\right)}{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x^2,x, algorithm="giac")
```

```
[Out] 2*b*c*arctan(-(sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c))*sign(b*x +
a)/sqrt(-c) - a*sqrt(d)*ln(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sig
n(b*x + a) + sqrt(d*x^2 + c)*b*sign(b*x + a) + 2*a*c*sqrt(d)*sign
(b*x + a)/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)
```

3.45 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3}\,dx$

Optimal. Leaf size=161

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(a+2bx)\sqrt{c+dx^2}}{2x^2(a+bx)}+\frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx}-\frac{ad\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}(a+bx)}$$

```
[Out] -((a + 2*b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*x
^2*(a + b*x)) + (b*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[
(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(a + b*x) - (a*d*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*Sqrt[c]*(a + b*x
))
```

Rubi [A] time = 0.380228, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(a+2bx)\sqrt{c+dx^2}}{2x^2(a+bx)}+\frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx}-\frac{ad\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}(a+bx)}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^3,x]
```

```
[Out] -((a + 2*b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*x
^2*(a + b*x)) + (b*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[
(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(a + b*x) - (a*d*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*Sqrt[c]*(a + b*x
))
```

Rubi in Sympy [A] time = 41.5783, size = 151, normalized size = 0.94

$$-\frac{ad\sqrt{a^2+2abx+b^2x^2}\,\text{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}\,(a+bx)}+\frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\,\text{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx}-\frac{(2a+4bx)\sqrt{c+dx^2}\sqrt{a^2+2abx+b^2x^2}}{4x^2\,(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x**3,x)
```

```
[Out] -a*d*sqrt(a**2 + 2*a*b*x + b**2*x**2)*atanh(sqrt(c + d*x**2)/sqrt
(c))/(2*sqrt(c)*(a + b*x)) + b*sqrt(d)*sqrt(a**2 + 2*a*b*x + b**2
*x**2)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(a + b*x) - (2*a + 4*b*x
)*sqrt(c + d*x**2)*sqrt(a**2 + 2*a*b*x + b**2*x**2)/(4*x**2*(a +
b*x))
```

Mathematica [A] time = 0.179201, size = 125, normalized size = 0.78

$$\frac{\sqrt{(a+bx)^2}\left(\sqrt{c}\left(2b\sqrt{d}x^2\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)-(a+2bx)\sqrt{c+dx^2}\right)-adx^2\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)+adx^2\log(x)\right)}{2\sqrt{c}x^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^3,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(a*d*x^2*Log[x] - a*d*x^2*Log[c + Sqrt[c]*Sqrt
[c + d*x^2]] + Sqrt[c]*(-((a + 2*b*x)*Sqrt[c + d*x^2]) + 2*b*Sqrt
[d]*x^2*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])))/(2*Sqrt[c]*x^2*(a +
 b*x))
```

Maple [C] time = 0.018, size = 141, normalized size = 0.9

$$\frac{csgn\,(bx+a)}{2\,x^2}\left(2\,b\sqrt{d}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)x^2c^{3/2}+2\,bdx^3\sqrt{dx^2+c}\sqrt{c}-ad\ln\left(2\,\frac{\sqrt{c}\sqrt{dx^2+c}+c}{x}\right)x^2c-2\,b\left(dx^2+c\right)^{3/2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x)
```

```
[Out] 1/2*csgn(b*x+a)*(2*b*d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))*x^2*c^
(3/2)+2*b*d*x^3*(d*x^2+c)^(1/2)*c^(1/2)-a*d*ln(2*(c^(1/2)*(d*x^2+
c)^(1/2)+c)/x)*x^2*c-2*b*(d*x^2+c)^(3/2)*x*c^(1/2)+a*d*(d*x^2+c)^
(1/2)*x^2*c^(1/2)-a*(d*x^2+c)^(3/2)*c^(1/2))/x^2/c^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.334976, size = 1, normalized size = 0.01

$$\left[\frac{2\,b\sqrt{c}\sqrt{d}x^2\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+adx^2\log\left(-\frac{(dx^2+2\,c)\sqrt{c}-2\,\sqrt{dx^2+c}c}{x^2}\right)-2\,\sqrt{dx^2+c}(2\,bx+a)\sqrt{c}}{4\,\sqrt{c}x^2},\ \frac{4\,b\sqrt{c}\sqrt{-dx}}{}\right.$$

$$-\frac{adx^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-b\sqrt{-c}\sqrt{d}x^2\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+\sqrt{dx^2+c}(2\,bx+a)\sqrt{-c}}{2\,\sqrt{-c}x^2},\ \frac{2\,b\sqrt{-c}\sqrt{-d}x^2\arctan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*sqrt(c)*sqrt(d)*x^2*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sq
rt(d)*x - c) + a*d*x^2*log(-((d*x^2 + 2*c)*sqrt(c) - 2*sqrt(d*x^2
 + c)*c)/x^2) - 2*sqrt(d*x^2 + c)*(2*b*x + a)*sqrt(c))/(sqrt(c)*x
^2), 1/4*(4*b*sqrt(c)*sqrt(-d)*x^2*arctan(d*x/(sqrt(d*x^2 + c)*sq
rt(-d))) + a*d*x^2*log(-((d*x^2 + 2*c)*sqrt(c) - 2*sqrt(d*x^2 + c
)*c)/x^2) - 2*sqrt(d*x^2 + c)*(2*b*x + a)*sqrt(c))/(sqrt(c)*x^2),
 -1/2*(a*d*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - b*sqrt(-c)*sqrt
(d)*x^2*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + sqrt(d*
x^2 + c)*(2*b*x + a)*sqrt(-c))/(sqrt(-c)*x^2), 1/2*(2*b*sqrt(-c)*
sqrt(-d)*x^2*arctan(d*x/(sqrt(d*x^2 + c)*sqrt(-d))) - a*d*x^2*arc
tan(sqrt(-c)/sqrt(d*x^2 + c)) - sqrt(d*x^2 + c)*(2*b*x + a)*sqrt(
-c))/(sqrt(-c)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^2}\sqrt{(a + bx)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x**3, x)
```

GIAC/XCAS [A] time = 0.282307, size = 269, normalized size = 1.67

$$\frac{ad\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^2+c}}{\sqrt{-c}}\right)\operatorname{sign}(bx+a)}{\sqrt{-c}} - b\sqrt{d}\ln\left(\left|-\sqrt{d}x + \sqrt{dx^2+c}\right|\right)\operatorname{sign}(bx+a) + \frac{\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^3 ad\operatorname{sign}(bx+a) + 2\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 bc\sqrt{d}\operatorname{sign}(bx+a) + \left(\sqrt{d}x - \sqrt{dx^2+c}\right)acd\operatorname{sign}(bx+a) - 2\,bc}{\left(\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 - c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x^3,x, algorithm="giac")
```

```
[Out] a*d*arctan(-(sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c))*sign(b*x + a)
/sqrt(-c) - b*sqrt(d)*ln(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sign(
b*x + a) + ((sqrt(d)*x - sqrt(d*x^2 + c))^3*a*d*sign(b*x + a) + 2
*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c*sqrt(d)*sign(b*x + a) + (sqr
t(d)*x - sqrt(d*x^2 + c))*a*c*d*sign(b*x + a) - 2*b*c^2*sqrt(d)*s
ign(b*x + a))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^2
```

3.46 $\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$

Optimal. Leaf size=317

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(2dx+e)\sqrt{c+dx^2+ex}\left(2ad\left(4cd-5e^2\right)-b\left(12cde-7e^3\right)\right)}{128d^4(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2+ex\right)^{3/2}\left(-6dx(10ad-7be)+50ade+32bcd-35be^2\right)}{240d^3(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\left(4cd-e^2\right)\left(8acd^2-10ade^2-12bcde+7be^3\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{256d^{9/2}(a+bx)}+\frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2+ex\right)^{3/2}}{5d(a+bx)}$$

```
[Out] -((2*a*d*(4*c*d - 5*e^2) - b*(12*c*d*e - 7*e^3))*(e + 2*d*x)*Sqrt
[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/(128*d^4*(a + b*
x)) + (b*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3/2
))/(5*d*(a + b*x)) - ((32*b*c*d + 50*a*d*e - 35*b*e^2 - 6*d*(10*a
*d - 7*b*e)*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3
/2))/(240*d^3*(a + b*x)) - ((4*c*d - e^2)*(8*a*c*d^2 - 12*b*c*d*e
 - 10*a*d*e^2 + 7*b*e^3)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(e
 + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(256*d^(9/2)*(a + b
*x))
```

Rubi [A] time = 0.833034, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2+ex\right)^{3/2}\left(-6dx(10ad-7be)+50ade+32bcd-35be^2\right)}{240d^3(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\left(4cd-e^2\right)\left(8acd^2-10ade^2-12bcde+7be^3\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{256d^{9/2}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}(2dx+e)\sqrt{c+dx^2+ex}\left(8acd^2-10ade^2-12bcde+7be^3\right)}{128d^4(a+bx)}+\frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2+ex\right)^{3/2}}{5d(a+bx)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] -((8*a*c*d^2 - 12*b*c*d*e - 10*a*d*e^2 + 7*b*e^3)*(e + 2*d*x)*Sqr
t[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/(128*d^4*(a + b
*x)) + (b*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3/
2))/(5*d*(a + b*x)) - ((32*b*c*d + 50*a*d*e - 35*b*e^2 - 6*d*(10*
a*d - 7*b*e)*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(
3/2))/(240*d^3*(a + b*x)) - ((4*c*d - e^2)*(8*a*c*d^2 - 12*b*c*d*
e - 10*a*d*e^2 + 7*b*e^3)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(
e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(256*d^(9/2)*(a +
b*x))
```

Rubi in Sympy [A] time = 71.2277, size = 313, normalized size = 0.99

$$\frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2+ex\right)^{\frac{3}{2}}}{5d\,(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2+ex\right)^{\frac{3}{2}}\left(-25ade-16bcd+\frac{35be^2}{2}+3dx\,(10ad-7be)\right)}{120d^3\,(a+bx)} - \frac{(2dx+e)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}\left(8acd^2-10ade^2-12bcde+7be^3\right)}{128d^4\,(a+bx)} + \frac{\left(-4cd+e^2\right)\sqrt{a^2+2abx+b^2x^2}\left(8acd^2-10ade^2-12bcde+7be^3\right)\operatorname{atanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{256d^{\frac{9}{2}}\,(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
```

```
[Out] b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2)*(c + d*x**2 + e*x)**(3/2)
/(5*d*(a + b*x)) + sqrt(a**2 + 2*a*b*x + b**2*x**2)*(c + d*x**2 +
 e*x)**(3/2)*(-25*a*d*e - 16*b*c*d + 35*b*e**2/2 + 3*d*x*(10*a*d
- 7*b*e))/(120*d**3*(a + b*x)) - (2*d*x + e)*sqrt(a**2 + 2*a*b*x
+ b**2*x**2)*sqrt(c + d*x**2 + e*x)*(8*a*c*d**2 - 10*a*d*e**2 - 1
2*b*c*d*e + 7*b*e**3)/(128*d**4*(a + b*x)) + (-4*c*d + e**2)*sqrt
(a**2 + 2*a*b*x + b**2*x**2)*(8*a*c*d**2 - 10*a*d*e**2 - 12*b*c*d
*e + 7*b*e**3)*atanh((2*d*x + e)/(2*sqrt(d)*sqrt(c + d*x**2 + e*x
)))/(256*d**(9/2)*(a + b*x))
```

Mathematica [A] time = 0.431116, size = 236, normalized size = 0.74

$$\frac{\sqrt{(a+bx)^2}\left(\frac{2\sqrt{c+x(dx+e)}\left(10ad\left(4cd(6dx-13e)+48d^3x^3+8d^2ex^2-10de^2x+15e^3\right)+b\left(-256c^2d^2+4cd\left(32d^2x^2-58dex+115e^2\right)+384d^4x^4+48d^3ex^3-56d^2e^2x^2\right.\right.}{15d^4}\right.}{256(a+bx)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*((2*Sqrt[c + x*(e + d*x)]*(10*a*d*(15*e^3 - 10
*d*e^2*x + 8*d^2*e*x^2 + 48*d^3*x^3 + 4*c*d*(-13*e + 6*d*x)) + b*
(-256*c^2*d^2 - 105*e^4 + 70*d*e^3*x - 56*d^2*e^2*x^2 + 48*d^3*e*
x^3 + 384*d^4*x^4 + 4*c*d*(115*e^2 - 58*d*e*x + 32*d^2*x^2))))/(1
5*d^4) - ((4*c*d - e^2)*(2*a*d*(4*c*d - 5*e^2) + b*(-12*c*d*e + 7
*e^3))*Log[e + 2*d*x + 2*Sqrt[d]*Sqrt[c + x*(e + d*x)]])/d^(9/2))
)/(256*(a + b*x))
```

Maple [C] time = 0.022, size = 532, normalized size = 1.7

$$-\frac{\operatorname{csgn}(bx+a)}{3840}\left(-768\,bx^2\left(dx^2+ex+c\right)^{3/2}d^{15/2}-960\,ax\left(dx^2+ex+c\right)^{3/2}d^{15/2}+672\,bex\left(dx^2+ex+c\right)^{3/2}d^{13/2}+800\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x)
```

```
[Out] -1/3840*csgn(b*x+a)*(-768*b*x^2*(d*x^2+e*x+c)^(3/2)*d^(15/2)-960*
a*x*(d*x^2+e*x+c)^(3/2)*d^(15/2)+672*b*e*x*(d*x^2+e*x+c)^(3/2)*d^
(13/2)+800*a*e*(d*x^2+e*x+c)^(3/2)*d^(13/2)+512*b*c*(d*x^2+e*x+c)
```

```
^(3/2)*d^(13/2)-560*b*e^2*(d*x^2+e*x+c)^(3/2)*d^(11/2)+480*a*c*(d
*x^2+e*x+c)^(1/2)*x*d^(15/2)-600*a*e^2*(d*x^2+e*x+c)^(1/2)*x*d^(1
3/2)-720*b*e*c*(d*x^2+e*x+c)^(1/2)*x*d^(13/2)+420*b*e^3*(d*x^2+e*
x+c)^(1/2)*x*d^(11/2)+240*a*c*(d*x^2+e*x+c)^(1/2)*e*d^(13/2)-300*
a*e^3*(d*x^2+e*x+c)^(1/2)*d^(11/2)-360*b*e^2*c*(d*x^2+e*x+c)^(1/2
)*d^(11/2)+210*b*e^4*(d*x^2+e*x+c)^(1/2)*d^(9/2)+480*a*c^2*ln(1/2
*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*d^7-720*a*e^2*l
n(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*c*d^6-720*
b*e*c^2*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*d
^6+150*a*e^4*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/
2))*d^5+600*b*e^3*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/
d^(1/2))*c*d^5-105*b*e^5*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*
d*x+e)/d^(1/2))*d^4)/d^(17/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.353961, size = 1, normalized size = 0.

$$\left[\frac{4\left(384\,bd^4x^4 - 256\,bc^2d^2 - 520\,acd^2e + 460\,bcde^2 + 150\,ade^3 - 105\,be^4 + 48\left(10\,ad^4 + bd^3e\right)x^3 + 8\left(16\,bcd^3 + 10\,ad^3e - 7\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)*x^2,x, algorithm="fricas")
```

```
[Out] [1/7680*(4*(384*b*d^4*x^4 - 256*b*c^2*d^2 - 520*a*c*d^2*e + 460*b
*c*d*e^2 + 150*a*d*e^3 - 105*b*e^4 + 48*(10*a*d^4 + b*d^3*e)*x^3
+ 8*(16*b*c*d^3 + 10*a*d^3*e - 7*b*d^2*e^2)*x^2 + 2*(120*a*c*d^3
- 116*b*c*d^2*e - 50*a*d^2*e^2 + 35*b*d*e^3)*x)*sqrt(d*x^2 + e*x
+ c)*sqrt(d) - 15*(32*a*c^2*d^3 - 48*b*c^2*d^2*e - 48*a*c*d^2*e^2
 + 40*b*c*d*e^3 + 10*a*d*e^4 - 7*b*e^5)*log(4*(2*d^2*x + d*e)*sqr
t(d*x^2 + e*x + c) + (8*d^2*x^2 + 8*d*e*x + 4*c*d + e^2)*sqrt(d))
)/d^(9/2), 1/3840*(2*(384*b*d^4*x^4 - 256*b*c^2*d^2 - 520*a*c*d^2
*e + 460*b*c*d*e^2 + 150*a*d*e^3 - 105*b*e^4 + 48*(10*a*d^4 + b*d
^3*e)*x^3 + 8*(16*b*c*d^3 + 10*a*d^3*e - 7*b*d^2*e^2)*x^2 + 2*(12
0*a*c*d^3 - 116*b*c*d^2*e - 50*a*d^2*e^2 + 35*b*d*e^3)*x)*sqrt(d*
x^2 + e*x + c)*sqrt(-d) - 15*(32*a*c^2*d^3 - 48*b*c^2*d^2*e - 48*
a*c*d^2*e^2 + 40*b*c*d*e^3 + 10*a*d*e^4 - 7*b*e^5)*arctan(1/2*(2*
d*x + e)*sqrt(-d)/(sqrt(d*x^2 + e*x + c)*d)))/(sqrt(-d)*d^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.289018, size = 497, normalized size = 1.57

$$\frac{1}{1920}\sqrt{dx^2+xe+c}\left(2\left(4\left(6\left(8\,bx\text{sign}\left(bx+a\right)+\frac{10\,ad^4\text{sign}\left(bx+a\right)+bd^3e\text{sign}\left(bx+a\right)}{d^4}\right)x+\frac{16\,bcd^3\text{sign}\left(bx+a\right)+10\,ad^3 \cdots}{\cdots}\right.\right.\right.$$

$$+\frac{\left(32\,ac^2d^3\text{sign}\left(bx+a\right)-48\,bc^2d^2e\text{sign}\left(bx+a\right)-48\,acd^2e^2\text{sign}\left(bx+a\right)+40\,bcde^3\text{sign}\left(bx+a\right)+10\,ade^4\text{sign}\left(bx+a\right)-\cdots\right.}{256\,d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)*x^2,x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(d*x^2 + x*e + c)*(2*(4*(6*(8*b*x*sign(b*x + a) + (10*
a*d^4*sign(b*x + a) + b*d^3*e*sign(b*x + a))/d^4)*x + (16*b*c*d^3
*sign(b*x + a) + 10*a*d^3*e*sign(b*x + a) - 7*b*d^2*e^2*sign(b*x
+ a))/d^4)*x + (120*a*c*d^3*sign(b*x + a) - 116*b*c*d^2*e*sign(b*
x + a) - 50*a*d^2*e^2*sign(b*x + a) + 35*b*d*e^3*sign(b*x + a))/d
^4)*x - (256*b*c^2*d^2*sign(b*x + a) + 520*a*c*d^2*e*sign(b*x + a
) - 460*b*c*d*e^2*sign(b*x + a) - 150*a*d*e^3*sign(b*x + a) + 105
*b*e^4*sign(b*x + a))/d^4) + 1/256*(32*a*c^2*d^3*sign(b*x + a) -
48*b*c^2*d^2*e*sign(b*x + a) - 48*a*c*d^2*e^2*sign(b*x + a) + 40*
b*c*d*e^3*sign(b*x + a) + 10*a*d*e^4*sign(b*x + a) - 7*b*e^5*sign
(b*x + a))*ln(abs(-2*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))*sqrt(d)
- e))/d^(9/2)
```

3.47 $\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$

Optimal. Leaf size=227

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\left(4cd-e^2\right)\left(8ade+4bcd-5be^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{128d^{7/2}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}(2dx+e)\sqrt{c+dx^2+ex}\left(8ade+4bcd-5be^2\right)}{64d^3(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2+ex\right)^{3/2}(8ad+6bdx-5be)}{24d^2(a+bx)}$$

```
[Out] -((4*b*c*d + 8*a*d*e - 5*b*e^2)*(e + 2*d*x)*Sqrt[a^2 + 2*a*b*x +
b^2*x^2]*Sqrt[c + e*x + d*x^2])/(64*d^3*(a + b*x)) + ((8*a*d - 5*
b*e + 6*b*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3
/2))/(24*d^2*(a + b*x)) - ((4*c*d - e^2)*(4*b*c*d + 8*a*d*e - 5*b
*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d
]*Sqrt[c + e*x + d*x^2])])/(128*d^(7/2)*(a + b*x))
```

Rubi [A] time = 0.376417, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\left(4cd-e^2\right)\left(8ade+4bcd-5be^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{128d^{7/2}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}(2dx+e)\sqrt{c+dx^2+ex}\left(8ade+4bcd-5be^2\right)}{64d^3(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2+ex\right)^{3/2}(8ad+6bdx-5be)}{24d^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] -((4*b*c*d + 8*a*d*e - 5*b*e^2)*(e + 2*d*x)*Sqrt[a^2 + 2*a*b*x +
b^2*x^2]*Sqrt[c + e*x + d*x^2])/(64*d^3*(a + b*x)) + ((8*a*d - 5*
b*e + 6*b*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3
/2))/(24*d^2*(a + b*x)) - ((4*c*d - e^2)*(4*b*c*d + 8*a*d*e - 5*b
*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d
]*Sqrt[c + e*x + d*x^2])])/(128*d^(7/2)*(a + b*x))
```

Rubi in Sympy [A] time = 47.4351, size = 221, normalized size = 0.97

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2+ex\right)^{\frac{3}{2}}(8ad+6bdx-5be)}{24d^2\,(a+bx)}+\frac{(2dx+e)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}\left(-8ade-4bcd+5be^2\right)}{64d^3\,(a+bx)}-\frac{\left(-4cd+e^2\right)\sqrt{a^2+2abx+b^2x^2}\left(-8ade-4bcd+5be^2\right)\operatorname{atanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{128d^{\frac{7}{2}}\,(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
```

```
[Out] sqrt(a**2 + 2*a*b*x + b**2*x**2)*(c + d*x**2 + e*x)**(3/2)*(8*a*d
 + 6*b*d*x - 5*b*e)/(24*d**2*(a + b*x)) + (2*d*x + e)*sqrt(a**2 +
```

```
 2*a*b*x + b**2*x**2)*sqrt(c + d*x**2 + e*x)*(-8*a*d*e - 4*b*c*d
+ 5*b*e**2)/(64*d**3*(a + b*x)) - (-4*c*d + e**2)*sqrt(a**2 + 2*a
*b*x + b**2*x**2)*(-8*a*d*e - 4*b*c*d + 5*b*e**2)*atanh((2*d*x +
e)/(2*sqrt(d)*sqrt(c + d*x**2 + e*x)))/(128*d**(7/2)*(a + b*x))
```

Mathematica [A] time = 0.35998, size = 176, normalized size = 0.78

$$\frac{\sqrt{(a+bx)^2}\left(2\sqrt{d}\sqrt{c+x(dx+e)}\left(8ad\left(8cd+8d^2x^2+2dex-3e^2\right)+b\left(4cd(6dx-13e)+48d^3x^3+8d^2ex^2-10de^2x+15e^3\right)\right)\right.}{384d^{7/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(2*Sqrt[d]*Sqrt[c + x*(e + d*x)]*(8*a*d*(8*c*d
 - 3*e^2 + 2*d*e*x + 8*d^2*x^2) + b*(15*e^3 - 10*d*e^2*x + 8*d^2*
e*x^2 + 48*d^3*x^3 + 4*c*d*(-13*e + 6*d*x))) - 3*(4*c*d - e^2)*(4
*b*c*d + 8*a*d*e - 5*b*e^2)*Log[e + 2*d*x + 2*Sqrt[d]*Sqrt[c + x*
(e + d*x)]]))/(384*d^(7/2)*(a + b*x))
```

Maple [C] time = 0.016, size = 383, normalized size = 1.7

$$-\frac{csgn\left(bx+a\right)}{384}\left(-96\,bx\left(dx^2+ex+c\right)^{3/2}d^{11/2}-128\,a\left(dx^2+ex+c\right)^{3/2}d^{11/2}+80\,be\left(dx^2+ex+c\right)^{3/2}d^{9/2}+96\,aex\sqrt{dx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x)
```

```
[Out] -1/384*csgn(b*x+a)*(-96*b*x*(d*x^2+e*x+c)^(3/2)*d^(11/2)-128*a*(d
*x^2+e*x+c)^(3/2)*d^(11/2)+80*b*e*(d*x^2+e*x+c)^(3/2)*d^(9/2)+96*
a*e*x*(d*x^2+e*x+c)^(1/2)*d^(11/2)+48*b*c*x*(d*x^2+e*x+c)^(1/2)*d
^(11/2)-60*b*e^2*x*(d*x^2+e*x+c)^(1/2)*d^(9/2)+48*a*e^2*(d*x^2+e*
x+c)^(1/2)*d^(9/2)+24*b*c*(d*x^2+e*x+c)^(1/2)*e*d^(9/2)-30*b*e^3*
(d*x^2+e*x+c)^(1/2)*d^(7/2)+96*a*e*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*
d^(1/2)+2*d*x+e)/d^(1/2))*c*d^5+48*b*c^2*ln(1/2*(2*(d*x^2+e*x+c)^
(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*d^5-24*a*e^3*ln(1/2*(2*(d*x^2+e*x
+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*d^4-72*b*e^2*ln(1/2*(2*(d*x^2
+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*c*d^4+15*b*e^4*ln(1/2*(2*
(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*d^3)/d^(13/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.336252, size = 1, normalized size = 0.

$$\left[\frac{4\left(48\,bd^3x^3+64\,acd^2-52\,bcde-24\,ade^2+15\,be^3+8\left(8\,ad^3+bd^2e\right)x^2+2\left(12\,bcd^2+8\,ad^2e-5\,bde^2\right)x\right)\sqrt{dx^2+ex+c}\sqrt{}}{76}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)*x,x, algorithm="fricas")
```

```
[Out] [1/768*(4*(48*b*d^3*x^3 + 64*a*c*d^2 - 52*b*c*d*e - 24*a*d*e^2 +
15*b*e^3 + 8*(8*a*d^3 + b*d^2*e)*x^2 + 2*(12*b*c*d^2 + 8*a*d^2*e
- 5*b*d*e^2)*x)*sqrt(d*x^2 + e*x + c)*sqrt(d) + 3*(16*b*c^2*d^2 +
 32*a*c*d^2*e - 24*b*c*d*e^2 - 8*a*d*e^3 + 5*b*e^4)*log(-4*(2*d^2
*x + d*e)*sqrt(d*x^2 + e*x + c) + (8*d^2*x^2 + 8*d*e*x + 4*c*d +
e^2)*sqrt(d)))/d^(7/2), 1/384*(2*(48*b*d^3*x^3 + 64*a*c*d^2 - 52*
b*c*d*e - 24*a*d*e^2 + 15*b*e^3 + 8*(8*a*d^3 + b*d^2*e)*x^2 + 2*(
12*b*c*d^2 + 8*a*d^2*e - 5*b*d*e^2)*x)*sqrt(d*x^2 + e*x + c)*sqrt
(-d) - 3*(16*b*c^2*d^2 + 32*a*c*d^2*e - 24*b*c*d*e^2 - 8*a*d*e^3
+ 5*b*e^4)*arctan(1/2*(2*d*x + e)*sqrt(-d)/(sqrt(d*x^2 + e*x + c)
*d)))/(sqrt(-d)*d^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.287873, size = 362, normalized size = 1.59

$$\frac{1}{192}\sqrt{dx^2+xe+c}\left(2\left(4\left(6\,bx\mathrm{sign}\left(bx+a\right)+\frac{8\,ad^3\mathrm{sign}\left(bx+a\right)+bd^2e\mathrm{sign}\left(bx+a\right)}{d^3}\right)x+\frac{12\,bcd^2\mathrm{sign}\left(bx+a\right)+8\,ad^2e\mathrm{sign}\left(\right.}{d^3}\right.\right.$$
$$+\frac{\left(16\,bc^2d^2\mathrm{sign}\left(bx+a\right)+32\,acd^2e\mathrm{sign}\left(bx+a\right)-24\,bcde^2\mathrm{sign}\left(bx+a\right)-8\,ade^3\mathrm{sign}\left(bx+a\right)+5\,be^4\mathrm{sign}\left(bx+a\right)\right)\ln\left(\left|-2\left(\right.\right.\right.}{128\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)*x,x, algorithm="giac")
```

```
[Out] 1/192*sqrt(d*x^2 + x*e + c)*(2*(4*(6*b*x*sign(b*x + a) + (8*a*d^3
*sign(b*x + a) + b*d^2*e*sign(b*x + a))/d^3)*x + (12*b*c*d^2*sign
(b*x + a) + 8*a*d^2*e*sign(b*x + a) - 5*b*d*e^2*sign(b*x + a))/d^
3)*x + (64*a*c*d^2*sign(b*x + a) - 52*b*c*d*e*sign(b*x + a) - 24*
a*d*e^2*sign(b*x + a) + 15*b*e^3*sign(b*x + a))/d^3) + 1/128*(16*
b*c^2*d^2*sign(b*x + a) + 32*a*c*d^2*e*sign(b*x + a) - 24*b*c*d*e
^2*sign(b*x + a) - 8*a*d*e^3*sign(b*x + a) + 5*b*e^4*sign(b*x + a
))*ln(abs(-2*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))*sqrt(d) - e))/d^
(7/2)
```

3.48 $\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$

Optimal. Leaf size=198

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(4cd-e^2\right)(2ad-be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{16d^{5/2}(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}(2dx+e)(2ad-be)\sqrt{c+dx^2+ex}}{8d^2(a+bx)}+\frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2+ex\right)^{3/2}}{3d(a+bx)}$$

```
[Out] ((2*a*d - b*e)*(e + 2*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c +
 e*x + d*x^2])/(8*d^2*(a + b*x)) + (b*Sqrt[a^2 + 2*a*b*x + b^2*x^
2]*(c + e*x + d*x^2)^(3/2))/(3*d*(a + b*x)) + ((2*a*d - b*e)*(4*c
*d - e^2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sq
rt[d]*Sqrt[c + e*x + d*x^2])])/(16*d^(5/2)*(a + b*x))
```

Rubi [A] time = 0.257692, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(4cd-e^2\right)(2ad-be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{16d^{5/2}(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}(2dx+e)(2ad-be)\sqrt{c+dx^2+ex}}{8d^2(a+bx)}+\frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2+ex\right)^{3/2}}{3d(a+bx)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] ((2*a*d - b*e)*(e + 2*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c +
 e*x + d*x^2])/(8*d^2*(a + b*x)) + (b*Sqrt[a^2 + 2*a*b*x + b^2*x^
2]*(c + e*x + d*x^2)^(3/2))/(3*d*(a + b*x)) + ((2*a*d - b*e)*(4*c
*d - e^2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sq
rt[d]*Sqrt[c + e*x + d*x^2])])/(16*d^(5/2)*(a + b*x))
```

Rubi in Sympy [A] time = 44.7745, size = 180, normalized size = 0.91

$$\frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2+ex\right)^{\frac{3}{2}}}{3d\,(a+bx)}+\frac{(2ad-be)\,(2dx+e)\,\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}}{8d^2\,(a+bx)}-\frac{(2ad-be)\left(-4cd+e^2\right)\sqrt{a^2+2abx+b^2x^2}\,\operatorname{atanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{16d^{\frac{5}{2}}\,(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
```

```
[Out] b*sqrt(a**2 + 2*a*b*x + b**2*x**2)*(c + d*x**2 + e*x)**(3/2)/(3*d
*(a + b*x)) + (2*a*d - b*e)*(2*d*x + e)*sqrt(a**2 + 2*a*b*x + b**
2*x**2)*sqrt(c + d*x**2 + e*x)/(8*d**2*(a + b*x)) - (2*a*d - b*e)
*(-4*c*d + e**2)*sqrt(a**2 + 2*a*b*x + b**2*x**2)*atanh((2*d*x +
e)/(2*sqrt(d)*sqrt(c + d*x**2 + e*x)))/(16*d**(5/2)*(a + b*x))
```

Mathematica [A] time = 0.247302, size = 132, normalized size = 0.67

$$\frac{\sqrt{(a+bx)^2}\left(2\sqrt{d}\sqrt{c+x(dx+e)}\left(6ad(2dx+e)+b\left(8cd+8d^2x^2+2dex-3e^2\right)\right)+3\left(4cd-e^2\right)(2ad-be)\log\left(2\sqrt{d}\sqrt{c+x(d\right.\right.}{48d^{5/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(2*Sqrt[d]*Sqrt[c + x*(e + d*x)]*(6*a*d*(e + 2
*d*x) + b*(8*c*d - 3*e^2 + 2*d*e*x + 8*d^2*x^2)) + 3*(2*a*d - b*e
)*(4*c*d - e^2)*Log[e + 2*d*x + 2*Sqrt[d]*Sqrt[c + x*(e + d*x)]])
)/(48*d^(5/2)*(a + b*x))
```

Maple [C] time = 0.014, size = 259, normalized size = 1.3

$$\frac{csgn(bx+a)}{48}\left(16\,b\left(dx^2+ex+c\right)^{3/2}d^{7/2}+24\,a\sqrt{dx^2+ex+c}xd^{9/2}-12\,be\sqrt{dx^2+ex+c}xd^{7/2}+24\,a\ln\left(1/2\,\frac{2\,\sqrt{dx^2+ex}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x)
```

```
[Out] 1/48*csgn(b*x+a)*(16*b*(d*x^2+e*x+c)^(3/2)*d^(7/2)+24*a*(d*x^2+e*
x+c)^(1/2)*x*d^(9/2)-12*b*e*(d*x^2+e*x+c)^(1/2)*x*d^(7/2)+24*a*ln
(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*c*d^4+12*a*
(d*x^2+e*x+c)^(1/2)*e*d^(7/2)-6*b*e^2*(d*x^2+e*x+c)^(1/2)*d^(5/2)
-6*a*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*e^2*
d^3-12*b*e*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2)
)*c*d^3+3*b*e^3*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^
(1/2))*d^2)/d^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.329206, size = 1, normalized size = 0.01

$$\left[\frac{4\left(8\,bd^2x^2+8\,bcd+6\,ade-3\,be^2+2\left(6\,ad^2+bde\right)x\right)\sqrt{dx^2+ex+c}\sqrt{d}+3\left(8\,acd^2-4\,bcde-2\,ade^2+be^3\right)\log\left(4\left(2\,d^2x\right.\right.}{96\,d^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*b*d^2*x^2 + 8*b*c*d + 6*a*d*e - 3*b*e^2 + 2*(6*a*d^2
+ b*d*e)*x)*sqrt(d*x^2 + e*x + c)*sqrt(d) + 3*(8*a*c*d^2 - 4*b*c*
d*e - 2*a*d*e^2 + b*e^3)*log(4*(2*d^2*x + d*e)*sqrt(d*x^2 + e*x +
 c) + (8*d^2*x^2 + 8*d*e*x + 4*c*d + e^2)*sqrt(d)))/d^(5/2), 1/48
*(2*(8*b*d^2*x^2 + 8*b*c*d + 6*a*d*e - 3*b*e^2 + 2*(6*a*d^2 + b*d
*e)*x)*sqrt(d*x^2 + e*x + c)*sqrt(-d) + 3*(8*a*c*d^2 - 4*b*c*d*e
- 2*a*d*e^2 + b*e^3)*arctan(1/2*(2*d*x + e)*sqrt(-d)/(sqrt(d*x^2
+ e*x + c)*d)))/(sqrt(-d)*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c + dx^2 + ex}\sqrt{(a + bx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)
```

GIAC/XCAS [A] time = 0.282361, size = 250, normalized size = 1.26

$$\frac{1}{24}\sqrt{dx^2 + xe + c}\left(2\left(4\, bx\text{sign}\,(bx + a) + \frac{6\, ad^2\text{sign}\,(bx + a) + bde\text{sign}\,(bx + a)}{d^2}\right)x + \frac{8\, bcd\text{sign}\,(bx + a) + 6\, ade\text{sign}\,(bx + a)}{d^2}\right.$$
$$-\frac{\left(8\, acd^2\text{sign}\,(bx + a) - 4\, bcde\text{sign}\,(bx + a) - 2\, ade^2\text{sign}\,(bx + a) + be^3\text{sign}\,(bx + a)\right)\ln\left(\left|-2\left(\sqrt{d}x - \sqrt{dx^2 + xe + c}\right)\sqrt{d} - e\right.\right.}{16\, d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(d*x^2 + x*e + c)*(2*(4*b*x*sign(b*x + a) + (6*a*d^2*sig
n(b*x + a) + b*d*e*sign(b*x + a))/d^2)*x + (8*b*c*d*sign(b*x + a)
 + 6*a*d*e*sign(b*x + a) - 3*b*e^2*sign(b*x + a))/d^2) - 1/16*(8*
a*c*d^2*sign(b*x + a) - 4*b*c*d*e*sign(b*x + a) - 2*a*d*e^2*sign(
b*x + a) + b*e^3*sign(b*x + a))*ln(abs(-2*(sqrt(d)*x - sqrt(d*x^2
 + x*e + c))*sqrt(d) - e))/d^(5/2)
```

3.49 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x}\,dx$

Optimal. Leaf size=211

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(4ade+4bcd-be^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{8d^{3/2}(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}(4ad+2bdx+be)}{4d(a+bx)}-\frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{a+bx}$$

```
[Out] ((4*a*d + b*e + 2*b*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e
*x + d*x^2])/(4*d*(a + b*x)) + ((4*b*c*d + 4*a*d*e - b*e^2)*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c +
e*x + d*x^2])])/(8*d^(3/2)*(a + b*x)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a
*b*x + b^2*x^2]*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x
^2])])/(a + b*x)
```

Rubi [A] time = 0.537125, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(4ade+4bcd-be^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{8d^{3/2}(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}(4ad+2bdx+be)}{4d(a+bx)}-\frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x,x]
```

```
[Out] ((4*a*d + b*e + 2*b*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e
*x + d*x^2])/(4*d*(a + b*x)) + ((4*b*c*d + 4*a*d*e - b*e^2)*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c +
e*x + d*x^2])])/(8*d^(3/2)*(a + b*x)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a
*b*x + b^2*x^2]*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x
^2])])/(a + b*x)
```

Rubi in Sympy [A] time = 68.4531, size = 197, normalized size = 0.93

$$-\frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\,\mathrm{atanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{a+bx}+\frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}\,(4ad+2bdx+be)}{4d\,(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\left(-4ade-4bcd+be^2\right)\mathrm{atanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{8d^{\frac{3}{2}}\,(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x,x)
```

```
[Out] -a*sqrt(c)*sqrt(a**2 + 2*a*b*x + b**2*x**2)*atanh((2*c + e*x)/(2*
sqrt(c)*sqrt(c + d*x**2 + e*x)))/(a + b*x) + sqrt(a**2 + 2*a*b*x
+ b**2*x**2)*sqrt(c + d*x**2 + e*x)*(4*a*d + 2*b*d*x + b*e)/(4*d*
(a + b*x)) - sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-4*a*d*e - 4*b*c*d
 + b*e**2)*atanh((2*d*x + e)/(2*sqrt(d)*sqrt(c + d*x**2 + e*x)))/
(8*d**(3/2)*(a + b*x))
```

Mathematica [A] time = 0.273794, size = 246, normalized size = 1.17

$$\sqrt{(a+bx)^2}\left(8ad^{3/2}\sqrt{c+x(dx+e)}-8a\sqrt{c}d^{3/2}\log\left(2\sqrt{c}\sqrt{c+x(dx+e)}+2c+ex\right)+8a\sqrt{c}d^{3/2}\log(x)+4ade\log\left(2\sqrt{d}\sqrt{c+x\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(8*a*d^(3/2)*Sqrt[c + x*(e + d*x)] + 2*b*Sqrt[
d]*e*Sqrt[c + x*(e + d*x)] + 4*b*d^(3/2)*x*Sqrt[c + x*(e + d*x)]
+ 8*a*Sqrt[c]*d^(3/2)*Log[x] - 8*a*Sqrt[c]*d^(3/2)*Log[2*c + e*x
+ 2*Sqrt[c]*Sqrt[c + x*(e + d*x)]] + 4*b*c*d*Log[e + 2*d*x + 2*Sq
rt[d]*Sqrt[c + x*(e + d*x)]] + 4*a*d*e*Log[e + 2*d*x + 2*Sqrt[d]*
Sqrt[c + x*(e + d*x)]] - b*e^2*Log[e + 2*d*x + 2*Sqrt[d]*Sqrt[c +
 x*(e + d*x)]]))/(8*d^(3/2)*(a + b*x))
```

Maple [C] time = 0.015, size = 214, normalized size = 1.

$$-\frac{csgn(bx+a)}{8}\left(8a\sqrt{c}\ln\left(\frac{2c+ex+2\sqrt{c}\sqrt{dx^2+ex+c}}{x}\right)d^{5/2}-4b\sqrt{dx^2+ex+c}xd^{5/2}-4ae\ln\left(1/2\frac{2\sqrt{dx^2+ex+c}\sqrt{d}+}{\sqrt{d}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x)
```

```
[Out] -1/8*csgn(b*x+a)*(8*a*c^(1/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)
^(1/2))/x)*d^(5/2)-4*b*(d*x^2+e*x+c)^(1/2)*x*d^(5/2)-4*a*e*ln(1/2
*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*d^2-4*b*ln(1/2*
(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*c*d^2-8*a*(d*x^2
+e*x+c)^(1/2)*d^(5/2)-2*b*(d*x^2+e*x+c)^(1/2)*e*d^(3/2)+b*ln(1/2*
(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*e^2*d)/d^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.995471, size = 1, normalized size = 0.

$$\left[\frac{8\,a\sqrt{c}d^{\frac{3}{2}}\log\left(\frac{8\,cex+\left(4\,cd+e^2\right)x^2-4\,\sqrt{dx^2+ex+c}(ex+2\,c)\sqrt{c}+8\,c^2}{x^2}\right)+4\,(2\,bdx+4\,ad+be)\sqrt{dx^2+ex+c}\sqrt{d}-\left(4\,bcd+4\,ade-be^2\right)\text{lo}}{16\,d^{\frac{3}{2}}}\right.$$

$$-\frac{16\,a\sqrt{-c}d^{\frac{3}{2}}\arctan\left(\frac{ex+2\,c}{2\,\sqrt{dx^2+ex+c}\sqrt{-c}}\right)-4\,(2\,bdx+4\,ad+be)\sqrt{dx^2+ex+c}\sqrt{d}+\left(4\,bcd+4\,ade-be^2\right)\log\left(-4\,\left(2\,d^2x+de\right)\right.}{16\,d^{\frac{3}{2}}}$$

$$-\frac{8\,a\sqrt{-c}\sqrt{-d}d\arctan\left(\frac{ex+2\,c}{2\,\sqrt{dx^2+ex+c}\sqrt{-c}}\right)-2\,(2\,bdx+4\,ad+be)\sqrt{dx^2+ex+c}\sqrt{-d}-\left(4\,bcd+4\,ade-be^2\right)\arctan\left(\frac{(2\,dx+e)\ldots}{2\,\sqrt{dx^2+ex\ldots}}\right.}{8\,\sqrt{-d}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x,x, algorithm="fricas")
```

```
[Out] [1/16*(8*a*sqrt(c)*d^(3/2)*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*s
qrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) + 4*(2*b*d
*x + 4*a*d + b*e)*sqrt(d*x^2 + e*x + c)*sqrt(d) - (4*b*c*d + 4*a*
d*e - b*e^2)*log(-4*(2*d^2*x + d*e)*sqrt(d*x^2 + e*x + c) + (8*d^
2*x^2 + 8*d*e*x + 4*c*d + e^2)*sqrt(d)))/d^(3/2), 1/8*(4*a*sqrt(c
)*sqrt(-d)*d*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*
x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) + 2*(2*b*d*x + 4*a*d + b
*e)*sqrt(d*x^2 + e*x + c)*sqrt(-d) + (4*b*c*d + 4*a*d*e - b*e^2)*
arctan(1/2*(2*d*x + e)*sqrt(-d)/(sqrt(d*x^2 + e*x + c)*d)))/(sqrt
(-d)*d), -1/16*(16*a*sqrt(-c)*d^(3/2)*arctan(1/2*(e*x + 2*c)/(sqr
t(d*x^2 + e*x + c)*sqrt(-c))) - 4*(2*b*d*x + 4*a*d + b*e)*sqrt(d*
x^2 + e*x + c)*sqrt(d) + (4*b*c*d + 4*a*d*e - b*e^2)*log(-4*(2*d^
2*x + d*e)*sqrt(d*x^2 + e*x + c) + (8*d^2*x^2 + 8*d*e*x + 4*c*d +
 e^2)*sqrt(d)))/d^(3/2), -1/8*(8*a*sqrt(-c)*sqrt(-d)*d*arctan(1/2
*(e*x + 2*c)/(sqrt(d*x^2 + e*x + c)*sqrt(-c))) - 2*(2*b*d*x + 4*a
*d + b*e)*sqrt(d*x^2 + e*x + c)*sqrt(-d) - (4*b*c*d + 4*a*d*e - b
*e^2)*arctan(1/2*(2*d*x + e)*sqrt(-d)/(sqrt(d*x^2 + e*x + c)*d)))
/(sqrt(-d)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2+ex}\sqrt{(a+bx)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x, x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.50 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx$

Optimal. Leaf size=202

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(a-bx)\sqrt{c+dx^2+ex}}{x(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}(2ad+be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{2\sqrt{d}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}(ae+2bc)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{2\sqrt{c}(a+bx)}$$

```
[Out] -(((a - b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])
/(x*(a + b*x))) + ((2*a*d + b*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Ar
cTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(2*Sqrt[d]*
(a + b*x)) - ((2*b*c + a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh
[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])/(2*Sqrt[c]*(a +
b*x))
```

Rubi [A] time = 0.500607, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(a-bx)\sqrt{c+dx^2+ex}}{x(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}(2ad+be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{2\sqrt{d}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}(ae+2bc)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{2\sqrt{c}(a+bx)}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^2,x]
```

```
[Out] -(((a - b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])
/(x*(a + b*x))) + ((2*a*d + b*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Ar
cTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(2*Sqrt[d]*
(a + b*x)) - ((2*b*c + a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh
[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])/(2*Sqrt[c]*(a +
b*x))
```

Rubi in Sympy [A] time = 63.0626, size = 189, normalized size = 0.94

$$-\frac{(2a-2bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}}{2x\,(a+bx)}+\frac{(2ad+be)\sqrt{a^2+2abx+b^2x^2}\,\operatorname{atanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{2\sqrt{d}\,(a+bx)}-\frac{(ae+2bc)\sqrt{a^2+2abx+b^2x^2}\,\operatorname{atanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{2\sqrt{c}\,(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x**2,x)
```

```
[Out] -(2*a - 2*b*x)*sqrt(a**2 + 2*a*b*x + b**2*x**2)*sqrt(c + d*x**2 +
 e*x)/(2*x*(a + b*x)) + (2*a*d + b*e)*sqrt(a**2 + 2*a*b*x + b**2*
x**2)*atanh((2*d*x + e)/(2*sqrt(d)*sqrt(c + d*x**2 + e*x)))/(2*sq
rt(d)*(a + b*x)) - (a*e + 2*b*c)*sqrt(a**2 + 2*a*b*x + b**2*x**2)
*atanh((2*c + e*x)/(2*sqrt(c)*sqrt(c + d*x**2 + e*x)))/(2*sqrt(c)
*(a + b*x))
```

Mathematica [A] time = 0.389941, size = 168, normalized size = 0.83

$$\frac{\sqrt{(a+bx)^2}\left(\sqrt{d}x\log(x)(ae+2bc)-\sqrt{d}x(ae+2bc)\log\left(2\sqrt{c}\sqrt{c+x(dx+e)}+2c+ex\right)+\sqrt{c}\left(2\sqrt{d}(bx-a)\sqrt{c+x(dx+e)}+\right.\right.}{2\sqrt{c}\sqrt{d}x(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^2,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(Sqrt[d]*(2*b*c + a*e)*x*Log[x] - Sqrt[d]*(2*b
*c + a*e)*x*Log[2*c + e*x + 2*Sqrt[c]*Sqrt[c + x*(e + d*x)]] + Sq
rt[c]*(2*Sqrt[d]*(-a + b*x)*Sqrt[c + x*(e + d*x)] + (2*a*d + b*e)
*x*Log[e + 2*d*x + 2*Sqrt[d]*Sqrt[c + x*(e + d*x)]])))/(2*Sqrt[c]
*Sqrt[d]*x*(a + b*x))
```

Maple [C] time = 0.018, size = 259, normalized size = 1.3

$$\frac{csgn\,(bx+a)}{2\,x}\left(2\,ad\ln\left(1/2\,\frac{2\,\sqrt{dx^2+ex+c}\sqrt{d}+2\,dx+e}{\sqrt{d}}\right)c^{3/2}x-2\,bc^2\ln\left(\frac{2\,c+ex+2\,\sqrt{c}\sqrt{dx^2+ex+c}}{x}\right)x\sqrt{d}+2\,ad^{3/2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x)
```

```
[Out] 1/2*csgn(b*x+a)*(2*a*d*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*
x+e)/d^(1/2))*c^(3/2)*x-2*b*c^2*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+
c)^(1/2))/x)*x*d^(1/2)+2*a*d^(3/2)*(d*x^2+e*x+c)^(1/2)*x^2*c^(1/2
)+b*e*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*c^(
3/2)*x-2*a*(d*x^2+e*x+c)^(3/2)*c^(1/2)*d^(1/2)+2*a*e*(d*x^2+e*x+c
)^(1/2)*x*c^(1/2)*d^(1/2)+2*b*(d*x^2+e*x+c)^(1/2)*c^(3/2)*x*d^(1/
2)-a*e*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/2))/x)*c*x*d^(1/2))
/c^(3/2)/x/d^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.46141, size = 1, normalized size = 0.

$$\left[\frac{(2\,ad+be)\sqrt{c}x\log\left(4\left(2\,d^2x+de\right)\sqrt{dx^2+ex+c}+\left(8\,d^2x^2+8\,dex+4\,cd+e^2\right)\sqrt{d}\right)+(2\,bc+ae)\sqrt{d}x\log\left(-\frac{4\left(cex+2\,c^2\right)\sqrt{dx^2}}{}\right.}{4\sqrt{c}\sqrt{d}x}\right.$$

$$-\frac{2\,(2\,bc+ae)\sqrt{d}x\arctan\left(\frac{(ex+2\,c)\sqrt{-c}}{2\sqrt{dx^2+ex+c}c}\right)-(2\,ad+be)\sqrt{-c}x\log\left(4\left(2\,d^2x+de\right)\sqrt{dx^2+ex+c}+\left(8\,d^2x^2+8\,dex+4\,cd+e^2\right)\right.}{4\sqrt{-c}\sqrt{d}x}$$

$$\left.-\frac{(2\,bc+ae)\sqrt{-d}x\arctan\left(\frac{(ex+2\,c)\sqrt{-c}}{2\sqrt{dx^2+ex+c}c}\right)-(2\,ad+be)\sqrt{-c}x\arctan\left(\frac{(2\,dx+e)\sqrt{-d}}{2\sqrt{dx^2+ex+c}d}\right)-2\sqrt{dx^2+ex+c}(bx-a)\sqrt{-c}\sqrt{-d}}{2\sqrt{-c}\sqrt{-d}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*((2*a*d + b*e)*sqrt(c)*x*log(4*(2*d^2*x + d*e)*sqrt(d*x^2 +
e*x + c) + (8*d^2*x^2 + 8*d*e*x + 4*c*d + e^2)*sqrt(d)) + (2*b*c
+ a*e)*sqrt(d)*x*log(-(4*(c*e*x + 2*c^2)*sqrt(d*x^2 + e*x + c) -
(8*c*e*x + (4*c*d + e^2)*x^2 + 8*c^2)*sqrt(c))/x^2) + 4*sqrt(d*x^
2 + e*x + c)*(b*x - a)*sqrt(c)*sqrt(d))/(sqrt(c)*sqrt(d)*x), 1/4*
(2*(2*a*d + b*e)*sqrt(c)*x*arctan(1/2*(2*d*x + e)*sqrt(-d)/(sqrt(
d*x^2 + e*x + c)*d)) + (2*b*c + a*e)*sqrt(-d)*x*log(-(4*(c*e*x +
2*c^2)*sqrt(d*x^2 + e*x + c) - (8*c*e*x + (4*c*d + e^2)*x^2 + 8*c
^2)*sqrt(c))/x^2) + 4*sqrt(d*x^2 + e*x + c)*(b*x - a)*sqrt(c)*sqr
t(-d))/(sqrt(c)*sqrt(-d)*x), -1/4*(2*(2*b*c + a*e)*sqrt(d)*x*arct
an(1/2*(e*x + 2*c)*sqrt(-c)/(sqrt(d*x^2 + e*x + c)*c)) - (2*a*d +
 b*e)*sqrt(-c)*x*log(4*(2*d^2*x + d*e)*sqrt(d*x^2 + e*x + c) + (8
*d^2*x^2 + 8*d*e*x + 4*c*d + e^2)*sqrt(d)) - 4*sqrt(d*x^2 + e*x +
 c)*(b*x - a)*sqrt(-c)*sqrt(d))/(sqrt(-c)*sqrt(d)*x), -1/2*((2*b*
c + a*e)*sqrt(-d)*x*arctan(1/2*(e*x + 2*c)*sqrt(-c)/(sqrt(d*x^2 +
 e*x + c)*c)) - (2*a*d + b*e)*sqrt(-c)*x*arctan(1/2*(2*d*x + e)*s
qrt(-d)/(sqrt(d*x^2 + e*x + c)*d)) - 2*sqrt(d*x^2 + e*x + c)*(b*x
 - a)*sqrt(-c)*sqrt(-d))/(sqrt(-c)*sqrt(-d)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c+dx^2+ex}\sqrt{(a+bx)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x**2, x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.51 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx$

Optimal. Leaf size=215

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,(4acd-ae^2+4bce)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{8c^{3/2}(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}(x(ae+4bc)+2ac)}{4cx^2(a+bx)} + \frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{a+bx}$$

```
[Out] -((2*a*c + (4*b*c + a*e)*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c
+ e*x + d*x^2])/(4*c*x^2*(a + b*x)) + (b*Sqrt[d]*Sqrt[a^2 + 2*a*b
*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2
])])/(a + b*x) - ((4*a*c*d + 4*b*c*e - a*e^2)*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2]*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])]
)/(8*c^(3/2)*(a + b*x))
```

Rubi [A] time = 0.53663, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,(4acd-ae^2+4bce)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{8c^{3/2}(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}(x(ae+4bc)+2ac)}{4cx^2(a+bx)} + \frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^3,x]
```

```
[Out] -((2*a*c + (4*b*c + a*e)*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c
+ e*x + d*x^2])/(4*c*x^2*(a + b*x)) + (b*Sqrt[d]*Sqrt[a^2 + 2*a*b
*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2
])])/(a + b*x) - ((4*a*c*d + 4*b*c*e - a*e^2)*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2]*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])]
)/(8*c^(3/2)*(a + b*x))
```

Rubi in Sympy [A] time = 66.6844, size = 201, normalized size = 0.93

$$\frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\,\operatorname{atanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{a+bx} - \frac{(2ac+x\,(ae+4bc))\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}}{4cx^2\,(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2}\,(-4acd+ae^2-4bce)\operatorname{atanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{8c^{\frac{3}{2}}\,(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x**3,x)
```

```
[Out] b*sqrt(d)*sqrt(a**2 + 2*a*b*x + b**2*x**2)*atanh((2*d*x + e)/(2*s
qrt(d)*sqrt(c + d*x**2 + e*x)))/(a + b*x) - (2*a*c + x*(a*e + 4*b
*c))*sqrt(a**2 + 2*a*b*x + b**2*x**2)*sqrt(c + d*x**2 + e*x)/(4*c
*x**2*(a + b*x)) + sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-4*a*c*d + a
*e**2 - 4*b*c*e)*atanh((2*c + e*x)/(2*sqrt(c)*sqrt(c + d*x**2 + e
*x)))/(8*c**(3/2)*(a + b*x))
```

Mathematica [A] time = 0.446973, size = 177, normalized size = 0.82

$$\frac{\sqrt{(a+bx)^2}\left(x^2\log(x)\left(4acd-ae^2+4bce\right)+x^2\left(a\left(e^2-4cd\right)-4bce\right)\log\left(2\sqrt{c}\sqrt{c+x(dx+e)}+2c+ex\right)-2\sqrt{c}\left(\sqrt{c+x(dx\right.\right.}{8c^{3/2}x^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^3,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*((4*a*c*d + 4*b*c*e - a*e^2)*x^2*Log[x] + (-4*
b*c*e + a*(-4*c*d + e^2))*x^2*Log[2*c + e*x + 2*Sqrt[c]*Sqrt[c +
x*(e + d*x)]] - 2*Sqrt[c]*((2*a*c + 4*b*c*x + a*e*x)*Sqrt[c + x*(
e + d*x)] - 4*b*c*Sqrt[d]*x^2*Log[e + 2*d*x + 2*Sqrt[d]*Sqrt[c +
x*(e + d*x)]])))/(8*c^(3/2)*x^2*(a + b*x))
```

Maple [C] time = 0.019, size = 345, normalized size = 1.6

$$\frac{csgn(bx+a)}{8x^2}\left(8b\sqrt{d}\ln\left(1/2\,\frac{2\sqrt{dx^2+ex+c}\sqrt{d}+2dx+e}{\sqrt{d}}\right)x^2c^{7/2}-2aed\sqrt{dx^2+ex+c}x^3c^{3/2}+8bd\sqrt{dx^2+ex+c}x^3c^{5/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x)
```

```
[Out] 1/8*csgn(b*x+a)*(8*b*d^(1/2)*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2
)+2*d*x+e)/d^(1/2))*x^2*c^(7/2)-2*a*e*d*(d*x^2+e*x+c)^(1/2)*x^3*c
^(3/2)+8*b*d*(d*x^2+e*x+c)^(1/2)*x^3*c^(5/2)+2*a*e*(d*x^2+e*x+c)^
(3/2)*x*c^(3/2)-8*b*(d*x^2+e*x+c)^(3/2)*x*c^(5/2)+4*a*d*(d*x^2+e*
x+c)^(1/2)*x^2*c^(5/2)-2*a*e^2*(d*x^2+e*x+c)^(1/2)*x^2*c^(3/2)+8*
b*e*(d*x^2+e*x+c)^(1/2)*x^2*c^(5/2)-4*a*d*ln((2*c+e*x+2*c^(1/2)*(
d*x^2+e*x+c)^(1/2))/x)*x^2*c^3-4*b*e*ln((2*c+e*x+2*c^(1/2)*(d*x^2
+e*x+c)^(1/2))/x)*x^2*c^3-4*a*(d*x^2+e*x+c)^(3/2)*c^(5/2)+a*e^2*l
n((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/2))/x)*x^2*c^2)/x^2/c^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.600436, size = 1, normalized size = 0.

$$\left[\frac{8\,bc^{\frac{3}{2}}\sqrt{d}x^{2}\log\left(8\,d^{2}x^{2}+8\,dex+4\,\sqrt{dx^{2}+ex+c}(2\,dx+e)\sqrt{d}+4\,cd+e^{2}\right)-\left(4\,acd+4\,bce-ae^{2}\right)x^{2}\log\left(\frac{4\,(cex+2\,c^{2})\sqrt{dx^{2}+e}}{}\right.}{16\,c^{\frac{3}{2}}x^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*b*c^(3/2)*sqrt(d)*x^2*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d
*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) - (4*a*c*d + 4
*b*c*e - a*e^2)*x^2*log((4*(c*e*x + 2*c^2)*sqrt(d*x^2 + e*x + c)
+ (8*c*e*x + (4*c*d + e^2)*x^2 + 8*c^2)*sqrt(c))/x^2) - 4*sqrt(d*
x^2 + e*x + c)*(2*a*c + (4*b*c + a*e)*x)*sqrt(c))/(c^(3/2)*x^2),
1/16*(16*b*c^(3/2)*sqrt(-d)*x^2*arctan(1/2*(2*d*x + e)/(sqrt(d*x^
2 + e*x + c)*sqrt(-d))) - (4*a*c*d + 4*b*c*e - a*e^2)*x^2*log((4*
(c*e*x + 2*c^2)*sqrt(d*x^2 + e*x + c) + (8*c*e*x + (4*c*d + e^2)*
x^2 + 8*c^2)*sqrt(c))/x^2) - 4*sqrt(d*x^2 + e*x + c)*(2*a*c + (4*
b*c + a*e)*x)*sqrt(c))/(c^(3/2)*x^2), 1/8*(4*b*sqrt(-c)*c*sqrt(d)
*x^2*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e
)*sqrt(d) + 4*c*d + e^2) - (4*a*c*d + 4*b*c*e - a*e^2)*x^2*arctan
(1/2*(e*x + 2*c)*sqrt(-c)/(sqrt(d*x^2 + e*x + c)*c)) - 2*sqrt(d*x
^2 + e*x + c)*(2*a*c + (4*b*c + a*e)*x)*sqrt(-c))/(sqrt(-c)*c*x^2
), 1/8*(8*b*sqrt(-c)*c*sqrt(-d)*x^2*arctan(1/2*(2*d*x + e)/(sqrt(
d*x^2 + e*x + c)*sqrt(-d))) - (4*a*c*d + 4*b*c*e - a*e^2)*x^2*arc
tan(1/2*(e*x + 2*c)*sqrt(-c)/(sqrt(d*x^2 + e*x + c)*c)) - 2*sqrt(
d*x^2 + e*x + c)*(2*a*c + (4*b*c + a*e)*x)*sqrt(-c))/(sqrt(-c)*c*
x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.52 $\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=452

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(af^2+2c\left(e^2-df\right)\right)}{2\sqrt{c}f^3}-\frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-2df\right)\right)-2df\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(e\left(\sqrt{e^2-4df}+e\right)\left(af^2+c\left(e^2-2df\right)\right)-2df\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

```
[Out] -((2*e - f*x)*Sqrt[a + c*x^2])/(2*f^2) + ((a*f^2 + 2*c*(e^2 - d*f
))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*f^3) - ((e*(e
 - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*d*f*(a*f^2 +
c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sq
rt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[
a + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^
2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((e*(e + Sqrt[e^2 - 4*d*f])*
(a*f^2 + c*(e^2 - 2*d*f)) - 2*d*f*(a*f^2 + c*(e^2 - d*f)))*ArcTan
h[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c
*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]
*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2
 - 4*d*f])])
```

Rubi [A] time = 4.09631, antiderivative size = 452, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(af^2+2c\left(e^2-df\right)\right)}{2\sqrt{c}f^3}-\frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-2df\right)\right)-2df\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(e\left(\sqrt{e^2-4df}+e\right)\left(af^2+c\left(e^2-2df\right)\right)-2df\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] -((2*e - f*x)*Sqrt[a + c*x^2])/(2*f^2) + ((a*f^2 + 2*c*(e^2 - d*f
))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*f^3) - ((e*(e
 - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*d*f*(a*f^2 +
```

```
c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sq
rt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[
a + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^
2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((e*(e + Sqrt[e^2 - 4*d*f])*
(a*f^2 + c*(e^2 - 2*d*f)) - 2*d*f*(a*f^2 + c*(e^2 - d*f)))*ArcTan
h[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c
*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]
*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2
 - 4*d*f])])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 3.32588, size = 882, normalized size = 1.95

$$\frac{f\sqrt{cx^2+a}(fx-2e)-\frac{\sqrt{2}\left(a\left(-e^2+\sqrt{e^2-4df}e+2df\right)f^2+c\left(-e^4+\sqrt{e^2-4df}e^3+4dfe^2-2df\sqrt{e^2-4df}e-2d^2f^2\right)\right)\log\left(-e-2fx+\sqrt{e^2-4df}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-\sqrt{e^2-4df}e-2df\right)}}-\frac{\sqrt{2}\left(a\left(e^2+\sqrt{e\cdots\right.\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] (f*(-2*e + f*x)*Sqrt[a + c*x^2] - (Sqrt[2]*(a*f^2*(-e^2 + 2*d*f +
 e*Sqrt[e^2 - 4*d*f]) + c*(-e^4 + 4*d*e^2*f - 2*d^2*f^2 + e^3*Sqr
t[e^2 - 4*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]))*Log[-e + Sqrt[e^2 -
4*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 - e*Sqrt[e^2 - 4*d*f])]) - (Sqrt[2]*(a*f^2*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f]) + c*(e^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt[e^2 -
4*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]))*Log[e + Sqrt[e^2 - 4*d*f] +
2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])]) + ((a*f^2 + 2*c*(e^2 - d*f))*Log[c*x + Sqrt[c]*S
qrt[a + c*x^2]])/Sqrt[c] + (Sqrt[2]*(a*f^2*(-e^2 + 2*d*f + e*Sqrt
[e^2 - 4*d*f]) + c*(-e^4 + 4*d*e^2*f - 2*d^2*f^2 + e^3*Sqrt[e^2 -
 4*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]))*Log[2*a*f*Sqrt[e^2 - 4*d*f]
 + c*(e^2 - 4*d*f - e*Sqrt[e^2 - 4*d*f])*x + Sqrt[2]*Sqrt[e^2 - 4
*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[
a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e
*Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*(a*f^2*(e^2 - 2*d*f + e*Sqrt[e^2
 - 4*d*f]) + c*(e^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d*
f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]))*Log[2*a*f*Sqrt[e^2 - 4*d*f] - c*
(e^2 - 4*d*f + e*Sqrt[e^2 - 4*d*f])*x + Sqrt[2]*Sqrt[e^2 - 4*d*f]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c
*x^2]])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])]))/(2*f^3)
```

Maple [B] time = 0.077, size = 7739, normalized size = 17.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)*x^2/(f*x^2 + e*x + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)*x^2/(f*x^2 + e*x + d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sqrt{a + cx^2}}{d + ex + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**2*sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```

GIAC/XCAS [A] time = 0.667244, size = 1, normalized size = 0.

$$\mathit{Done}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^2 + a)*x^2/(f*x^2 + e*x + d),x, algorithm="giac")
```

```
[Out] Done
```

3.53 $\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=395

$$-\frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2cdef-\left(\sqrt{e^2-4df}+e\right)\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2}+\frac{\sqrt{a+cx^2}}{f}$$

```
[Out] Sqrt[a + c*x^2]/f - (Sqrt[c]*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2
]])/f^2 - ((2*c*d*e*f - (e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 -
 d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sq
rt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^
2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*
f - e*Sqrt[e^2 - 4*d*f])]) + ((2*c*d*e*f - (e + Sqrt[e^2 - 4*d*f]
)*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d
*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*
f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 3.70065, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28

$$-\frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2cdef-\left(\sqrt{e^2-4df}+e\right)\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2}+\frac{\sqrt{a+cx^2}}{f}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```


```
[Out] Sqrt[a + c*x^2]/f - (Sqrt[c]*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2
]])/f^2 - ((2*c*d*e*f - (e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 -
 d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sq
rt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^
2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*
f - e*Sqrt[e^2 - 4*d*f])]) + ((2*c*d*e*f - (e + Sqrt[e^2 - 4*d*f]
)*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d
*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*
f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.68396, size = 785, normalized size = 1.99

$$-\frac{\frac{\sqrt{2}\left(af^2\left(\sqrt{e^2-4df}-e\right)+c\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}+3def-e^3\right)\right)\log\left(\sqrt{2}\sqrt{a+cx^2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}+2af\sqrt{e^2-4df}+cx\left(-e\sqrt{e^2-4df}\cdots\right.\right.}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] (2*f*Sqrt[a + c*x^2] + (Sqrt[2]*(a*f^2*(-e + Sqrt[e^2 - 4*d*f]) +
 c*(-e^3 + 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f
]))*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[
2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*(a*f
^2*(e + Sqrt[e^2 - 4*d*f]) + c*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*
d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x]
)/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 -
 4*d*f])]) - 2*Sqrt[c]*e*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] - (Sq
rt[2]*(a*f^2*(-e + Sqrt[e^2 - 4*d*f]) + c*(-e^3 + 3*d*e*f + e^2*S
qrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[2*a*f*Sqrt[e^2 - 4
*d*f] + c*(e^2 - 4*d*f - e*Sqrt[e^2 - 4*d*f])*x + Sqrt[2]*Sqrt[e^
2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*
Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*
f - e*Sqrt[e^2 - 4*d*f])]) - (Sqrt[2]*(a*f^2*(e + Sqrt[e^2 - 4*d*
f]) + c*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4
*d*f]))*Log[2*a*f*Sqrt[e^2 - 4*d*f] - c*(e^2 - 4*d*f + e*Sqrt[e^2
 - 4*d*f])*x + Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 -
2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f
]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]))/(2*f^2)
```

Maple [B] time = 0.02, size = 5581, normalized size = 14.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^2 + a)*x/(f*x^2 + e*x + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^2 + a)*x/(f*x^2 + e*x + d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{a + cx^2}}{d + ex + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x*sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}x}{fx^2 + ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^2 + a)*x/(f*x^2 + e*x + d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*x/(f*x^2 + e*x + d), x)
```

3.54 $\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2} \, dx$

Optimal. Leaf size=298

$$-\frac{\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\frac{\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f}$$

```
[Out] (Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f - (Sqrt[2*a*f^2
+ c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e -
Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*
Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*
f]) + (Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcT
anh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 +
 c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[
2]*f*Sqrt[e^2 - 4*d*f])
```

Rubi [A] time = 0.893806, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$-\frac{\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\frac{\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f - (Sqrt[2*a*f^2
+ c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e -
Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*
Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*
f]) + (Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcT
anh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 +
 c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[
2]*f*Sqrt[e^2 - 4*d*f])
```

Rubi in Sympy [A] time = 77.5853, size = 296, normalized size = 0.99

$$\frac{\sqrt{c}\operatorname{atanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f} - \frac{\sqrt{2}\sqrt{2af^2-2cdf+ce^2-ce\sqrt{-4df+e^2}}\operatorname{atanh}\left(\frac{\sqrt{2}\left(2af-cx\left(e-\sqrt{-4df+e^2}\right)\right)}{2\sqrt{a+cx^2}\sqrt{2af^2-2cdf+ce^2-ce\sqrt{-4df+e^2}}}\right)}{2f\sqrt{-4df+e^2}} + \frac{\sqrt{2}\sqrt{2af^2-2cdf+ce^2+ce\sqrt{-4df+e^2}}\operatorname{atanh}\left(\frac{\sqrt{2}\left(2af-cx\left(e+\sqrt{-4df+e^2}\right)\right)}{2\sqrt{a+cx^2}\sqrt{2af^2-2cdf+ce^2+ce\sqrt{-4df+e^2}}}\right)}{2f\sqrt{-4df+e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] sqrt(c)*atanh(sqrt(c)*x/sqrt(a + c*x**2))/f - sqrt(2)*sqrt(2*a*f*
*2 - 2*c*d*f + c*e**2 - c*e*sqrt(-4*d*f + e**2))*atanh(sqrt(2)*(2
*a*f - c*x*(e - sqrt(-4*d*f + e**2)))/(2*sqrt(a + c*x**2)*sqrt(2*
a*f**2 - 2*c*d*f + c*e**2 - c*e*sqrt(-4*d*f + e**2))))/(2*f*sqrt(
-4*d*f + e**2)) + sqrt(2)*sqrt(2*a*f**2 - 2*c*d*f + c*e**2 + c*e*
sqrt(-4*d*f + e**2))*atanh(sqrt(2)*(2*a*f - c*x*(e + sqrt(-4*d*f
+ e**2)))/(2*sqrt(a + c*x**2)*sqrt(2*a*f**2 - 2*c*d*f + c*e**2 +
c*e*sqrt(-4*d*f + e**2))))/(2*f*sqrt(-4*d*f + e**2))
```

Mathematica [A] time = 2.38001, size = 472, normalized size = 1.58

$$\frac{-\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}\log\left(\sqrt{2}\sqrt{a+cx^2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}+2af\sqrt{e^2-4c\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Log[-e
+ Sqrt[e^2 - 4*d*f] - 2*f*x] - Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f])]*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x] + 2*Sqrt
[c]*Sqrt[e^2 - 4*d*f]*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] - Sqrt[4
*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Log[2*a*f*Sqrt
[e^2 - 4*d*f] + c*(e^2 - 4*d*f - e*Sqrt[e^2 - 4*d*f])*x + Sqrt[2]
*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4
*d*f])]*Sqrt[a + c*x^2]] + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sq
rt[e^2 - 4*d*f])]*Log[2*a*f*Sqrt[e^2 - 4*d*f] - c*(e^2 - 4*d*f +
e*Sqrt[e^2 - 4*d*f])*x + Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 +
 c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]])/(2*f*Sq
rt[e^2 - 4*d*f])
```

Maple [B] time = 0.019, size = 3249, normalized size = 10.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] 1/2/(-4*d*f+e^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4
*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(
```

```
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)+1/2*c^(
1/2)/f*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+c*(x-1/2*(-e+(-4*d*f+e
^2)^(1/2))/f))/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(
e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))-1/2/(-4*
d*f+e^2)^(1/2)*c^(1/2)/f*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+c*(x
-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^
2)^(1/2))*e+1/2/f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c
*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^
(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e
^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-
(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(
-4*d*f+e^2)^(1/2))/f))*c*e-1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2)
)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*
f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*
d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2
*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*a+1/(-4*d*f+e^
2)^(1/2)/f*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*
c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)
/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2
)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e
^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^
2)^(1/2))/f))*c*d-1/2/(-4*d*f+e^2)^(1/2)/f^2*2^(1/2)/((-(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f
*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e
^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)
/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^2*c-1/2/(-4*d*f
+e^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*
f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)+1/2/(-4*d*f+e^2)^(1/2
)*c^(1/2)/f*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-
c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))*e+1/2*c
^(1/2)/f*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(
e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))+1/2/f^2*2^
(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*
ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-4*d
*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c
*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*e+1/(
-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e
^2*c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2)
)/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/
f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f
+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f))*a-1/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/(((-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*d+1/2/(-4*d*f+e^2)^(1/2)/f^2*2
```

```
^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)
*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-4*
d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^2*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^2 + a)/(f*x^2 + e*x + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^2 + a)/(f*x^2 + e*x + d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}}{d + ex + fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}}{fx^2 + ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^2 + a)/(f*x^2 + e*x + d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)/(f*x^2 + e*x + d), x)
```

3.55 $\int \frac{\sqrt{a+cx^2}}{x(d+ex+fx^2)}\,dx$

Optimal. Leaf size=358

$$\frac{\left(\left(e-\sqrt{e^2-4df}\right)(cd-af)+2aef\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\left(\sqrt{e^2-4df}+e\right)(cd-af)+2aef\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

```
[Out] ((2*a*e*f + (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f -
 c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*
d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^
2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])
 - ((2*a*e*f + (c*d - a*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*
f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 -
 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt
[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]
)]) - (Sqrt[a]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
```

Rubi [A] time = 2.70918, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{\left(\left(e-\sqrt{e^2-4df}\right)(cd-af)+2aef\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\left(\sqrt{e^2-4df}+e\right)(cd-af)+2aef\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + c*x^2]/(x*(d + e*x + f*x^2)),x]
```


```
[Out] ((2*a*e*f + (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f -
 c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*
d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^
2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])
 - ((2*a*e*f + (c*d - a*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*
f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 -
 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt
[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]
)]) - (Sqrt[a]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a)**(1/2)/x/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.20709, size = 667, normalized size = 1.86

$$-\frac{\frac{\sqrt{2}\left(cd\left(\sqrt{e^2-4df}-e\right)-af\left(\sqrt{e^2-4df}+e\right)\right)\log\left(\sqrt{2}\sqrt{a+cx^2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}+2af\sqrt{e^2-4df}+cx\left(-e\sqrt{e^2-4df}-4df+e^2\right)\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{2}\left(af\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] (2*Sqrt[a]*Log[x] + (Sqrt[2]*(c*d*(-e + Sqrt[e^2 - 4*d*f]) - a*f*
(e + Sqrt[e^2 - 4*d*f]))*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x])/(Sq
rt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*
f])]) + (Sqrt[2]*(a*f*(e - Sqrt[e^2 - 4*d*f]) + c*d*(e + Sqrt[e^2
 - 4*d*f]))*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[e^2 - 4*d*f
]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - 2*Sqrt
[a]*Log[a + Sqrt[a]*Sqrt[a + c*x^2]] - (Sqrt[2]*(c*d*(-e + Sqrt[e
^2 - 4*d*f]) - a*f*(e + Sqrt[e^2 - 4*d*f]))*Log[2*a*f*Sqrt[e^2 -
4*d*f] + c*(e^2 - 4*d*f - e*Sqrt[e^2 - 4*d*f])*x + Sqrt[2]*Sqrt[e
^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]
*Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d
*f - e*Sqrt[e^2 - 4*d*f])]) - (Sqrt[2]*(a*f*(e - Sqrt[e^2 - 4*d*f
]) + c*d*(e + Sqrt[e^2 - 4*d*f]))*Log[2*a*f*Sqrt[e^2 - 4*d*f] - c
*(e^2 - 4*d*f + e*Sqrt[e^2 - 4*d*f])*x + Sqrt[2]*Sqrt[e^2 - 4*d*f
]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a +
c*x^2]])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqr
t[e^2 - 4*d*f])]))/(2*d)
```

Maple [B] time = 0.022, size = 3544, normalized size = 9.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x)
```

```
[Out] 4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*a^(1/2)*ln((2*
a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-
4*d*f+e^2)^(1/2))*(c*x^2+a)^(1/2)+f/(-e+(-4*d*f+e^2)^(1/2))/(-4*d
*f+e^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4
*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)+1/(-e+(-4*d*f+e^
2)^(1/2))*c^(1/2)*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+c*(x-1/2*(-
e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))
/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))
/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2
))-1/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*c^(1/2)*ln((-1/2*
c*(e-(-4*d*f+e^2)^(1/2))/f+c*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))/c
^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(
```

```
1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))*e+1/f/(-e+(-4*d*f+e^2)^(
1/2))*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^
2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-
c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*
2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/
2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(
1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1
/2))/f))*c*e-2*f/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/
2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln
(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e-(-4*d*
f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-
1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*a+
2/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f
*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e
^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)
/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*d-1/f/(-e+(-4*d
*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/
f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)
)/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^2*c+f/(e+(-4*d*f+e^2)^(1/2
))/(-4*d*f+e^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c
*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)-1/(e+(-4*d*
f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^
2)^(1/2))/f+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+e^2*c)/f^2)^(1/2))*e-1/(e+(-4*d*f+e^2)^(1/2))*c^(1/2)*ln((-1/2
*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))/c
^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1
/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))-1/f/(e+(-4*d*f+e^2)^(1/2))*
2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2
)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-4
*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(
((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*e-2
*f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(
1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*a+2/(e+(-4*d*f+e^2)^(1/2)
)/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c
*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+e^2*c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*
c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*
d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f))*c*d-1/f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)
*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/
2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-
4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*
(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+
```

```
1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^2*
c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}}{(fx^2 + ex + d)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x), x)
```

Fricas [A] time = 39.9145, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d
^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f))*log
((2*a*c*d*e*x - a^2*e^2 + sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^
2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*
a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d
^2*e^2 - 4*d^3*f)) - (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^
2 - 4*d^5*f)))/x) - sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (
d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 -
4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 - sqrt(2)*(d^3*e^2 - 4*d^4*f
)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2
 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 -
4*d^5*f)))/(d^2*e^2 - 4*d^3*f)) - (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^
2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - sqrt(2)*d*sqrt((2*c*d^2 + a*e^2
- 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))
)/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 + sqrt(2)*(d^3*
e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*
sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^
2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f)) + (a*d^2*e^2 - 4*a*d
^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) + sqrt(2)*d*sqrt((2*c
*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^
2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 -
sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqr
t(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f
)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f)) + (a*d^
2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - 2*sqrt
(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2))/d, -1/4*
(sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*
sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f))*log((2*a*
c*d*e*x - a^2*e^2 + sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4
*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f
+ (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2
 - 4*d^3*f)) - (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*
d^5*f)))/x) - sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^
2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*
f))*log((2*a*c*d*e*x - a^2*e^2 - sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt
(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e
^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*
f)))/(d^2*e^2 - 4*d^3*f)) - (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/
(d^4*e^2 - 4*d^5*f)))/x) - sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*
```

```
d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2
*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 + sqrt(2)*(d^3*e^2 -
4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((
2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4
*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f)) + (a*d^2*e^2 - 4*a*d^3*f)*
sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) + sqrt(2)*d*sqrt((2*c*d^2 +
 a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*
d^5*f)))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 - sqrt(2
)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^
2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt
(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f)) + (a*d^2*e^2
- 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) + 4*sqrt(-a)*a
rctan(a/(sqrt(c*x^2 + a)*sqrt(-a))))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}}{x\left(d + ex + fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+a)**(1/2)/x/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x),x, algorithm="giac")
```

```
[Out] Timed out
```

3.56 $\int \frac{\sqrt{a+cx^2}}{x^2(d+ex+fx^2)}\,dx$

Optimal. Leaf size=382

$$-\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(a\left(-e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{\sqrt{a+cx^2}}{dx}$$

```
[Out] -(Sqrt[a + c*x^2]/(d*x)) - (f*(2*c*d^2 + a*(e^2 - 2*d*f + e*Sqrt[
e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqr
t[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a
 + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2
 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(2*c*d^2 + a*(e^2 - 2*d*f
- e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f]
)*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]
)]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2
 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[a]*e*ArcTanh[S
qrt[a + c*x^2]/Sqrt[a]])/d^2
```

Rubi [A] time = 3.12612, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444

$$-\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(a\left(-e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{\sqrt{a+cx^2}}{dx}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```


```
[Out] -(Sqrt[a + c*x^2]/(d*x)) - (f*(2*c*d^2 + a*(e^2 - 2*d*f + e*Sqrt[
e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqr
t[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a
 + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2
 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(2*c*d^2 + a*(e^2 - 2*d*f
- e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f]
)*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]
)]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2
 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[a]*e*ArcTanh[S
qrt[a + c*x^2]/Sqrt[a]])/d^2
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a)**(1/2)/x**2/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.15938, size = 600, normalized size = 1.57

$$-\frac{\sqrt{2}f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{4af^2-2ce\sqrt{e^2-4df}-4cdf+2ce^2}+2af+cx\left(\sqrt{e^2-4df}-e\right)\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{2}f\left(a\left(e\sqrt{e^2-4df}+2df-e^2\right)-2cd^2\right)\log\left(-\right.}{\sqrt{e^2-4d}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] ((-2*d*Sqrt[a + c*x^2])/x - 2*Sqrt[a]*e*Log[x] + (Sqrt[2]*f*(2*c*
d^2 + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Log[-e + Sqrt[e^2 -
4*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 - e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*f*(-2*c*d^2 + a*(-e^2 + 2*d*
f + e*Sqrt[e^2 - 4*d*f]))*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sq
rt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*
f])]) + 2*Sqrt[a]*e*Log[a + Sqrt[a]*Sqrt[a + c*x^2]] - (Sqrt[2]*f
*(2*c*d^2 + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Log[2*a*f + c*
(-e + Sqrt[e^2 - 4*d*f])*x + Sqrt[2*c*e^2 - 4*c*d*f + 4*a*f^2 - 2
*c*e*Sqrt[e^2 - 4*d*f]]*Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (Sqrt[2]*f*(
-2*c*d^2 + a*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Log[-2*a*f + c
*e*x + c*Sqrt[e^2 - 4*d*f]*x - Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*Sqrt[2
*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]))/(2*d^2)
```

Maple [B] time = 0.051, size = 3703, normalized size = 9.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x)
```

```
[Out] 4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x*(c*x^2+a)^
(3/2)-4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c/a*x*(c
*x^2+a)^(1/2)-4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*
c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+2*f^2/(-e+(-4*d*f+e^2)^(1/2
))^2/(-4*d*f+e^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-
4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*
(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)+2*f/(-
e+(-4*d*f+e^2)^(1/2))^2*c^(1/2)*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))
/f+c*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x-1/2*(-e+(-4*d
*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d
*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^
2*c)/f^2)^(1/2))-2*f/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)
*c^(1/2)*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+c*(x-1/2*(-e+(-4*d*f
```

```
+e^2)^(1/2))/f))/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c
*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))*e+2/(-
e+(-4*d*f+e^2)^(1/2))^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e
^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c
*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(
-e+(-4*d*f+e^2)^(1/2))/f))*c*e-4*f^2/(-e+(-4*d*f+e^2)^(1/2))^2/(-
4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/
2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*
c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4
*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*
d*f+e^2)^(1/2))/f))*a+4*f/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^
(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^
2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-
c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*
2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/
2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(
1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1
/2))/f))*c*d-2/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/
2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln
(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e-(-4*d*
f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-
1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^
2*c-2*f^2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*(4*(x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+e^2*c)/f^2)^(1/2)+2*f/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1
/2)*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-
c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))*e+2*f/(
e+(-4*d*f+e^2)^(1/2))^2*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))
/f+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)
/f^2)^(1/2))+2/(e+(-4*d*f+e^2)^(1/2))^2*2^(1/2)/(((-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))
/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2)
)/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))
/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*e+4*f^2/(e+(-4*d*f+e^2)^(1/2
))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c
*d*f+e^2*c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e
^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(
-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f))*a-4*f/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1
/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^
(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e
+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/
2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f
*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c
*d+2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))
```

```
/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^
2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^2*c+16*f^2*e/(-e+(-
4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2*a^(1/2)*ln((2*a+2*a^
(1/2)*(c*x^2+a)^(1/2))/x)-16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(
-4*d*f+e^2)^(1/2))^2*(c*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}}{(fx^2 + ex + d)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}}{x^2 \left(d + ex + fx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+a)**(1/2)/x**2/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x**2*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.57 $\int \frac{\sqrt{a+cx^2}}{x^3(d+ex+fx^2)}\,dx$

Optimal. Leaf size=507

$$-\frac{\sqrt{a}\,(e^2-df)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{f\left(a\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+cd^2\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(a\left(-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}-3def+e^3\right)+cd^2\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{a+cx^2}}{2dx^2}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

```
[Out] -Sqrt[a + c*x^2]/(2*d*x^2) + (e*Sqrt[a + c*x^2])/(d^2*x) + (f*(c*
d^2*(e + Sqrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4
*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2
- 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
- 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(c*d^2*(e
 - Sqrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f]
+ d*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*
f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*
f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f
^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (c*ArcTanh[Sqrt[a
+ c*x^2]/Sqrt[a]])/(2*Sqrt[a]*d) - (Sqrt[a]*(e^2 - d*f)*ArcTanh[S
qrt[a + c*x^2]/Sqrt[a]])/d^3
```

Rubi [A] time = 4.10989, antiderivative size = 507, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$

$$-\frac{\sqrt{a}\,(e^2-df)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{f\left(a\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+cd^2\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(a\left(-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}-3def+e^3\right)+cd^2\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{a+cx^2}}{2dx^2}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + c*x^2]/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] -Sqrt[a + c*x^2]/(2*d*x^2) + (e*Sqrt[a + c*x^2])/(d^2*x) + (f*(c*
d^2*(e + Sqrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4
```

```
*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2
- 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
- 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(c*d^2*(e
 - Sqrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f]
+ d*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*
f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*
f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f
^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (c*ArcTanh[Sqrt[a
+ c*x^2]/Sqrt[a]])/(2*Sqrt[a]*d) - (Sqrt[a]*(e^2 - d*f)*ArcTanh[S
qrt[a + c*x^2]/Sqrt[a]])/d^3
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a)**(1/2)/x**3/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.6914, size = 767, normalized size = 1.51

$$\frac{\frac{\log\left(\sqrt{a}\sqrt{a+cx^2}+a\right)\left(2adf-2ae^2-cd^2\right)}{\sqrt{a}}+\frac{\log(x)\left(2a\left(e^2-df\right)+cd^2\right)}{\sqrt{a}}+\frac{\sqrt{2}f\left(a\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+cd^2\left(\sqrt{e^2-4df}+e\right)\right)\log\left(\sqrt{a+cx^2}\sqrt{4af^2-\cdots}\right.}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df\cdots\right.}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + c*x^2]/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] ((d*(-d + 2*e*x)*Sqrt[a + c*x^2])/x^2 + ((c*d^2 + 2*a*(e^2 - d*f)
)*Log[x])/Sqrt[a] - (Sqrt[2]*f*(c*d^2*(e + Sqrt[e^2 - 4*d*f]) + a
*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))
*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*f*(c*d^
2*(e - Sqrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d
*f] + d*f*Sqrt[e^2 - 4*d*f]))*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])
/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 -
4*d*f])]) + ((-(c*d^2) - 2*a*e^2 + 2*a*d*f)*Log[a + Sqrt[a]*Sqrt[
a + c*x^2]])/Sqrt[a] + (Sqrt[2]*f*(c*d^2*(e + Sqrt[e^2 - 4*d*f])
+ a*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f
]))*Log[2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x + Sqrt[2*c*e^2 - 4*c
*d*f + 4*a*f^2 - 2*c*e*Sqrt[e^2 - 4*d*f]]*Sqrt[a + c*x^2]])/(Sqrt
[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]
)]) + (Sqrt[2]*f*(c*d^2*(-e + Sqrt[e^2 - 4*d*f]) + a*(-e^3 + 3*d*
e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[-2*a*f
+ c*e*x + c*Sqrt[e^2 - 4*d*f]*x - Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f
 + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*Sqr
t[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]))/(2*d^3)
```

Maple [B] time = 0.026, size = 3993, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x)
```

```
[Out] 2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x^2*(c*x^2+a
)^(3/2)+2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c/a^(1
/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-2*f/(-e+(-4*d*f+e^2)^(1
/2))/(e+(-4*d*f+e^2)^(1/2))*c/a*(c*x^2+a)^(1/2)+4*f^3/(-e+(-4*d*f
+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2
))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1
/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1
/2)+4*f^2/(-e+(-4*d*f+e^2)^(1/2))^3*c^(1/2)*ln((-1/2*c*(e-(-4*d*f
+e^2)^(1/2))/f+c*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+e^2*c)/f^2)^(1/2))-4*f^2/(-e+(-4*d*f+e^2)^(1/2))^3/(-4*
d*f+e^2)^(1/2)*c^(1/2)*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+c*(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(
1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(
1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)
^(1/2))*e+4*f/(-e+(-4*d*f+e^2)^(1/2))^3*2^(1/2)/((-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(
1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)
^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*e-8*f^3/(-e+(-4*d*f+
e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f
)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))
/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))
/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*a+8*f^2/(-e+(-4*d*f+e^2)^(1/2
))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e
^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c
*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(
-e+(-4*d*f+e^2)^(1/2))/f))*c*d-4*f/(-e+(-4*d*f+e^2)^(1/2))^3/(-4*
d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^
2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)
/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d
*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*
f+e^2)^(1/2))/f))*e^2*c+4*f^3/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^
2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e
^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)-4*f^2/(e+(-4*d*f+e^2)^(1
/2))^3/(-4*d*f+e^2)^(1/2)*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2
))/f+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*
c)/f^2)^(1/2))*e-4*f^2/(e+(-4*d*f+e^2)^(1/2))^3*c^(1/2)*ln((-1/2*
c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))/c^
(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/
2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))-4*f/(e+(-4*d*f+e^2)^(1/2))^3
*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/
2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-
4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*
(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*e-
8*f^3/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+
```

```
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-4*d*f+e^2)^(1/2)
)/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f
^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*a+8*f^2/(e+(-4*d*f+e
^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*
d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-
4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f))*c*d-4*f/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d
*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^
2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c
)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)
^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2
)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f))*e^2*c+16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)
^(1/2))^2/a/x*(c*x^2+a)^(3/2)-16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/
(e+(-4*d*f+e^2)^(1/2))^2*c/a*x*(c*x^2+a)^(1/2)-16*f^2*e/(-e+(-4*d
*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2*c^(1/2)*ln(x*c^(1/2)+(c
*x^2+a)^(1/2))-64*f^4/(-e+(-4*d*f+e^2)^(1/2))^3/(e+(-4*d*f+e^2)^(
1/2))^3*a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)*d+64*f^3/(-
e+(-4*d*f+e^2)^(1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3*a^(1/2)*ln((2*a+
2*a^(1/2)*(c*x^2+a)^(1/2))/x)*e^2+64*f^4/(-e+(-4*d*f+e^2)^(1/2))^
3/(e+(-4*d*f+e^2)^(1/2))^3*(c*x^2+a)^(1/2)*d-64*f^3/(-e+(-4*d*f+e
^2)^(1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3*(c*x^2+a)^(1/2)*e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+a}}{(fx^2+ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}}{x^3\,(d+ex+fx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+a)**(1/2)/x**3/(f*x**2+e*x+d),x)

[Out] Integral(sqrt(a + c*x**2)/(x**3*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [A] time = 1.34518, size = 1, normalized size = 0.

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^3),x, algorithm="giac")

[Out] Done
```

3.58 $\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=795

$$-\frac{(4e-3fx)(cx^2+a)^{3/2}}{12f^2}-\frac{(8e(af^2+c(e^2-2df))-f(3af^2+4c(e^2-df))x)\sqrt{cx^2+a}}{8f^4}$$
$$+\frac{(3a^2f^4+12ac(e^2-df)f^2+8c^2(e^4-3dfe^2+d^2f^2))\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{cx^2+a}}\right)}{8\sqrt{c}f^5}$$
$$-\frac{\left(a^2\left(e^2-\sqrt{e^2-4df}e-2df\right)f^4+2ac\left(e^4-\sqrt{e^2-4df}e^3-4dfe^2+2df\sqrt{e^2-4df}e+2d^2f^2\right)f^2+c^2\left(e^6-\sqrt{e^2-4df}\right.\right.}{\sqrt{2}f^5\sqrt{e^2-4df}\sqrt{2af^2+c\left(\right.}}$$
$$+\frac{\left(a^2\left(e^2+\sqrt{e^2-4df}e-2df\right)f^4+2ac\left(e^4+\sqrt{e^2-4df}e^3-4dfe^2-2df\sqrt{e^2-4df}e+2d^2f^2\right)f^2+c^2\left(e^6+\sqrt{e^2-4df}e\right.\right.}{\sqrt{2}f^5\sqrt{e^2-4df}\sqrt{2af^2+c\left(\right.}}$$

```
[Out] -((8*e*(a*f^2 + c*(e^2 - 2*d*f)) - f*(3*a*f^2 + 4*c*(e^2 - d*f))*
x)*Sqrt[a + c*x^2])/(8*f^4) - ((4*e - 3*f*x)*(a + c*x^2)^(3/2))/(
12*f^2) + ((3*a^2*f^4 + 12*a*c*f^2*(e^2 - d*f) + 8*c^2*(e^4 - 3*d
*e^2*f + d^2*f^2))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*Sqrt[
c]*f^5) - ((a^2*f^4*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + 2*a*c*f
^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2 - e^3*Sqrt[e^2 - 4*d*f] + 2*d*e*f
*Sqrt[e^2 - 4*d*f]) + c^2*(e^6 - 6*d*e^4*f + 9*d^2*e^2*f^2 - 2*d^
3*f^3 - e^5*Sqrt[e^2 - 4*d*f] + 4*d*e^3*f*Sqrt[e^2 - 4*d*f] - 3*d
^2*e*f^2*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4
*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4
*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*f]*Sqrt[2*
a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((a^2*f^4*(e^2
- 2*d*f + e*Sqrt[e^2 - 4*d*f]) + 2*a*c*f^2*(e^4 - 4*d*e^2*f + 2*d
^2*f^2 + e^3*Sqrt[e^2 - 4*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]) + c^2
*(e^6 - 6*d*e^4*f + 9*d^2*e^2*f^2 - 2*d^3*f^3 + e^5*Sqrt[e^2 - 4*
d*f] - 4*d*e^3*f*Sqrt[e^2 - 4*d*f] + 3*d^2*e*f^2*Sqrt[e^2 - 4*d*f
]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2
*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]
)/(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 9.888, antiderivative size = 795, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259

$$-\frac{(4e-3fx)(cx^2+a)^{3/2}}{12f^2}-\frac{(8e(af^2+c(e^2-2df))-f(3af^2+4c(e^2-df))x)\sqrt{cx^2+a}}{8f^4}$$
$$+\frac{(3a^2f^4+12ac(e^2-df)f^2+8c^2(e^4-3dfe^2+d^2f^2))\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{cx^2+a}}\right)}{8\sqrt{c}f^5}$$
$$-\frac{\left(a^2\left(e^2-\sqrt{e^2-4df}e-2df\right)f^4+2ac\left(e^4-\sqrt{e^2-4df}e^3-4dfe^2+2df\sqrt{e^2-4df}e+2d^2f^2\right)f^2+c^2\left(e^6-\sqrt{e^2-4df}\right.\right.}{\sqrt{2}f^5\sqrt{e^2-4df}\sqrt{2af^2+c\left(\right.}}$$
$$+\frac{\left(a^2\left(e^2+\sqrt{e^2-4df}e-2df\right)f^4+2ac\left(e^4+\sqrt{e^2-4df}e^3-4dfe^2-2df\sqrt{e^2-4df}e+2d^2f^2\right)f^2+c^2\left(e^6+\sqrt{e^2-4df}e\right.\right.}{\sqrt{2}f^5\sqrt{e^2-4df}\sqrt{2af^2+c\left(\right.}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] -((8*e*(a*f^2 + c*(e^2 - 2*d*f)) - f*(3*a*f^2 + 4*c*(e^2 - d*f))*
x)*Sqrt[a + c*x^2])/(8*f^4) - ((4*e - 3*f*x)*(a + c*x^2)^(3/2))/(
12*f^2) + ((3*a^2*f^4 + 12*a*c*f^2*(e^2 - d*f) + 8*c^2*(e^4 - 3*d
*e^2*f + d^2*f^2))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*Sqrt[
c]*f^5) - ((a^2*f^4*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + 2*a*c*f
^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2 - e^3*Sqrt[e^2 - 4*d*f] + 2*d*e*f
*Sqrt[e^2 - 4*d*f]) + c^2*(e^6 - 6*d*e^4*f + 9*d^2*e^2*f^2 - 2*d^
3*f^3 - e^5*Sqrt[e^2 - 4*d*f] + 4*d*e^3*f*Sqrt[e^2 - 4*d*f] - 3*d
^2*e*f^2*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4
*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4
*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*f]*Sqrt[2*
a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((a^2*f^4*(e^2
- 2*d*f + e*Sqrt[e^2 - 4*d*f]) + 2*a*c*f^2*(e^4 - 4*d*e^2*f + 2*d
^2*f^2 + e^3*Sqrt[e^2 - 4*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]) + c^2
*(e^6 - 6*d*e^4*f + 9*d^2*e^2*f^2 - 2*d^3*f^3 + e^5*Sqrt[e^2 - 4*
d*f] - 4*d*e^3*f*Sqrt[e^2 - 4*d*f] + 3*d^2*e*f^2*Sqrt[e^2 - 4*d*f
]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2
*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]
)/(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f])])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 6.22491, size = 1565, normalized size = 1.97

$$\sqrt{cx^2+a}\left(\frac{cx^3}{4f}-\frac{cex^2}{3f^2}+\frac{\left(4ce^2+5af^2-4cdf\right)x}{8f^3}-\frac{e\left(3ce^2+4af^2-6cdf\right)}{3f^4}\right)$$

$$-\frac{\left(-c^2e^6+c^2\sqrt{e^2-4df}e^5-2acf^2e^4+6c^2dfe^4+2acf^2\sqrt{e^2-4df}e^3-4c^2df\sqrt{e^2-4df}e^3-a^2f^4e^2+8acdf^3e^2-9c^2d^2f\right.}{\sqrt{2}f^5\sqrt{e^2-4df}\sqrt{ce^2}}$$

$$-\frac{\left(c^2e^6+c^2\sqrt{e^2-4df}e^5+2acf^2e^4-6c^2dfe^4+2acf^2\sqrt{e^2-4df}e^3-4c^2df\sqrt{e^2-4df}e^3+a^2f^4e^2-8acdf^3e^2+9c^2d^2f^2e\right.}{\sqrt{2}f^5\sqrt{e^2-4df}\sqrt{ce^2+}}$$

$$+\frac{\left(8c^2e^4+12acf^2e^2-24c^2dfe^2+3a^2f^4-12acdf^3+8c^2d^2f^2\right)\log\left(cx+\sqrt{c}\sqrt{cx^2+a}\right)}{8\sqrt{c}f^5}$$

$$+\frac{\left(-c^2e^6+c^2\sqrt{e^2-4df}e^5-2acf^2e^4+6c^2dfe^4+2acf^2\sqrt{e^2-4df}e^3-4c^2df\sqrt{e^2-4df}e^3-a^2f^4e^2+8acdf^3e^2-9c^2d^2f^2\right.}{}$$

$$+\frac{\left(c^2e^6+c^2\sqrt{e^2-4df}e^5+2acf^2e^4-6c^2dfe^4+2acf^2\sqrt{e^2-4df}e^3-4c^2df\sqrt{e^2-4df}e^3+a^2f^4e^2-8acdf^3e^2+9c^2d^2f^2e^2\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] Sqrt[a + c*x^2]*(-(e*(3*c*e^2 - 6*c*d*f + 4*a*f^2))/(3*f^4) + ((4
*c*e^2 - 4*c*d*f + 5*a*f^2)*x)/(8*f^3) - (c*e*x^2)/(3*f^2) + (c*x
^3)/(4*f)) - ((-(c^2*e^6) + 6*c^2*d*e^4*f - 9*c^2*d^2*e^2*f^2 - 2
*a*c*e^4*f^2 + 2*c^2*d^3*f^3 + 8*a*c*d*e^2*f^3 - 4*a*c*d^2*f^4 -
a^2*e^2*f^4 + 2*a^2*d*f^5 + c^2*e^5*Sqrt[e^2 - 4*d*f] - 4*c^2*d*e
^3*f*Sqrt[e^2 - 4*d*f] + 3*c^2*d^2*e*f^2*Sqrt[e^2 - 4*d*f] + 2*a*
c*e^3*f^2*Sqrt[e^2 - 4*d*f] - 4*a*c*d*e*f^3*Sqrt[e^2 - 4*d*f] + a
^2*e*f^4*Sqrt[e^2 - 4*d*f])*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x])/
(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f + 2*a*f^2 - c
*e*Sqrt[e^2 - 4*d*f]]) - ((c^2*e^6 - 6*c^2*d*e^4*f + 9*c^2*d^2*e^
2*f^2 + 2*a*c*e^4*f^2 - 2*c^2*d^3*f^3 - 8*a*c*d*e^2*f^3 + 4*a*c*d
^2*f^4 + a^2*e^2*f^4 - 2*a^2*d*f^5 + c^2*e^5*Sqrt[e^2 - 4*d*f] -
4*c^2*d*e^3*f*Sqrt[e^2 - 4*d*f] + 3*c^2*d^2*e*f^2*Sqrt[e^2 - 4*d*
f] + 2*a*c*e^3*f^2*Sqrt[e^2 - 4*d*f] - 4*a*c*d*e*f^3*Sqrt[e^2 - 4
*d*f] + a^2*e*f^4*Sqrt[e^2 - 4*d*f])*Log[e + Sqrt[e^2 - 4*d*f] +
2*f*x])/(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f + 2*a
*f^2 + c*e*Sqrt[e^2 - 4*d*f]]) + ((8*c^2*e^4 - 24*c^2*d*e^2*f + 8
*c^2*d^2*f^2 + 12*a*c*e^2*f^2 - 12*a*c*d*f^3 + 3*a^2*f^4)*Log[c*x
 + Sqrt[c]*Sqrt[a + c*x^2]])/(8*Sqrt[c]*f^5) + ((-(c^2*e^6) + 6*c
^2*d*e^4*f - 9*c^2*d^2*e^2*f^2 - 2*a*c*e^4*f^2 + 2*c^2*d^3*f^3 +
8*a*c*d*e^2*f^3 - 4*a*c*d^2*f^4 - a^2*e^2*f^4 + 2*a^2*d*f^5 + c^2
*e^5*Sqrt[e^2 - 4*d*f] - 4*c^2*d*e^3*f*Sqrt[e^2 - 4*d*f] + 3*c^2*
d^2*e*f^2*Sqrt[e^2 - 4*d*f] + 2*a*c*e^3*f^2*Sqrt[e^2 - 4*d*f] - 4
*a*c*d*e*f^3*Sqrt[e^2 - 4*d*f] + a^2*e*f^4*Sqrt[e^2 - 4*d*f])*Log
[2*a*f*Sqrt[e^2 - 4*d*f] + c*e^2*x - 4*c*d*f*x - c*e*Sqrt[e^2 - 4
*d*f]*x + Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f + 2*a*f^
2 - c*e*Sqrt[e^2 - 4*d*f]]*Sqrt[a + c*x^2]])/(Sqrt[2]*f^5*Sqrt[e^
2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f]
]) + ((c^2*e^6 - 6*c^2*d*e^4*f + 9*c^2*d^2*e^2*f^2 + 2*a*c*e^4*f^
2 - 2*c^2*d^3*f^3 - 8*a*c*d*e^2*f^3 + 4*a*c*d^2*f^4 + a^2*e^2*f^4
 - 2*a^2*d*f^5 + c^2*e^5*Sqrt[e^2 - 4*d*f] - 4*c^2*d*e^3*f*Sqrt[e
^2 - 4*d*f] + 3*c^2*d^2*e*f^2*Sqrt[e^2 - 4*d*f] + 2*a*c*e^3*f^2*S
qrt[e^2 - 4*d*f] - 4*a*c*d*e*f^3*Sqrt[e^2 - 4*d*f] + a^2*e*f^4*Sq
rt[e^2 - 4*d*f])*Log[2*a*f*Sqrt[e^2 - 4*d*f] - c*e^2*x + 4*c*d*f*
x - c*e*Sqrt[e^2 - 4*d*f]*x + Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^
2 - 2*c*d*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f]]*Sqrt[a + c*x^2]])/
(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f + 2*a*f^2 + c
*e*Sqrt[e^2 - 4*d*f]])
```

Maple [B] time = 0.043, size = 19148, normalized size = 24.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + a)^(3/2)*x^2/(f*x^2 + e*x + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + a)^(3/2)*x^2/(f*x^2 + e*x + d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(a + cx^2\right)^{\frac{3}{2}}}{d + ex + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**2*(a + c*x**2)**(3/2)/(d + e*x + f*x**2), x)
```

GIAC/XCAS [A] time = 0.918341, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + a)^(3/2)*x^2/(f*x^2 + e*x + d),x, algorithm="giac")
```

```
[Out] Done
```

3.59 $\int \frac{x\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=591

$$-\frac{\left(2cdef\left(2af^2+c\left(e^2-2df\right)\right)-\left(e-\sqrt{e^2-4df}\right)\left(a^2f^4+2acf^2\left(e^2-df\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right)\right)\tanh^{-1}\left(\frac{\cdots}{\sqrt{2}\sqrt{a+cx\cdots}}\right.}{\sqrt{2}f^4\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$+\frac{\left(2cdef\left(2af^2+c\left(e^2-2df\right)\right)-\left(\sqrt{e^2-4df}+e\right)\left(a^2f^4+2acf^2\left(e^2-df\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right)\right)\tanh^{-1}\left(\frac{2\cdots}{\sqrt{2}\sqrt{a+cx^2}\cdots}\right.}{\sqrt{2}f^4\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$-\frac{\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(af^2+c\left(e^2-2df\right)\right)}{f^4}+\frac{\sqrt{a+cx^2}\left(af^2+c\left(e^2-df\right)\right)}{f^3}$$
$$-\frac{cex\sqrt{a+cx^2}}{2f^2}-\frac{a\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2f^2}+\frac{\left(a+cx^2\right)^{3/2}}{3f}$$

```
[Out] ((a*f^2 + c*(e^2 - d*f))*Sqrt[a + c*x^2])/f^3 - (c*e*x*Sqrt[a + c
*x^2])/(2*f^2) + (a + c*x^2)^(3/2)/(3*f) - (a*Sqrt[c]*e*ArcTanh[(
Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*f^2) - (Sqrt[c]*e*(a*f^2 + c*(e^2
 - 2*d*f))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^4 - ((2*c*d*e*
f*(2*a*f^2 + c*(e^2 - 2*d*f)) - (e - Sqrt[e^2 - 4*d*f])*(a^2*f^4
+ 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*d*e^2*f + d^2*f^2)))*ArcTa
nh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 +
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2
]*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^
2 - 4*d*f])]) + ((2*c*d*e*f*(2*a*f^2 + c*(e^2 - 2*d*f)) - (e + Sq
rt[e^2 - 4*d*f])*(a^2*f^4 + 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*
d*e^2*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x
)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*
Sqrt[a + c*x^2])])/(Sqrt[2]*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 +
c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 7.02553, antiderivative size = 591, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.36$

$$-\frac{\left(2cdef\left(2af^2+c\left(e^2-2df\right)\right)-\left(e-\sqrt{e^2-4df}\right)\left(a^2f^4+2acf^2\left(e^2-df\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right)\right)\tanh^{-1}\left(\frac{\cdots}{\sqrt{2}\sqrt{a+cx\cdots}}\right.}{\sqrt{2}f^4\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$+\frac{\left(2cdef\left(2af^2+c\left(e^2-2df\right)\right)-\left(\sqrt{e^2-4df}+e\right)\left(a^2f^4+2acf^2\left(e^2-df\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right)\right)\tanh^{-1}\left(\frac{2\cdots}{\sqrt{2}\sqrt{a+cx^2}\cdots}\right.}{\sqrt{2}f^4\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$-\frac{\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(af^2+c\left(e^2-2df\right)\right)}{f^4}+\frac{\sqrt{a+cx^2}\left(af^2+c\left(e^2-df\right)\right)}{f^3}$$
$$-\frac{cex\sqrt{a+cx^2}}{2f^2}-\frac{a\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2f^2}+\frac{\left(a+cx^2\right)^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] ((a*f^2 + c*(e^2 - d*f))*Sqrt[a + c*x^2])/f^3 - (c*e*x*Sqrt[a + c
*x^2])/(2*f^2) + (a + c*x^2)^(3/2)/(3*f) - (a*Sqrt[c]*e*ArcTanh[(
Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*f^2) - (Sqrt[c]*e*(a*f^2 + c*(e^2
 - 2*d*f))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^4 - ((2*c*d*e*
f*(2*a*f^2 + c*(e^2 - 2*d*f)) - (e - Sqrt[e^2 - 4*d*f])*(a^2*f^4
+ 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*d*e^2*f + d^2*f^2)))*ArcTa
nh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 +
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2
]*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^
2 - 4*d*f])]) + ((2*c*d*e*f*(2*a*f^2 + c*(e^2 - 2*d*f)) - (e + Sq
rt[e^2 - 4*d*f])*(a^2*f^4 + 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*
d*e^2*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x
)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*
Sqrt[a + c*x^2])])/(Sqrt[2]*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 +
c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 4.08487, size = 1176, normalized size = 1.99

$$f\sqrt{cx^2+a}\left(8af^2+c\left(6e^2-3fxe+2f\left(fx^2-3d\right)\right)\right)+\frac{3\sqrt{2}\left(a^2\left(\sqrt{e^2-4df}-e\right)f^4-2ac\left(e^3-\sqrt{e^2-4df}\,e^2-3dfe+df\sqrt{e^2-4df}\right)f^2+c^2\left(-e^5+\sqrt{e^2-\cdots}\right.\right.}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\cdots\right.}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] (f*Sqrt[a + c*x^2]*(8*a*f^2 + c*(6*e^2 - 3*e*f*x + 2*f*(-3*d + f*
x^2))) + (3*Sqrt[2]*(a^2*f^4*(-e + Sqrt[e^2 - 4*d*f]) - 2*a*c*f^2
*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f])
+ c^2*(-e^5 + 5*d*e^3*f - 5*d^2*e*f^2 + e^4*Sqrt[e^2 - 4*d*f] - 3
*d*e^2*f*Sqrt[e^2 - 4*d*f] + d^2*f^2*Sqrt[e^2 - 4*d*f]))*Log[-e +
 Sqrt[e^2 - 4*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*
(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (3*Sqrt[2]*(a^2*f^4*(e +
Sqrt[e^2 - 4*d*f]) + 2*a*c*f^2*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*
d*f] - d*f*Sqrt[e^2 - 4*d*f]) + c^2*(e^5 - 5*d*e^3*f + 5*d^2*e*f^
2 + e^4*Sqrt[e^2 - 4*d*f] - 3*d*e^2*f*Sqrt[e^2 - 4*d*f] + d^2*f^2
*Sqrt[e^2 - 4*d*f]))*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[e^
2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
 - 3*Sqrt[c]*e*(3*a*f^2 + 2*c*(e^2 - 2*d*f))*Log[c*x + Sqrt[c]*Sq
rt[a + c*x^2]] - (3*Sqrt[2]*(a^2*f^4*(-e + Sqrt[e^2 - 4*d*f]) - 2
*a*c*f^2*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 -
4*d*f]) + c^2*(-e^5 + 5*d*e^3*f - 5*d^2*e*f^2 + e^4*Sqrt[e^2 - 4*
d*f] - 3*d*e^2*f*Sqrt[e^2 - 4*d*f] + d^2*f^2*Sqrt[e^2 - 4*d*f]))*
Log[2*a*f*Sqrt[e^2 - 4*d*f] + c*(e^2 - 4*d*f - e*Sqrt[e^2 - 4*d*f
])*x + Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f -
e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*Sqrt[2
*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (3*Sqrt[2]*(a^
2*f^4*(e + Sqrt[e^2 - 4*d*f]) + 2*a*c*f^2*(e^3 - 3*d*e*f + e^2*Sq
rt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]) + c^2*(e^5 - 5*d*e^3*f +
 5*d^2*e*f^2 + e^4*Sqrt[e^2 - 4*d*f] - 3*d*e^2*f*Sqrt[e^2 - 4*d*f
] + d^2*f^2*Sqrt[e^2 - 4*d*f]))*Log[2*a*f*Sqrt[e^2 - 4*d*f] - c*(
```

```
e^2 - 4*d*f + e*Sqrt[e^2 - 4*d*f])*x + Sqrt[2]*Sqrt[e^2 - 4*d*f]*
Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*
x^2]])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[
e^2 - 4*d*f])]))/(6*f^4)
```

Maple [B] time = 0.024, size = 14709, normalized size = 24.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + a)^(3/2)*x/(f*x^2 + e*x + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + a)^(3/2)*x/(f*x^2 + e*x + d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a + cx^2\right)^{\frac{3}{2}}}{d + ex + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x*(a + c*x**2)**(3/2)/(d + e*x + f*x**2), x)
```

GIAC/XCAS [A] time = 0.825786, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(3/2)*x/(f*x^2 + e*x + d),x, algorithm="giac")
```

```
[Out] Done
```

3.60 $\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=484

$$-\frac{\left(ce\left(e-\sqrt{e^2-4df}\right)\left(2af^2+c\left(e^2-2df\right)\right)-2f\left(-a^2f^3+2acdf^2+c^2d\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(ce\left(\sqrt{e^2-4df}+e\right)\left(2af^2+c\left(e^2-2df\right)\right)-2f\left(-a^2f^3+2acdf^2+c^2d\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3af^2+2c\left(e^2-df\right)\right)}{2f^3}-\frac{c\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

```
[Out] -(c*(2*e - f*x)*Sqrt[a + c*x^2])/(2*f^2) + (Sqrt[c]*(3*a*f^2 + 2*
c*(e^2 - d*f))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*f^3) - ((
c*e*(e - Sqrt[e^2 - 4*d*f])*(2*a*f^2 + c*(e^2 - 2*d*f)) - 2*f*(2*
a*c*d*f^2 - a^2*f^3 + c^2*d*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e -
 Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e
*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4
*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((
c*e*(e + Sqrt[e^2 - 4*d*f])*(2*a*f^2 + c*(e^2 - 2*d*f)) - 2*f*(2*
a*c*d*f^2 - a^2*f^3 + c^2*d*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e +
 Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e
*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4
*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 8.82711, antiderivative size = 482, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\left(-2a^2f^4-ce\left(e-\sqrt{e^2-4df}\right)\left(2af^2+c\left(e^2-2df\right)\right)+4acdf^3+2c^2df\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(-2a^2f^4-ce\left(\sqrt{e^2-4df}+e\right)\left(2af^2+c\left(e^2-2df\right)\right)+4acdf^3+2c^2df\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3af^2+2c\left(e^2-df\right)\right)}{2f^3}-\frac{c\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

```
[Out] -(c*(2*e - f*x)*Sqrt[a + c*x^2])/(2*f^2) + (Sqrt[c]*(3*a*f^2 + 2*
c*(e^2 - d*f))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*f^3) + ((
4*a*c*d*f^3 - 2*a^2*f^4 + 2*c^2*d*f*(e^2 - d*f) - c*e*(e - Sqrt[e
^2 - 4*d*f])*(2*a*f^2 + c*(e^2 - 2*d*f)))*ArcTanh[(2*a*f - c*(e -
 Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e
*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4
*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((
4*a*c*d*f^3 - 2*a^2*f^4 + 2*c^2*d*f*(e^2 - d*f) - c*e*(e + Sqrt[e
```

```
^2 - 4*d*f])*(2*a*f^2 + c*(e^2 - 2*d*f)))*ArcTanh[(2*a*f - c*(e +
 Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e
*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4
*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.71635, size = 932, normalized size = 1.93

$$\frac{cf\sqrt{cx^2+a}(fx-2e)-\frac{\sqrt{2}\left(-2a^2f^4+2ac\left(-e^2+\sqrt{e^2-4df}e+2df\right)f^2+c^2\left(-e^4+\sqrt{e^2-4df}e^3+4dfe^2-2df\sqrt{e^2-4df}e-2d^2f^2\right)\right)\log\left(-e-2fx+\sqrt{e^2-4df}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-\sqrt{e^2-4df}e-2df\right)}}-}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

```
[Out] (c*f*(-2*e + f*x)*Sqrt[a + c*x^2] - (Sqrt[2]*(-2*a^2*f^4 + 2*a*c*
f^2*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]) + c^2*(-e^4 + 4*d*e^2*f
- 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]))
*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (Sqrt[2]*(2*a^2*
f^4 + 2*a*c*f^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + c^2*(e^4 -
4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d*f] - 2*d*e*f*Sqrt[e^2
- 4*d*f]))*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[e^2 - 4*d*f]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) + Sqrt[c]
*(3*a*f^2 + 2*c*(e^2 - d*f))*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] +
 (Sqrt[2]*(-2*a^2*f^4 + 2*a*c*f^2*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*
d*f]) + c^2*(-e^4 + 4*d*e^2*f - 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d*f]
 - 2*d*e*f*Sqrt[e^2 - 4*d*f]))*Log[2*a*f*Sqrt[e^2 - 4*d*f] + c*(e
^2 - 4*d*f - e*Sqrt[e^2 - 4*d*f])*x + Sqrt[2]*Sqrt[e^2 - 4*d*f]*S
qrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x
^2]])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f])]) + (Sqrt[2]*(2*a^2*f^4 + 2*a*c*f^2*(e^2 - 2*d*f + e
*Sqrt[e^2 - 4*d*f]) + c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt
[e^2 - 4*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]))*Log[2*a*f*Sqrt[e^2 -
4*d*f] - c*(e^2 - 4*d*f + e*Sqrt[e^2 - 4*d*f])*x + Sqrt[2]*Sqrt[e
^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]
*Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d
*f + e*Sqrt[e^2 - 4*d*f])]))/(2*f^3)
```

Maple [B] time = 0.022, size = 8954, normalized size = 18.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + a)^(3/2)/(f*x^2 + e*x + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + a)^(3/2)/(f*x^2 + e*x + d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + cx^{2}\right)^{\frac{3}{2}}}{d + ex + fx^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(d + e*x + f*x**2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + a)^(3/2)/(f*x^2 + e*x + d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.61 $\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)}\,dx$

Optimal. Leaf size=496

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}-\frac{\left(2ef\left(c^2d^2-a^2f^2\right)-\left(e-\sqrt{e^2-4df}\right)\left(c^2de^2-f(cd-af)^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2ef\left(c^2d^2-a^2f^2\right)-\left(\sqrt{e^2-4df}+e\right)\left(c^2de^2-f(cd-af)^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{c^{3/2}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2}+\frac{\sqrt{a+cx^2}(cd-af)}{df}+\frac{a\sqrt{a+cx^2}}{d}$$

```
[Out] (a*Sqrt[a + c*x^2])/d + ((c*d - a*f)*Sqrt[a + c*x^2])/(d*f) - (c^
(3/2)*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^2 - ((2*e*f*(c^2*
d^2 - a^2*f^2) - (c^2*d*e^2 - f*(c*d - a*f)^2)*(e - Sqrt[e^2 - 4*
d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqr
t[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2
])])/(Sqrt[2]*d*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d
*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*e*f*(c^2*d^2 - a^2*f^2) - (c^2*
d*e^2 - f*(c*d - a*f)^2)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f
- c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2
*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*f^2*Sq
rt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*
f])]) - (a^(3/2)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
```

Rubi [A] time = 5.34175, antiderivative size = 496, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}-\frac{\left(2ef\left(c^2d^2-a^2f^2\right)-\left(e-\sqrt{e^2-4df}\right)\left(c^2de^2-f(cd-af)^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2ef\left(c^2d^2-a^2f^2\right)-\left(\sqrt{e^2-4df}+e\right)\left(c^2de^2-f(cd-af)^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{c^{3/2}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2}+\frac{\sqrt{a+cx^2}(cd-af)}{df}+\frac{a\sqrt{a+cx^2}}{d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)),x]
```

```
[Out] (a*Sqrt[a + c*x^2])/d + ((c*d - a*f)*Sqrt[a + c*x^2])/(d*f) - (c^
(3/2)*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^2 - ((2*e*f*(c^2*
```

```
d^2 - a^2*f^2) - (c^2*d*e^2 - f*(c*d - a*f)^2)*(e - Sqrt[e^2 - 4*
d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqr
t[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2
])])/(Sqrt[2]*d*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d
*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*e*f*(c^2*d^2 - a^2*f^2) - (c^2*
d*e^2 - f*(c*d - a*f)^2)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f
- c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2
*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*f^2*Sq
rt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*
f])]) - (a^(3/2)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a)**(3/2)/x/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.07136, size = 956, normalized size = 1.93

$$\frac{\log(x)a^{3/2}}{d}-\frac{\log\left(a+\sqrt{cx^2+a}\sqrt{a}\right)a^{3/2}}{d}$$

$$-\frac{\left(a^2\left(e+\sqrt{e^2-4df}\right)f^3+2acd\left(e-\sqrt{e^2-4df}\right)f^2+c^2d\left(e^3-\sqrt{e^2-4df}e^2-3dfe+df\sqrt{e^2-4df}\right)\right)\log\left(-e-2fx+\right.}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-\sqrt{e^2-4df}e-2df\right)}}$$

$$+\frac{\left(a^2\left(e-\sqrt{e^2-4df}\right)f^3+2acd\left(e+\sqrt{e^2-4df}\right)f^2+c^2d\left(e^3+\sqrt{e^2-4df}e^2-3dfe-df\sqrt{e^2-4df}\right)\right)\log\left(e+2fx+\sqrt{\right.}}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2+\sqrt{e^2-4df}e-2df\right)}}$$

$$-\frac{c^{3/2}e\log\left(cx+\sqrt{c}\sqrt{cx^2+a}\right)}{f^2}$$

$$+\frac{\left(a^2\left(e+\sqrt{e^2-4df}\right)f^3+2acd\left(e-\sqrt{e^2-4df}\right)f^2+c^2d\left(e^3-\sqrt{e^2-4df}e^2-3dfe+df\sqrt{e^2-4df}\right)\right)\log\left(2a\sqrt{e^2-4d}\right.}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-\sqrt{e^2}\right.}}$$

$$-\frac{\left(a^2\left(e-\sqrt{e^2-4df}\right)f^3+2acd\left(e+\sqrt{e^2-4df}\right)f^2+c^2d\left(e^3+\sqrt{e^2-4df}e^2-3dfe-df\sqrt{e^2-4df}\right)\right)\log\left(2a\sqrt{e^2-4d}\right.}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2+\sqrt{e^2}\right.}}$$

$$+\frac{c\sqrt{cx^2+a}}{f}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)),x]
```

```
[Out] (c*Sqrt[a + c*x^2])/f + (a^(3/2)*Log[x])/d - ((2*a*c*d*f^2*(e - S
qrt[e^2 - 4*d*f]) + a^2*f^3*(e + Sqrt[e^2 - 4*d*f]) + c^2*d*(e^3
- 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f]))*Log[-
e + Sqrt[e^2 - 4*d*f] - 2*f*x])/(Sqrt[2]*d*f^2*Sqrt[e^2 - 4*d*f]*
Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((a^2*f^
3*(e - Sqrt[e^2 - 4*d*f]) + 2*a*c*d*f^2*(e + Sqrt[e^2 - 4*d*f]) +
 c^2*d*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*
```

```
d*f]))*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[2]*d*f^2*Sqrt[e^
2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
 - (a^(3/2)*Log[a + Sqrt[a]*Sqrt[a + c*x^2]])/d - (c^(3/2)*e*Log[
c*x + Sqrt[c]*Sqrt[a + c*x^2]])/f^2 + ((2*a*c*d*f^2*(e - Sqrt[e^2
 - 4*d*f]) + a^2*f^3*(e + Sqrt[e^2 - 4*d*f]) + c^2*d*(e^3 - 3*d*e
*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f]))*Log[2*a*f*Sq
rt[e^2 - 4*d*f] + c*(e^2 - 4*d*f - e*Sqrt[e^2 - 4*d*f])*x + Sqrt[
2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 -
 4*d*f])]*Sqrt[a + c*x^2]])/(Sqrt[2]*d*f^2*Sqrt[e^2 - 4*d*f]*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((a^2*f^3*(e
 - Sqrt[e^2 - 4*d*f]) + 2*a*c*d*f^2*(e + Sqrt[e^2 - 4*d*f]) + c^2
*d*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]
))*Log[2*a*f*Sqrt[e^2 - 4*d*f] - c*(e^2 - 4*d*f + e*Sqrt[e^2 - 4*
d*f])*x + Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]])/(Sqrt[2]*d*f^2*Sqrt[e^
2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Maple [B] time = 0.027, size = 9728, normalized size = 19.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2+a)^{\frac{3}{2}}}{(fx^2+ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+cx^2)^{\frac{3}{2}}}{x\,(d+ex+fx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+a)**(3/2)/x/(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(x*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [A] time = 1.31481, size = 1, normalized size = 0.

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x),x, algorithm="giac")
```

```
[Out] Done
```

3.62 $\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)}\,dx$

Optimal. Leaf size=604

$$\frac{a^{3/2}e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{\left(a^2f^2\left(e\sqrt{e^2-4df}-2df+e^2\right)+4acd^2f^2+c^2d^2\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$+\frac{\left(a^2f^2\left(-e\sqrt{e^2-4df}-2df+e^2\right)+4acd^2f^2+c^2d^2\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$-\frac{ae\sqrt{a+cx^2}}{d^2}+\frac{\sqrt{a+cx^2}(2ae-cdx)}{2d^2}+\frac{\sqrt{c}(2cd-3af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2df}$$
$$-\frac{\left(a+cx^2\right)^{3/2}}{dx}+\frac{3cx\sqrt{a+cx^2}}{2d}+\frac{3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2d}$$

```
[Out] -((a*e*Sqrt[a + c*x^2])/d^2) + (3*c*x*Sqrt[a + c*x^2])/(2*d) + ((
2*a*e - c*d*x)*Sqrt[a + c*x^2])/(2*d^2) - (a + c*x^2)^(3/2)/(d*x)
 + (3*a*Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*d) + (Sq
rt[c]*(2*c*d - 3*a*f)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*d*
f) - ((4*a*c*d^2*f^2 + c^2*d^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]
) + a^2*f^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f -
 c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*
d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*f*Sqr
t[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f
])]) + ((4*a*c*d^2*f^2 + a^2*f^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*
f]) + c^2*d^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f
 - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 -
2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*f*S
qrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f])]) + (a^(3/2)*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^2
```

Rubi [A] time = 5.93298, antiderivative size = 604, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.518

$$\frac{a^{3/2}e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{\left(a^2f^2\left(e\sqrt{e^2-4df}-2df+e^2\right)+4acd^2f^2+c^2d^2\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$+\frac{\left(a^2f^2\left(-e\sqrt{e^2-4df}-2df+e^2\right)+4acd^2f^2+c^2d^2\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$-\frac{ae\sqrt{a+cx^2}}{d^2}+\frac{\sqrt{a+cx^2}(2ae-cdx)}{2d^2}+\frac{\sqrt{c}(2cd-3af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2df}$$
$$-\frac{\left(a+cx^2\right)^{3/2}}{dx}+\frac{3cx\sqrt{a+cx^2}}{2d}+\frac{3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] -((a*e*Sqrt[a + c*x^2])/d^2) + (3*c*x*Sqrt[a + c*x^2])/(2*d) + ((
2*a*e - c*d*x)*Sqrt[a + c*x^2])/(2*d^2) - (a + c*x^2)^(3/2)/(d*x)
 + (3*a*Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*d) + (Sq
rt[c]*(2*c*d - 3*a*f)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*d*
f) - ((4*a*c*d^2*f^2 + c^2*d^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]
) + a^2*f^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f -
 c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*
d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*f*Sqr
t[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f
])]) + ((4*a*c*d^2*f^2 + a^2*f^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*
f]) + c^2*d^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f
 - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 -
2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*f*S
qrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f])]) + (a^(3/2)*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^2
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a)**(3/2)/x**2/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.49819, size = 880, normalized size = 1.46

$$-\frac{e\log(x)a^{3/2}}{d^2}+\frac{e\log\left(a+\sqrt{cx^2+a}\sqrt{a}\right)a^{3/2}}{d^2}-\frac{\sqrt{cx^2+a}a}{dx}$$
$$-\frac{\left(-4acf^2d^2+c^2\left(-e^2+\sqrt{e^2-4df}e+2df\right)d^2+a^2f^2\left(-e^2-\sqrt{e^2-4df}e+2df\right)\right)\log\left(-e-2fx+\sqrt{e^2-4df}\right)}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-\sqrt{e^2-4df}e-2df\right)}}$$
$$-\frac{\left(4acf^2d^2+c^2\left(e^2+\sqrt{e^2-4df}e-2df\right)d^2+a^2f^2\left(e^2-\sqrt{e^2-4df}e-2df\right)\right)\log\left(e+2fx+\sqrt{e^2-4df}\right)}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2+\sqrt{e^2-4df}e-2df\right)}}$$
$$+\frac{c^{3/2}\log\left(cx+\sqrt{c}\sqrt{cx^2+a}\right)}{f}$$
$$+\frac{\left(-4acf^2d^2+c^2\left(-e^2+\sqrt{e^2-4df}e+2df\right)d^2+a^2f^2\left(-e^2-\sqrt{e^2-4df}e+2df\right)\right)\log\left(2a\sqrt{e^2-4df}f+c\left(e^2-\sqrt{e^2-4\right.\right.}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-\sqrt{e^2-4df}e-\right.}}$$
$$+\frac{\left(4acf^2d^2+c^2\left(e^2+\sqrt{e^2-4df}e-2df\right)d^2+a^2f^2\left(e^2-\sqrt{e^2-4df}e-2df\right)\right)\log\left(2a\sqrt{e^2-4df}f-c\left(e^2+\sqrt{e^2-4df}e\right.\right.}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2+\sqrt{e^2-4df}e-2d\right.}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] -((a*Sqrt[a + c*x^2])/(d*x)) - (a^(3/2)*e*Log[x])/d^2 - ((-4*a*c*
d^2*f^2 + a^2*f^2*(-e^2 + 2*d*f - e*Sqrt[e^2 - 4*d*f]) + c^2*d^2*
(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Log[-e + Sqrt[e^2 - 4*d*f]
- 2*f*x])/(Sqrt[2]*d^2*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2
- 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((4*a*c*d^2*f^2 + a^2*f^2*(e^2
 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + c^2*d^2*(e^2 - 2*d*f + e*Sqrt[e
^2 - 4*d*f]))*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[2]*d^2*f*
Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*
d*f])]) + (a^(3/2)*e*Log[a + Sqrt[a]*Sqrt[a + c*x^2]])/d^2 + (c^(
3/2)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/f + ((-4*a*c*d^2*f^2 + a
^2*f^2*(-e^2 + 2*d*f - e*Sqrt[e^2 - 4*d*f]) + c^2*d^2*(-e^2 + 2*d
*f + e*Sqrt[e^2 - 4*d*f]))*Log[2*a*f*Sqrt[e^2 - 4*d*f] + c*(e^2 -
 4*d*f - e*Sqrt[e^2 - 4*d*f])*x + Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[
2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]]
)/(Sqrt[2]*d^2*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
- e*Sqrt[e^2 - 4*d*f])]) + ((4*a*c*d^2*f^2 + a^2*f^2*(e^2 - 2*d*f
 - e*Sqrt[e^2 - 4*d*f]) + c^2*d^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f]))*Log[2*a*f*Sqrt[e^2 - 4*d*f] - c*(e^2 - 4*d*f + e*Sqrt[e^2 -
 4*d*f])*x + Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*
d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]])/(Sqrt[2]*d^2*f*Sqrt
[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]
)])
```

Maple [B] time = 0.029, size = 9912, normalized size = 16.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + a\right)^{\frac{3}{2}}}{\left(fx^2 + ex + d\right)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + cx^2\right)^{\frac{3}{2}}}{x^2\left(d + ex + fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+a)**(3/2)/x**2/(f*x**2+e*x+d),x)

[Out] Integral((a + c*x**2)**(3/2)/(x**2*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [A] time = 0.664234, size = 1, normalized size = 0.

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^2),x, algorithm="giac")

[Out] Done
```

3.63 $\int \frac{\left(a+cx^2\right)^{3/2}}{x^3\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=668

$$-\frac{a^{3/2}\left(e^2-df\right)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+\frac{\left(a^2f\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+2acd^2f\left(\sqrt{e^2-4df}+e\right)+c^2d^3\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2\cdots}{\sqrt{2}\sqrt{a+cx^2}\cdots}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(a^2f\left(-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}-3def+e^3\right)+2acd^2f\left(e-\sqrt{e^2-4df}\right)+c^2d^3\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{\cdots}{\sqrt{2}\sqrt{a+c\cdots}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{a\sqrt{a+cx^2}\left(e^2-df\right)}{d^3}+\frac{e\left(a+cx^2\right)^{3/2}}{d^2x}-\frac{3cex\sqrt{a+cx^2}}{2d^2}-\frac{\sqrt{a+cx^2}\left(2\left(a\left(e^2-df\right)+cd^2\right)-cdex\right)}{2d^3}-\frac{\left(a+cx^2\right)^{3/2}}{2dx^2}+\frac{3c\sqrt{a+cx^2}}{2d}-\frac{3\sqrt{a}c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2d}$$

```
[Out] (3*c*Sqrt[a + c*x^2])/(2*d) + (a*(e^2 - d*f)*Sqrt[a + c*x^2])/d^3
 - (3*c*e*x*Sqrt[a + c*x^2])/(2*d^2) - ((2*(c*d^2 + a*(e^2 - d*f)
) - c*d*e*x)*Sqrt[a + c*x^2])/(2*d^3) - (a + c*x^2)^(3/2)/(2*d*x^
2) + (e*(a + c*x^2)^(3/2))/(d^2*x) + ((c^2*d^3*(e - Sqrt[e^2 - 4*
d*f]) + 2*a*c*d^2*f*(e + Sqrt[e^2 - 4*d*f]) + a^2*f*(e^3 - 3*d*e*
f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*
f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 -
 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^3*Sq
rt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*
f])]) - ((2*a*c*d^2*f*(e - Sqrt[e^2 - 4*d*f]) + c^2*d^3*(e + Sqrt
[e^2 - 4*d*f]) + a^2*f*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d
*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])
*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2
+ c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (3*Sqrt[a]*c*ArcTanh[
Sqrt[a + c*x^2]/Sqrt[a]])/(2*d) - (a^(3/2)*(e^2 - d*f)*ArcTanh[Sq
rt[a + c*x^2]/Sqrt[a]])/d^3
```

Rubi [A] time = 6.97277, antiderivative size = 668, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 15, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$

$$-\frac{a^{3/2}\left(e^2-df\right)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+\frac{\left(a^2f\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+2acd^2f\left(\sqrt{e^2-4df}+e\right)+c^2d^3\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2\cdots}{\sqrt{2}\sqrt{a+cx^2}\cdots}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(a^2f\left(-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}-3def+e^3\right)+2acd^2f\left(e-\sqrt{e^2-4df}\right)+c^2d^3\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{\cdots}{\sqrt{2}\sqrt{a+c\cdots}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{a\sqrt{a+cx^2}\left(e^2-df\right)}{d^3}+\frac{e\left(a+cx^2\right)^{3/2}}{d^2x}-\frac{3cex\sqrt{a+cx^2}}{2d^2}-\frac{\sqrt{a+cx^2}\left(2\left(a\left(e^2-df\right)+cd^2\right)-cdex\right)}{2d^3}-\frac{\left(a+cx^2\right)^{3/2}}{2dx^2}+\frac{3c\sqrt{a+cx^2}}{2d}-\frac{3\sqrt{a}c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] (3*c*Sqrt[a + c*x^2])/(2*d) + (a*(e^2 - d*f)*Sqrt[a + c*x^2])/d^3
 - (3*c*e*x*Sqrt[a + c*x^2])/(2*d^2) - ((2*(c*d^2 + a*(e^2 - d*f)
) - c*d*e*x)*Sqrt[a + c*x^2])/(2*d^3) - (a + c*x^2)^(3/2)/(2*d*x^
2) + (e*(a + c*x^2)^(3/2))/(d^2*x) + ((c^2*d^3*(e - Sqrt[e^2 - 4*
d*f]) + 2*a*c*d^2*f*(e + Sqrt[e^2 - 4*d*f]) + a^2*f*(e^3 - 3*d*e*
f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*
f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 -
 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^3*Sq
rt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*
f])]) - ((2*a*c*d^2*f*(e - Sqrt[e^2 - 4*d*f]) + c^2*d^3*(e + Sqrt
[e^2 - 4*d*f]) + a^2*f*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d
*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])
*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2
+ c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (3*Sqrt[a]*c*ArcTanh[
Sqrt[a + c*x^2]/Sqrt[a]])/(2*d) - (a^(3/2)*(e^2 - d*f)*ArcTanh[Sq
rt[a + c*x^2]/Sqrt[a]])/d^3
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a)**(3/2)/x**3/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 4.53441, size = 949, normalized size = 1.42

$$\frac{\frac{ad\sqrt{cx^2+a}(2ex-d)}{x^2} - \sqrt{a}\left(-3cd^2+2afd-2ae^2\right)\log(x) - \frac{\sqrt{2}\left(c^2\left(e-\sqrt{e^2-4df}\right)d^3+2acf\left(e+\sqrt{e^2-4df}\right)d^2+a^2f\left(e^3+\sqrt{e^2-4df}\,e^2-3dfe-df\sqrt{e^2-4d\ldots}\right.\right.}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-\sqrt{e^2-4df}\,e-2df\right)}}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] ((a*d*(-d + 2*e*x)*Sqrt[a + c*x^2])/x^2 - Sqrt[a]*(-3*c*d^2 - 2*a
*e^2 + 2*a*d*f)*Log[x] - (Sqrt[2]*(c^2*d^3*(e - Sqrt[e^2 - 4*d*f]
) + 2*a*c*d^2*f*(e + Sqrt[e^2 - 4*d*f]) + a^2*f*(e^3 - 3*d*e*f +
e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[-e + Sqrt[e^2
 - 4*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*
d*f - e*Sqrt[e^2 - 4*d*f])]) - (Sqrt[2]*(2*a*c*d^2*f*(-e + Sqrt[e
^2 - 4*d*f]) - c^2*d^3*(e + Sqrt[e^2 - 4*d*f]) - a^2*f*(e^3 - 3*d
*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f]))*Log[e + Sq
rt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^
2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) + Sqrt[a]*(-3*c*d^2 - 2*a*e^2
+ 2*a*d*f)*Log[a + Sqrt[a]*Sqrt[a + c*x^2]] + (Sqrt[2]*(c^2*d^3*(
e - Sqrt[e^2 - 4*d*f]) + 2*a*c*d^2*f*(e + Sqrt[e^2 - 4*d*f]) + a^
2*f*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f
]))*Log[2*a*f*Sqrt[e^2 - 4*d*f] + c*(e^2 - 4*d*f - e*Sqrt[e^2 - 4
*d*f])*x + Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*
f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*Sq
rt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*(
2*a*c*d^2*f*(-e + Sqrt[e^2 - 4*d*f]) - c^2*d^3*(e + Sqrt[e^2 - 4*
d*f]) - a^2*f*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e
```

```
^2 - 4*d*f]))*Log[2*a*f*Sqrt[e^2 - 4*d*f] - c*(e^2 - 4*d*f + e*Sq
rt[e^2 - 4*d*f])*x + Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]])/(Sqrt[e^2 -
 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]))/(
2*d^3)
```

Maple [B] time = 0.03, size = 10298, normalized size = 15.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^{\frac{3}{2}}}{(fx^2+ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^3),x, algorithm="maxima")

[Out] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^3),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+a)**(3/2)/x**3/(f*x**2+e*x+d),x)

[Out] Timed out
```

GIAC/XCAS [A] time = 1.61857, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^3),x, algorithm="giac")
```

```
[Out] Done
```

3.64 $\int \frac{x^3}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=380

$$-\frac{\left(2def-(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2def-(e^2-df)\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f^2}+\frac{\sqrt{a+cx^2}}{cf}$$

```
[Out] Sqrt[a + c*x^2]/(c*f) - (e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/
(Sqrt[c]*f^2) - ((2*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*
ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f
^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(S
qrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sq
rt[e^2 - 4*d*f])]) + ((2*d*e*f - (e^2 - d*f)*(e + Sqrt[e^2 - 4*d*
f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[
2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])
])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f +
 e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 2.42913, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{\left(2def-(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2def-(e^2-df)\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f^2}+\frac{\sqrt{a+cx^2}}{cf}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```


```
[Out] Sqrt[a + c*x^2]/(c*f) - (e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/
(Sqrt[c]*f^2) - ((2*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*
ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f
^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(S
qrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sq
rt[e^2 - 4*d*f])]) + ((2*d*e*f - (e^2 - d*f)*(e + Sqrt[e^2 - 4*d*
f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[
2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])
])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f +
```

```
e*Sqrt[e^2 - 4*d*f])])
```

Rubi in Sympy [A] time = 145.077, size = 371, normalized size = 0.98

$$\frac{\sqrt{2}\left(-2def+\left(e-\sqrt{-4df+e^2}\right)\left(-df+e^2\right)\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(2af-cx\left(e-\sqrt{-4df+e^2}\right)\right)}{2\sqrt{a+cx^2}\sqrt{2af^2-2cdf+ce^2-ce\sqrt{-4df+e^2}}}\right)}{2f^2\sqrt{-4df+e^2}\sqrt{2af^2-2cdf+ce^2-ce\sqrt{-4df+e^2}}} - \frac{\sqrt{2}\left(-2def+\left(e+\sqrt{-4df+e^2}\right)\left(-df+e^2\right)\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(2af-cx\left(e+\sqrt{-4df+e^2}\right)\right)}{2\sqrt{a+cx^2}\sqrt{2af^2-2cdf+ce^2+ce\sqrt{-4df+e^2}}}\right)}{2f^2\sqrt{-4df+e^2}\sqrt{2af^2-2cdf+ce^2+ce\sqrt{-4df+e^2}}} + \frac{\sqrt{a+cx^2}}{cf} - \frac{e\operatorname{atanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] sqrt(2)*(-2*d*e*f + (e - sqrt(-4*d*f + e**2))*(-d*f + e**2))*atan
h(sqrt(2)*(2*a*f - c*x*(e - sqrt(-4*d*f + e**2)))/(2*sqrt(a + c*x
**2)*sqrt(2*a*f**2 - 2*c*d*f + c*e**2 - c*e*sqrt(-4*d*f + e**2)))
)/(2*f**2*sqrt(-4*d*f + e**2)*sqrt(2*a*f**2 - 2*c*d*f + c*e**2 -
c*e*sqrt(-4*d*f + e**2))) - sqrt(2)*(-2*d*e*f + (e + sqrt(-4*d*f
+ e**2))*(-d*f + e**2))*atanh(sqrt(2)*(2*a*f - c*x*(e + sqrt(-4*d
*f + e**2)))/(2*sqrt(a + c*x**2)*sqrt(2*a*f**2 - 2*c*d*f + c*e**2
 + c*e*sqrt(-4*d*f + e**2))))/(2*f**2*sqrt(-4*d*f + e**2)*sqrt(2*
a*f**2 - 2*c*d*f + c*e**2 + c*e*sqrt(-4*d*f + e**2))) + sqrt(a +
c*x**2)/(c*f) - e*atanh(sqrt(c)*x/sqrt(a + c*x**2))/(sqrt(c)*f**2
)
```

Mathematica [A] time = 1.23548, size = 696, normalized size = 1.83

$$-\frac{\sqrt{2}\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}+3def-e^3\right)\log\left(\sqrt{2}\sqrt{a+cx^2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}+2af\sqrt{e^2-4df}+cx\left(-e\sqrt{e^2-4df}-4df+e^2\right)\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} - \sqrt{2}\left(e^2 \ldots\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*f*Sqrt[a + c*x^2])/c + (Sqrt[2]*(-e^3 + 3*d*e*f + e^2*Sqrt[e^
2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])*Log[-e + Sqrt[e^2 - 4*d*f] -
2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt
[e^2 - 4*d*f])]) + (Sqrt[2]*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f
] - d*f*Sqrt[e^2 - 4*d*f])*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(S
qrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f])]) - (2*e*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/Sqrt[c] - (Sqrt
[2]*(-e^3 + 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*
f])*Log[2*a*f*Sqrt[e^2 - 4*d*f] + c*(e^2 - 4*d*f - e*Sqrt[e^2 - 4
*d*f])*x + Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*
f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*Sq
rt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (Sqrt[2]*(
e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])*Lo
g[2*a*f*Sqrt[e^2 - 4*d*f] - c*(e^2 - 4*d*f + e*Sqrt[e^2 - 4*d*f])
*x + Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*
Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a
```

```
*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]))/(2*f^2)
```

Maple [B] time = 0.037, size = 2397, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] (c*x^2+a)^(1/2)/c/f-1/f^2*e*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)
+1/2/f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/
f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2
-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*
2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2
)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/
2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f))*d-1/2/f^3*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^
2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c
)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)
^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2
)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f))*e^2+3/2/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))
/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2)
)/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))
/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*d*e-1/2/f^3/(-4*d*f+e^2)^(1/2)
*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/
2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-
4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*
(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^3+
1/2/f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/
f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^
2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/
2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(
1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)
^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^
(1/2))/f))*d-1/2/f^3*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e
-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f))*e^2-3/2/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2
)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(
-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*d*e+1/2
/f^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e
^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c
```

```
*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(
-e+(-4*d*f+e^2)^(1/2))/f))*e^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a + cx^2}\,(d + ex + fx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{cx^2 + a}(fx^2 + ex + d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)),x, algorithm="giac")
```

```
[Out] integrate(x^3/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)), x)
```

3.65 $\int \frac{x^2}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=344

$$-\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f}$$

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*f) - ((e^2 - 2*d*f
- e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])
*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])
]*Sqrt[a + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 +
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((2*d*f - e*(e + Sqrt[e
^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt
[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a
+ c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 -
2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 1.31063, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185

$$-\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```


```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*f) - ((e^2 - 2*d*f
- e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])
*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])
]*Sqrt[a + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 +
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((2*d*f - e*(e + Sqrt[e
^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt
[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a
+ c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 -
2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi in Sympy [A] time = 96.0856, size = 337, normalized size = 0.98

$$\frac{\sqrt{2}\left(2df-e\left(e-\sqrt{-4df+e^2}\right)\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(2af-cx\left(e-\sqrt{-4df+e^2}\right)\right)}{2\sqrt{a+cx^2}\sqrt{2af^2-2cdf+ce^2-ce\sqrt{-4df+e^2}}}\right)}{2f\sqrt{-4df+e^2}\sqrt{2af^2-2cdf+ce^2-ce\sqrt{-4df+e^2}}} - \frac{\sqrt{2}\left(2df-e\left(e+\sqrt{-4df+e^2}\right)\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(2af-cx\left(e+\sqrt{-4df+e^2}\right)\right)}{2\sqrt{a+cx^2}\sqrt{2af^2-2cdf+ce^2+ce\sqrt{-4df+e^2}}}\right)}{2f\sqrt{-4df+e^2}\sqrt{2af^2-2cdf+ce^2+ce\sqrt{-4df+e^2}}} + \frac{\operatorname{atanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x**2/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] sqrt(2)*(2*d*f - e*(e - sqrt(-4*d*f + e**2)))*atanh(sqrt(2)*(2*a*
f - c*x*(e - sqrt(-4*d*f + e**2)))/(2*sqrt(a + c*x**2)*sqrt(2*a*f
**2 - 2*c*d*f + c*e**2 - c*e*sqrt(-4*d*f + e**2))))/(2*f*sqrt(-4*
d*f + e**2)*sqrt(2*a*f**2 - 2*c*d*f + c*e**2 - c*e*sqrt(-4*d*f +
e**2))) - sqrt(2)*(2*d*f - e*(e + sqrt(-4*d*f + e**2)))*atanh(sqr
t(2)*(2*a*f - c*x*(e + sqrt(-4*d*f + e**2)))/(2*sqrt(a + c*x**2)*
sqrt(2*a*f**2 - 2*c*d*f + c*e**2 + c*e*sqrt(-4*d*f + e**2))))/(2*
f*sqrt(-4*d*f + e**2)*sqrt(2*a*f**2 - 2*c*d*f + c*e**2 + c*e*sqrt
(-4*d*f + e**2))) + atanh(sqrt(c)*x/sqrt(a + c*x**2))/(sqrt(c)*f)
```

Mathematica [A] time = 0.84151, size = 602, normalized size = 1.75

$$\frac{\frac{\sqrt{2}\left(e\sqrt{e^2-4df}+2df-e^2\right)\log\left(\sqrt{2}\sqrt{a+cx^2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}+2af\sqrt{e^2-4df}+cx\left(-e\sqrt{e^2-4df}-4df+e^2\right)\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{2}\left(e\sqrt{e^2-4df}-2df+e^2\right)}{}}{}$$

Antiderivative was successfully verified.

```
[In]   Integrate[x^2/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (-((Sqrt[2]*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*Log[-e + Sqrt[e^
2 - 4*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2
*d*f - e*Sqrt[e^2 - 4*d*f])])) - (Sqrt[2]*(e^2 - 2*d*f + e*Sqrt[e
^2 - 4*d*f])*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[e^2 - 4*d*
f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) + (2*Lo
g[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/Sqrt[c] + (Sqrt[2]*(-e^2 + 2*d*
f + e*Sqrt[e^2 - 4*d*f])*Log[2*a*f*Sqrt[e^2 - 4*d*f] + c*(e^2 - 4
*d*f - e*Sqrt[e^2 - 4*d*f])*x + Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*
a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]])/
(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4
*d*f])]) + (Sqrt[2]*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*Log[2*a*f
*Sqrt[e^2 - 4*d*f] - c*(e^2 - 4*d*f + e*Sqrt[e^2 - 4*d*f])*x + Sq
rt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^
2 - 4*d*f])]*Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 +
c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]))/(2*f)
```

Maple [B] time = 0.022, size = 1796, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/f*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)+1/2/f^2*2^(1/2)/((-(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/
2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*
d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-
4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e
^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e+1/f/(-4*d*
f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^
2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*
c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/
f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f
^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f
+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+
e^2)^(1/2))/f))*d-1/2/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f
*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e
^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)
/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^2+1/2/f^2*2^(1/
2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(
(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-4*d*f+
e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e-1/f/(-4*
d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e
^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*
c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2
)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^
2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f))*d+1/2/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))
/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{cx^2+a}(fx^2+ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)), x)
```

3.66 $\int \frac{x}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=294

$$\frac{\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

```
[Out] ((e - Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f
])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f
])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 +
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((e + Sqrt[e^2 - 4*d*f]
)*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/
(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])])
```

Rubi [A] time = 0.682133, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12

$$\frac{\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((e - Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f
])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f
])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 +
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((e + Sqrt[e^2 - 4*d*f]
)*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/
(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])])
```

Rubi in Sympy [A] time = 53.2378, size = 294, normalized size = 1.

$$\frac{\sqrt{2}\left(e-\sqrt{-4df+e^{2}}\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(2af-cx\left(e-\sqrt{-4df+e^{2}}\right)\right)}{2\sqrt{a+cx^{2}}\sqrt{2af^{2}-2cdf+ce^{2}-ce\sqrt{-4df+e^{2}}}}\right)}{2\sqrt{-4df+e^{2}}\sqrt{2af^{2}-2cdf+ce^{2}-ce\sqrt{-4df+e^{2}}}}-\frac{\sqrt{2}\left(e+\sqrt{-4df+e^{2}}\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(2af-cx\left(e+\sqrt{-4df+e^{2}}\right)\right)}{2\sqrt{a+cx^{2}}\sqrt{2af^{2}-2cdf+ce^{2}+ce\sqrt{-4df+e^{2}}}}\right)}{2\sqrt{-4df+e^{2}}\sqrt{2af^{2}-2cdf+ce^{2}+ce\sqrt{-4df+e^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] sqrt(2)*(e - sqrt(-4*d*f + e**2))*atanh(sqrt(2)*(2*a*f - c*x*(e -
 sqrt(-4*d*f + e**2)))/(2*sqrt(a + c*x**2)*sqrt(2*a*f**2 - 2*c*d*
f + c*e**2 - c*e*sqrt(-4*d*f + e**2))))/(2*sqrt(-4*d*f + e**2)*sq
rt(2*a*f**2 - 2*c*d*f + c*e**2 - c*e*sqrt(-4*d*f + e**2))) - sqrt
(2)*(e + sqrt(-4*d*f + e**2))*atanh(sqrt(2)*(2*a*f - c*x*(e + sqr
t(-4*d*f + e**2)))/(2*sqrt(a + c*x**2)*sqrt(2*a*f**2 - 2*c*d*f +
c*e**2 + c*e*sqrt(-4*d*f + e**2))))/(2*sqrt(-4*d*f + e**2)*sqrt(2
*a*f**2 - 2*c*d*f + c*e**2 + c*e*sqrt(-4*d*f + e**2)))
```

Mathematica [A] time = 4.29505, size = 484, normalized size = 1.65

$$\frac{-\frac{\left(\sqrt{e^{2}-4df}-e\right)\log\left(\sqrt{2}\sqrt{a+cx^{2}}\sqrt{e^{2}-4df}\sqrt{2af^{2}+c\left(-e\sqrt{e^{2}-4df}-2df+e^{2}\right)}+2af\sqrt{e^{2}-4df}+cx\left(-e\sqrt{e^{2}-4df}-4df+e^{2}\right)\right)}{\sqrt{2af^{2}+c\left(-e\sqrt{e^{2}-4df}-2df+e^{2}\right)}}-\frac{\left(\sqrt{e^{2}-4df}+e\right)\log\left(\sqrt{2}\sqrt{a+cx^{2}}\sqrt{e^{2}\cdots}\right.}{\cdots}}{\sqrt{2}\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (((-e + Sqrt[e^2 - 4*d*f])*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x])/S
qrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])] + ((e + Sqrt
[e^2 - 4*d*f])*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/Sqrt[2*a*f^2 +
 c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])] - ((-e + Sqrt[e^2 - 4*d*f
])*Log[2*a*f*Sqrt[e^2 - 4*d*f] + c*(e^2 - 4*d*f - e*Sqrt[e^2 - 4*
d*f])*x + Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]])/Sqrt[2*a*f^2 + c*(e^2
- 2*d*f - e*Sqrt[e^2 - 4*d*f])] - ((e + Sqrt[e^2 - 4*d*f])*Log[2*
a*f*Sqrt[e^2 - 4*d*f] - c*(e^2 - 4*d*f + e*Sqrt[e^2 - 4*d*f])*x +
 Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])]*Sqrt[a + c*x^2]])/Sqrt[2*a*f^2 + c*(e^2 - 2*d*f +
 e*Sqrt[e^2 - 4*d*f])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f])
```

Maple [B] time = 0.018, size = 1172, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/2/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
```

```
2*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+
e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*
c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*
(-e+(-4*d*f+e^2)^(1/2))/f))*e-1/2/f*2^(1/2)/((-(-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(
-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/
2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/
2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-1/2/(-4*d*f+e^2)^(1/2)/f*2
^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)
*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-4*
d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e-1/2/
f*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1
/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(
-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)
*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.875124, size = 6865, normalized size = 23.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 + a*c
*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3
*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*
c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*
c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2
+ a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4
)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^
2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*a*c*d^2*e*x - 2*a
^2*d*e^2 + sqrt(2)*(a^2*e^4 - 4*a^2*d*e^2*f - (2*c^3*d^4*e^2 + 3*
a*c^2*d^2*e^4 + a^2*c*e^6 + 8*a^3*d^2*f^4 - 6*(4*a^2*c*d^3 + a^3*
d*e^2)*f^3 + (24*a*c^2*d^4 + 22*a^2*c*d^2*e^2 + a^3*e^4)*f^2 - 2*
(4*c^3*d^5 + 9*a*c^2*d^3*e^2 + 4*a^2*c*d*e^4)*f)*sqrt(a^2*e^2/(c^
4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3
*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*
(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 +
 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((2*
c*d^2 + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 +
```

```
(8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(a
^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5
 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2
)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*
(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 +
 a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3
 + 3*a*c*d*e^2)*f)) + 2*(a*c^2*d^3*e^2 + a^2*c*d*e^4 - 4*a^3*d^2*
f^3 + (8*a^2*c*d^3 + a^3*d*e^2)*f^2 - 2*(2*a*c^2*d^4 + 3*a^2*c*d^
2*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e
^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d
^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3
*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f))
)/x) + 1/4*sqrt(2)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (c^2*d^2*e^2
 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d
^3 + 3*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4
+ a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(
2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^
2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2
*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 +
a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*a*c*d^2*e*x
 - 2*a^2*d*e^2 - sqrt(2)*(a^2*e^4 - 4*a^2*d*e^2*f - (2*c^3*d^4*e^
2 + 3*a*c^2*d^2*e^4 + a^2*c*e^6 + 8*a^3*d^2*f^4 - 6*(4*a^2*c*d^3
+ a^3*d*e^2)*f^3 + (24*a*c^2*d^4 + 22*a^2*c*d^2*e^2 + a^3*e^4)*f^
2 - 2*(4*c^3*d^5 + 9*a*c^2*d^3*e^2 + 4*a^2*c*d*e^4)*f)*sqrt(a^2*e
^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (
16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^
3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4
*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sq
rt((2*c*d^2 + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*
f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*
sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4
*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c
*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^
2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2
*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c
^2*d^3 + 3*a*c*d*e^2)*f)) + 2*(a*c^2*d^3*e^2 + a^2*c*d*e^4 - 4*a^
3*d^2*f^3 + (8*a^2*c*d^3 + a^3*d*e^2)*f^2 - 2*(2*a*c^2*d^4 + 3*a^
2*c*d^2*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2
*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2
*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2
 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^
4)*f)))/x) - 1/4*sqrt(2)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (c^2*d
^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2
*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^
2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4
- 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*
c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a
^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*
d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*a*c*d
^2*e*x - 2*a^2*d*e^2 + sqrt(2)*(a^2*e^4 - 4*a^2*d*e^2*f + (2*c^3*
d^4*e^2 + 3*a*c^2*d^2*e^4 + a^2*c*e^6 + 8*a^3*d^2*f^4 - 6*(4*a^2*
c*d^3 + a^3*d*e^2)*f^3 + (24*a*c^2*d^4 + 22*a^2*c*d^2*e^2 + a^3*e
^4)*f^2 - 2*(4*c^3*d^5 + 9*a*c^2*d^3*e^2 + 4*a^2*c*d*e^4)*f)*sqrt
(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f
^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e
^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 -
4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 +
 a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 + a*c*e^4 - 4*
a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^
2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 -
 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 +
 a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e
^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c
^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 -
2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) - 2*(a*c^2*d^3*e^2 + a^2*c*d*e^4
- 4*a^3*d^2*f^3 + (8*a^2*c*d^3 + a^3*d*e^2)*f^2 - 2*(2*a*c^2*d^4
+ 3*a^2*c*d^2*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4
 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*
(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d
^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^
2*d*e^4)*f)))/x) + 1/4*sqrt(2)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f -
(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2
```

```
- 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*
c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2
)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 1
1*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2
 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (
8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4
*a*c*d^2*e*x - 2*a^2*d*e^2 - sqrt(2)*(a^2*e^4 - 4*a^2*d*e^2*f + (
2*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4 + a^2*c*e^6 + 8*a^3*d^2*f^4 - 6*(
4*a^2*c*d^3 + a^3*d*e^2)*f^3 + (24*a*c^2*d^4 + 22*a^2*c*d^2*e^2 +
 a^3*e^4)*f^2 - 2*(4*c^3*d^5 + 9*a*c^2*d^3*e^2 + 4*a^2*c*d*e^4)*f
)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a
^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3
*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*
f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c
*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 + a*c*e^
4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*
c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2
*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2
*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a
^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f
)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*
f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) - 2*(a*c^2*d^3*e^2 + a^2*c*
d*e^4 - 4*a^3*d^2*f^3 + (8*a^2*c*d^3 + a^3*d*e^2)*f^2 - 2*(2*a*c^
2*d^4 + 3*a^2*c*d^2*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d
^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4
 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2
*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*
a^2*c^2*d*e^4)*f)))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a + cx^2}\,(d + ex + fx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{cx^2 + a}(fx^2 + ex + d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)), x)
```

3.67 $\int \frac{1}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=266

$$\frac{\sqrt{2}f\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{2}f\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

```
[Out] -((Sqrt[2]*f*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[
2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a +
 c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*S
qrt[e^2 - 4*d*f])])) + (Sqrt[2]*f*ArcTanh[(2*a*f - c*(e + Sqrt[e^
2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^
2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 +
 c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 0.452703, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\sqrt{2}f\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{2}f\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[2]*f*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[
2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a +
 c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*S
qrt[e^2 - 4*d*f])])) + (Sqrt[2]*f*ArcTanh[(2*a*f - c*(e + Sqrt[e^
2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^
2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 +
 c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi in Sympy [A] time = 45.554, size = 267, normalized size = 1.

$$\frac{\sqrt{2}f\,\mathrm{atanh}\left(\frac{\sqrt{2}\left(2af-cx\left(e+\sqrt{-4df+e^2}\right)\right)}{2\sqrt{a+cx^2}\sqrt{2af^2-2cdf+ce^2+ce\sqrt{-4df+e^2}}}\right)}{\sqrt{-4df+e^2}\sqrt{2af^2-2cdf+ce^2+ce\sqrt{-4df+e^2}}}-\frac{\sqrt{2}f\,\mathrm{atanh}\left(\frac{\sqrt{2}\left(2af-cx\left(e-\sqrt{-4df+e^2}\right)\right)}{2\sqrt{a+cx^2}\sqrt{2af^2-2cdf+ce^2-ce\sqrt{-4df+e^2}}}\right)}{\sqrt{-4df+e^2}\sqrt{2af^2-2cdf+ce^2-ce\sqrt{-4df+e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] sqrt(2)*f*atanh(sqrt(2)*(2*a*f - c*x*(e + sqrt(-4*d*f + e**2)))/(
2*sqrt(a + c*x**2)*sqrt(2*a*f**2 - 2*c*d*f + c*e**2 + c*e*sqrt(-4
*d*f + e**2))))/(sqrt(-4*d*f + e**2)*sqrt(2*a*f**2 - 2*c*d*f + c*
e**2 + c*e*sqrt(-4*d*f + e**2))) - sqrt(2)*f*atanh(sqrt(2)*(2*a*f
 - c*x*(e - sqrt(-4*d*f + e**2)))/(2*sqrt(a + c*x**2)*sqrt(2*a*f*
*2 - 2*c*d*f + c*e**2 - c*e*sqrt(-4*d*f + e**2))))/(sqrt(-4*d*f +
 e**2)*sqrt(2*a*f**2 - 2*c*d*f + c*e**2 - c*e*sqrt(-4*d*f + e**2)
```

```
))
```

Mathematica [A] time = 2.57428, size = 425, normalized size = 1.6

$$\frac{\sqrt{2}f\left(-\frac{\log\left(\sqrt{2}\sqrt{a+cx^2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}+2af\sqrt{e^2-4df}+cx\left(-e\sqrt{e^2-4df}-4df+e^2\right)\right)}{\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\log\left(\sqrt{2}\sqrt{a+cx^2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2\cdots\right.}\right.}{\sqrt{2af^2\cdots}}\right.}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (Sqrt[2]*f*(Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x]/Sqrt[2*a*f^2 + c*
(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])] - Log[e + Sqrt[e^2 - 4*d*f]
+ 2*f*x]/Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])] -
Log[2*a*f*Sqrt[e^2 - 4*d*f] + c*(e^2 - 4*d*f - e*Sqrt[e^2 - 4*d*f
])*x + Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f -
e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]]/Sqrt[2*a*f^2 + c*(e^2 - 2*
d*f - e*Sqrt[e^2 - 4*d*f])] + Log[2*a*f*Sqrt[e^2 - 4*d*f] - c*(e^
2 - 4*d*f + e*Sqrt[e^2 - 4*d*f])*x + Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sq
rt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^
2]]/Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]))/Sqrt[
e^2 - 4*d*f]
```

Maple [B] time = 0.017, size = 589, normalized size = 2.2

$$-\sqrt{2}\ln\left(1\left(\frac{1}{f^2}\left(-\sqrt{-4\,df+e^2}ce+2\,af^2-2\,cdf+e^2c\right)-\frac{c}{f}\left(e-\sqrt{-4\,df+e^2}\right)\left(x-\frac{1}{2\,f}\left(-e+\sqrt{-4\,df+e^2}\right)\right)+\frac{\sqrt{2}}{2}\sqrt{\frac{1}{f^2}\cdots}\right.\right.$$

$$+\sqrt{2}\ln\left(1\left(\frac{1}{f^2}\left(\sqrt{-4\,df+e^2}ce+2\,af^2-2\,cdf+e^2c\right)-\frac{c}{f}\left(e+\sqrt{-4\,df+e^2}\right)\left(x+\frac{1}{2\,f}\left(e+\sqrt{-4\,df+e^2}\right)\right)+\frac{\sqrt{2}}{2}\sqrt{\frac{1}{f^2}\left(\sqrt{\cdots}\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c
*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(
e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f))+1/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)+2*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^
(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.86457, size = 6849, normalized size = 25.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 + (c^2*d^2*e^2 + a*c
*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3
*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*
c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*
c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2
+ a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4
)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^
2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*c^2*d*e*f*x - 2*a
*c*e^2*f + sqrt(2)*(c^2*d*e^3 + 4*a*c*d*e*f^2 - (4*c^2*d^2*e + a*
c*e^3)*f - (c^3*d^3*e^3 + a*c^2*d*e^5 - 4*a^3*d*e*f^4 + (4*a^2*c*
d^2*e + a^3*e^3)*f^3 + (4*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2 - (4*c
^3*d^4*e + 5*a*c^2*d^2*e^3 - a^2*c*e^5)*f)*sqrt(c^2*e^2/(c^4*d^4*
e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2
 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c
^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c
^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((c*e^2 -
2*c*d*f + 2*a*f^2 + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c
*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2
/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16
*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3
+ 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d
^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e
^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a
*c*d*e^2)*f)) + 2*(4*a^3*d*f^4 - (8*a^2*c*d^2 + a^3*e^2)*f^3 + 2*
(2*a*c^2*d^3 + 3*a^2*c*d*e^2)*f^2 - (a*c^2*d^2*e^2 + a^2*c*e^4)*f
)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a
^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3
*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*
f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/x) + 1
/4*sqrt(2)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 + (c^2*d^2*e^2 + a*c*e
^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a
*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^
2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^
2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 +
a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*
f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)
*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*c^2*d*e*f*x - 2*a*c
*e^2*f - sqrt(2)*(c^2*d*e^3 + 4*a*c*d*e*f^2 - (4*c^2*d^2*e + a*c*
e^3)*f - (c^3*d^3*e^3 + a*c^2*d*e^5 - 4*a^3*d*e*f^4 + (4*a^2*c*d^
2*e + a^3*e^3)*f^3 + (4*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2 - (4*c^3
*d^4*e + 5*a*c^2*d^2*e^3 - a^2*c*e^5)*f)*sqrt(c^2*e^2/(c^4*d^4*e^
2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 +
 a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3
*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3
*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((c*e^2 - 2*
c*d*f + 2*a*f^2 + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d
^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(
c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a
^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 +
2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5
 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4
 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c
*d*e^2)*f)) + 2*(4*a^3*d*f^4 - (8*a^2*c*d^2 + a^3*e^2)*f^3 + 2*(2
```

```
*a*c^2*d^3 + 3*a^2*c*d*e^2)*f^2 - (a*c^2*d^2*e^2 + a^2*c*e^4)*f)*
sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4
*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c
*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^
2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/x) - 1/4
*sqrt(2)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 - (c^2*d^2*e^2 + a*c*e^4
 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c
*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*
e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*
d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^
3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)
))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f
^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*c^2*d*e*f*x - 2*a*c*e
^2*f + sqrt(2)*(c^2*d*e^3 + 4*a*c*d*e*f^2 - (4*c^2*d^2*e + a*c*e^
3)*f + (c^3*d^3*e^3 + a*c^2*d*e^5 - 4*a^3*d*e*f^4 + (4*a^2*c*d^2*
e + a^3*e^3)*f^3 + (4*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2 - (4*c^3*d
^4*e + 5*a*c^2*d^2*e^3 - a^2*c*e^5)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2
+ 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a
^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d
^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d
^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((c*e^2 - 2*c*
d*f + 2*a*f^2 - (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2
 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^
4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3
*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*
(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 +
 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 -
 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d
*e^2)*f)) - 2*(4*a^3*d*f^4 - (8*a^2*c*d^2 + a^3*e^2)*f^3 + 2*(2*a
*c^2*d^3 + 3*a^2*c*d*e^2)*f^2 - (a*c^2*d^2*e^2 + a^2*c*e^4)*f)*sq
rt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d
*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d
*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2
- 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/x) + 1/4*s
qrt(2)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 - (c^2*d^2*e^2 + a*c*e^4 -
 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d
*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^
6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^
3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*
c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))
/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2
 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*c^2*d*e*f*x - 2*a*c*e^2
*f - sqrt(2)*(c^2*d*e^3 + 4*a*c*d*e*f^2 - (4*c^2*d^2*e + a*c*e^3)
*f + (c^3*d^3*e^3 + a*c^2*d*e^5 - 4*a^3*d*e*f^4 + (4*a^2*c*d^2*e
+ a^3*e^3)*f^3 + (4*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2 - (4*c^3*d^4
*e + 5*a*c^2*d^2*e^3 - a^2*c*e^5)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 +
2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4
*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4
 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3
*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((c*e^2 - 2*c*d*
f + 2*a*f^2 - (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 +
 a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*
d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c
*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8
*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3
*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4
*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e
^2)*f)) - 2*(4*a^3*d*f^4 - (8*a^2*c*d^2 + a^3*e^2)*f^3 + 2*(2*a*c
^2*d^3 + 3*a^2*c*d*e^2)*f^2 - (a*c^2*d^2*e^2 + a^2*c*e^4)*f)*sqrt
(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f
^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e
^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 -
4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}(fx^2 + ex + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)), x)
```

3.68 $\int \frac{1}{x\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=330

$$\frac{f\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[Out] (f*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d
*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^
2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(e - Sqrt[e^2 -
4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sq
rt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^
2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
+ e*Sqrt[e^2 - 4*d*f])]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt
[a]*d)
```

Rubi [A] time = 1.85615, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\frac{f\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```


```
[Out] (f*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d
*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^
2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(e - Sqrt[e^2 -
4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sq
rt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^
2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
+ e*Sqrt[e^2 - 4*d*f])]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt
[a]*d)
```

Rubi in Sympy [A] time = 131.601, size = 325, normalized size = 0.98

$$-\frac{\sqrt{2}f\left(e-\sqrt{-4df+e^2}\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(2af-cx\left(e+\sqrt{-4df+e^2}\right)\right)}{2\sqrt{a+cx^2}\sqrt{2af^2-2cdf+ce^2+ce\sqrt{-4df+e^2}}}\right)}{2d\sqrt{-4df+e^2}\sqrt{2af^2-2cdf+ce^2+ce\sqrt{-4df+e^2}}}+\frac{\sqrt{2}f\left(e+\sqrt{-4df+e^2}\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(2af-cx\left(e-\sqrt{-4df+e^2}\right)\right)}{2\sqrt{a+cx^2}\sqrt{2af^2-2cdf+ce^2-ce\sqrt{-4df+e^2}}}\right)}{2d\sqrt{-4df+e^2}\sqrt{2af^2-2cdf+ce^2-ce\sqrt{-4df+e^2}}}-\frac{\operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(2)*f*(e - sqrt(-4*d*f + e**2))*atanh(sqrt(2)*(2*a*f - c*x*(
e + sqrt(-4*d*f + e**2)))/(2*sqrt(a + c*x**2)*sqrt(2*a*f**2 - 2*c
*d*f + c*e**2 + c*e*sqrt(-4*d*f + e**2))))/(2*d*sqrt(-4*d*f + e**
2)*sqrt(2*a*f**2 - 2*c*d*f + c*e**2 + c*e*sqrt(-4*d*f + e**2))) +
 sqrt(2)*f*(e + sqrt(-4*d*f + e**2))*atanh(sqrt(2)*(2*a*f - c*x*(
e - sqrt(-4*d*f + e**2)))/(2*sqrt(a + c*x**2)*sqrt(2*a*f**2 - 2*c
*d*f + c*e**2 - c*e*sqrt(-4*d*f + e**2))))/(2*d*sqrt(-4*d*f + e**
2)*sqrt(2*a*f**2 - 2*c*d*f + c*e**2 - c*e*sqrt(-4*d*f + e**2))) -
 atanh(sqrt(a + c*x**2)/sqrt(a))/(sqrt(a)*d)
```

Mathematica [A] time = 3.4225, size = 513, normalized size = 1.55

$$\frac{\frac{\sqrt{2}f\left(\sqrt{e^2-4df}+e\right)\log\left(\sqrt{a+cx^2}\sqrt{4af^2-2ce\sqrt{e^2-4df}-4cdf+2ce^2}+2af+cx\left(\sqrt{e^2-4df}-e\right)\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{2}f\left(\sqrt{e^2-4df}-e\right)\log\left(-\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df\right.}\right.}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df\right.}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*Log[x])/Sqrt[a] - (Sqrt[2]*f*(e + Sqrt[e^2 - 4*d*f])*Log[-e +
 Sqrt[e^2 - 4*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*
(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (Sqrt[2]*f*(-e + Sqrt[e^2
 - 4*d*f])*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[e^2 - 4*d*f]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (2*Log[
a + Sqrt[a]*Sqrt[a + c*x^2]])/Sqrt[a] + (Sqrt[2]*f*(e + Sqrt[e^2
- 4*d*f])*Log[2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x + Sqrt[2*c*e^2
 - 4*c*d*f + 4*a*f^2 - 2*c*e*Sqrt[e^2 - 4*d*f]]*Sqrt[a + c*x^2]])
/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 -
4*d*f])]) + (Sqrt[2]*f*(-e + Sqrt[e^2 - 4*d*f])*Log[-2*a*f + c*e*
x + c*Sqrt[e^2 - 4*d*f]*x - Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*S
qrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*
f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]))/(2*d)
```

Maple [B] time = 0.021, size = 681, normalized size = 2.1

$$4\,{\frac {f}{\left(-e+\sqrt {-4\,df+{e}^{2}}\right)\left(e+\sqrt {-4\,df+{e}^{2}}\right)\sqrt {a}}\ln \left({\frac {2\,a+2\,\sqrt {a}\sqrt {c{x}^{2}+a}}{x}} \right) }$$
$$-2\,{\frac {f\sqrt {2}}{\left(-e+\sqrt {-4\,df+{e}^{2}}\right)\sqrt {-4\,df+{e}^{2}}}\ln \left(1\left({\frac {-\sqrt {-4\,df+{e}^{2}}ce+2\,a{f}^{2}-2\,cdf+{e}^{2}c}{{f}^{2}}}-{\frac {c\left(e-\sqrt {-4\,df+{e}^{2}}\right)}{f}}\left(x-1/2\,{\frac {-e+ }{}}\right.\right.\right.}$$
$$-2\,{\frac {f\sqrt {2}}{\left(e+\sqrt {-4\,df+{e}^{2}}\right)\sqrt {-4\,df+{e}^{2}}}\ln \left(1\left({\frac {\sqrt {-4\,df+{e}^{2}}ce+2\,a{f}^{2}-2\,cdf+{e}^{2}c}{{f}^{2}}}-{\frac {c\left(e+\sqrt {-4\,df+{e}^{2}}\right)}{f}}\left(x+1/2\,{\frac {e+\sqrt {-4}}{f}}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a^(1/2)*ln((2*
a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-2*f/(-e+(-4*d*f+e^2)^(1/2))/(-4*d
*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e
^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2
*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))
/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/
f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*
f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f))-2*f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^
(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*
ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-4*d
*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c
*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}(fx^2 + ex + d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x),x, algorithm="giac")
```

```
[Out] Timed out
```

3.69 $\int \frac{1}{x^2\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=367

$$-\frac{f\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}-\frac{\sqrt{a+cx^2}}{adx}$$

```
[Out] -(Sqrt[a + c*x^2]/(a*d*x)) - (f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f
])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*
a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])
/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e
*Sqrt[e^2 - 4*d*f])]) + (f*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*Ar
cTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2
 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqr
t[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])]) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*
d^2)
```

Rubi [A] time = 2.67815, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$

$$-\frac{f\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}-\frac{\sqrt{a+cx^2}}{adx}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(a*d*x)) - (f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f
])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*
a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])
/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e
*Sqrt[e^2 - 4*d*f])]) + (f*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*Ar
cTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2
 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqr
t[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])]) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*
d^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.14071, size = 567, normalized size = 1.54

$$-\frac{\sqrt{2}f\left(e\sqrt{e^2-4df}-2df+e^2\right)\log\left(\sqrt{a+cx^2}\sqrt{4af^2-2ce\sqrt{e^2-4df}-4cdf+2ce^2}+2af+cx\left(\sqrt{e^2-4df}-e\right)\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{2}f\left(e\sqrt{e^2-4df}+2df-e^2\right)\log\left(-\sqrt{a+cx^2}\sqrt{4af^2+2c\cdots}\right.}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{\cdots}\right.}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((-2*d*Sqrt[a + c*x^2])/(a*x) - (2*e*Log[x])/Sqrt[a] + (Sqrt[2]*f
*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*Log[-e + Sqrt[e^2 - 4*d*f] -
 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqr
t[e^2 - 4*d*f])]) + (Sqrt[2]*f*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f
])*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*
a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) + (2*e*Log[a + Sq
rt[a]*Sqrt[a + c*x^2]])/Sqrt[a] - (Sqrt[2]*f*(e^2 - 2*d*f + e*Sqr
t[e^2 - 4*d*f])*Log[2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x + Sqrt[2
*c*e^2 - 4*c*d*f + 4*a*f^2 - 2*c*e*Sqrt[e^2 - 4*d*f]]*Sqrt[a + c*
x^2]])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[
e^2 - 4*d*f])]) - (Sqrt[2]*f*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])
*Log[-2*a*f + c*e*x + c*Sqrt[e^2 - 4*d*f]*x - Sqrt[4*a*f^2 + 2*c*
(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]])/(Sqrt[e^2
- 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]))/
(2*d^2)
```

Maple [B] time = 0.023, size = 736, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x*(c*x^2+a)^
(1/2)-4*f^2/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/
((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e
^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(
-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2
*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))+4*f^2
/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*
```

```
c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(
1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+16*f^2*e/(-e+(-4*d*f+e^2)
^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x
^2+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}(fx^2 + ex + d)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a + cx^2}\,(d + ex + fx^2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.70 $\int \frac{1}{x^3\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=457

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}-\frac{(e^2-df)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}+\frac{f\left(-(e^2-df)\left(e-\sqrt{e^2-4df}\right)-4def+2e^3\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(-(e^2-df)\left(\sqrt{e^2-4df}+e\right)-4def+2e^3\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{e\sqrt{a+cx^2}}{ad^2x}-\frac{\sqrt{a+cx^2}}{2adx^2}$$

```
[Out] -Sqrt[a + c*x^2]/(2*a*d*x^2) + (e*Sqrt[a + c*x^2])/(a*d^2*x) + (f
*(2*e^3 - 4*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[
(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d
^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 -
 4*d*f])]) - (f*(2*e^3 - 4*d*e*f - (e^2 - d*f)*(e + Sqrt[e^2 - 4*
d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqr
t[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2
])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 + e*Sqrt[e^2 - 4*d*f])]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/
(2*a^(3/2)*d) - ((e^2 - d*f)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(S
qrt[a]*d^3)
```

Rubi [A] time = 4.06833, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}-\frac{(e^2-df)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}+\frac{f\left(-(e^2-df)\left(e-\sqrt{e^2-4df}\right)-4def+2e^3\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(-(e^2-df)\left(\sqrt{e^2-4df}+e\right)-4def+2e^3\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{e\sqrt{a+cx^2}}{ad^2x}-\frac{\sqrt{a+cx^2}}{2adx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -Sqrt[a + c*x^2]/(2*a*d*x^2) + (e*Sqrt[a + c*x^2])/(a*d^2*x) + (f
*(2*e^3 - 4*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[
(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(
```

```
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d
^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 -
 4*d*f])]) - (f*(2*e^3 - 4*d*e*f - (e^2 - d*f)*(e + Sqrt[e^2 - 4*
d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqr
t[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2
])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 + e*Sqrt[e^2 - 4*d*f])]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/
(2*a^(3/2)*d) - ((e^2 - d*f)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(S
qrt[a]*d^3)
```

Rubi in Sympy [A] time = 177.497, size = 439, normalized size = 0.96

$$\frac{\sqrt{2}f\left(2e\left(-2df+e^2\right)-\left(e-\sqrt{-4df+e^2}\right)\left(-df+e^2\right)\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(2af-cx\left(e-\sqrt{-4df+e^2}\right)\right)}{2\sqrt{a+cx^2}\sqrt{2af^2-2cdf+ce^2-ce\sqrt{-4df+e^2}}}\right)}{2d^3\sqrt{-4df+e^2}\sqrt{2af^2-2cdf+ce^2-ce\sqrt{-4df+e^2}}} - \frac{\sqrt{2}f\left(2e\left(-2df+e^2\right)-\left(e+\sqrt{-4df+e^2}\right)\left(-df+e^2\right)\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(2af-cx\left(e+\sqrt{-4df+e^2}\right)\right)}{2\sqrt{a+cx^2}\sqrt{2af^2-2cdf+ce^2+ce\sqrt{-4df+e^2}}}\right)}{2d^3\sqrt{-4df+e^2}\sqrt{2af^2-2cdf+ce^2+ce\sqrt{-4df+e^2}}} - \frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} - \frac{\left(-df+e^2\right)\operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3} + \frac{c\operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] sqrt(2)*f*(2*e*(-2*d*f + e**2) - (e - sqrt(-4*d*f + e**2))*(-d*f
+ e**2))*atanh(sqrt(2)*(2*a*f - c*x*(e - sqrt(-4*d*f + e**2)))/(2
*sqrt(a + c*x**2)*sqrt(2*a*f**2 - 2*c*d*f + c*e**2 - c*e*sqrt(-4*
d*f + e**2))))/(2*d**3*sqrt(-4*d*f + e**2)*sqrt(2*a*f**2 - 2*c*d*
f + c*e**2 - c*e*sqrt(-4*d*f + e**2))) - sqrt(2)*f*(2*e*(-2*d*f +
 e**2) - (e + sqrt(-4*d*f + e**2))*(-d*f + e**2))*atanh(sqrt(2)*(
2*a*f - c*x*(e + sqrt(-4*d*f + e**2)))/(2*sqrt(a + c*x**2)*sqrt(2
*a*f**2 - 2*c*d*f + c*e**2 + c*e*sqrt(-4*d*f + e**2))))/(2*d**3*s
qrt(-4*d*f + e**2)*sqrt(2*a*f**2 - 2*c*d*f + c*e**2 + c*e*sqrt(-4
*d*f + e**2))) - sqrt(a + c*x**2)/(2*a*d*x**2) + e*sqrt(a + c*x**
2)/(a*d**2*x) - (-d*f + e**2)*atanh(sqrt(a + c*x**2)/sqrt(a))/(sq
rt(a)*d**3) + c*atanh(sqrt(a + c*x**2)/sqrt(a))/(2*a**(3/2)*d)
```

Mathematica [A] time = 1.89541, size = 678, normalized size = 1.48

$$\frac{\frac{\log\left(\sqrt{a}\sqrt{a+cx^2}+a\right)\left(2adf-2ae^2+cd^2\right)}{a^{3/2}} - \frac{\log(x)\left(2adf-2ae^2+cd^2\right)}{a^{3/2}} + \frac{\sqrt{2}f\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)\log\left(\sqrt{a+cx^2}\sqrt{4af^2-2ce\sqrt{e^2-4df}-4cdf+2\cdots}\right.}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((d*(-d + 2*e*x)*Sqrt[a + c*x^2])/(a*x^2) - ((c*d^2 - 2*a*e^2 + 2
*a*d*f)*Log[x])/a^(3/2) + (Sqrt[2]*f*(-e^3 + 3*d*e*f - e^2*Sqrt[e
^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f])*Log[-e + Sqrt[e^2 - 4*d*f] -
 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqr
t[e^2 - 4*d*f])]) + (Sqrt[2]*f*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*
d*f] + d*f*Sqrt[e^2 - 4*d*f])*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])
/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 -
4*d*f])]) + ((c*d^2 - 2*a*e^2 + 2*a*d*f)*Log[a + Sqrt[a]*Sqrt[a +
```

```
 c*x^2]])/a^(3/2) + (Sqrt[2]*f*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*
d*f] - d*f*Sqrt[e^2 - 4*d*f])*Log[2*a*f + c*(-e + Sqrt[e^2 - 4*d*
f])*x + Sqrt[2*c*e^2 - 4*c*d*f + 4*a*f^2 - 2*c*e*Sqrt[e^2 - 4*d*f
]]*Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2
*d*f - e*Sqrt[e^2 - 4*d*f])]) - (Sqrt[2]*f*(e^3 - 3*d*e*f - e^2*S
qrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f])*Log[-2*a*f + c*e*x + c*
Sqrt[e^2 - 4*d*f]*x - Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^
2 - 4*d*f])]*Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 +
c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]))/(2*d^3)
```

Maple [B] time = 0.025, size = 911, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x^2*(c*x^2+a
)^(1/2)-2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c/a^(3
/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-8*f^3/(-e+(-4*d*f+e^2)^
(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-2*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d
*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c
-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2
*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f))-8*f^3/(e+(-4*d*f+e^2)^(1/2))^3/(-4
*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2
*c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^
2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e
^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f))+16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/
2))^2/a/x*(c*x^2+a)^(1/2)-64*f^4/(-e+(-4*d*f+e^2)^(1/2))^3/(e+(-4
*d*f+e^2)^(1/2))^3/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)*
d+64*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3/a^(1/
2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)*e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}(fx^2 + ex + d)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(fx^2+ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^3),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^3), x)
```

3.71 $\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=499

$$\frac{cex\left(a\left(e^2-2df\right)+cd^2\right)+af\left(a\left(e^2-df\right)+cd^2\right)}{af^2\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}-\frac{\left(2adef-\left(e-\sqrt{e^2-4df}\right)\left(a\left(e^2-df\right)+cd^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2adef-\left(\sqrt{e^2-4df}+e\right)\left(a\left(e^2-df\right)+cd^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{ex}{af^2\sqrt{a+cx^2}}-\frac{1}{cf\sqrt{a+cx^2}}$$

```
[Out] -(1/(c*f*Sqrt[a + c*x^2])) - (e*x)/(a*f^2*Sqrt[a + c*x^2]) + (a*f
*(c*d^2 + a*(e^2 - d*f)) + c*e*(c*d^2 + a*(e^2 - 2*d*f))*x)/(a*f^
2*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) - ((2*a*d*e*f - (e -
 Sqrt[e^2 - 4*d*f])*(c*d^2 + a*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(
e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
- e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*
d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*
Sqrt[e^2 - 4*d*f])]) + ((2*a*d*e*f - (e + Sqrt[e^2 - 4*d*f])*(c*d
^2 + a*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x
)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*
Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a
*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 4.6387, antiderivative size = 499, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\frac{cex\left(a\left(e^2-2df\right)+cd^2\right)+af\left(a\left(e^2-df\right)+cd^2\right)}{af^2\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}-\frac{\left(2adef-\left(e-\sqrt{e^2-4df}\right)\left(a\left(e^2-df\right)+cd^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2adef-\left(\sqrt{e^2-4df}+e\right)\left(a\left(e^2-df\right)+cd^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{ex}{af^2\sqrt{a+cx^2}}-\frac{1}{cf\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -(1/(c*f*Sqrt[a + c*x^2])) - (e*x)/(a*f^2*Sqrt[a + c*x^2]) + (a*f
*(c*d^2 + a*(e^2 - d*f)) + c*e*(c*d^2 + a*(e^2 - 2*d*f))*x)/(a*f^
2*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) - ((2*a*d*e*f - (e -
 Sqrt[e^2 - 4*d*f])*(c*d^2 + a*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(
```

```
e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
- e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*
d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*
Sqrt[e^2 - 4*d*f])]) + ((2*a*d*e*f - (e + Sqrt[e^2 - 4*d*f])*(c*d
^2 + a*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x
)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*
Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a
*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.82614, size = 727, normalized size = 1.46

$$-\frac{\frac{\sqrt{2}\left(a\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}+3def-e^3\right)+cd^2\left(\sqrt{e^2-4df}-e\right)\right)\log\left(\sqrt{a+cx^2}\sqrt{4af^2-2ce\sqrt{e^2-4df}-4cdf+2ce^2}+2af+cx\left(\sqrt{e^2-4df}-e\right)\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{2}\left(a\left(e^2\sqrt{e^2-\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] ((-2*a*(-(c*d) + a*f + c*e*x))/(c*Sqrt[a + c*x^2]) + (Sqrt[2]*(c*
d^2*(-e + Sqrt[e^2 - 4*d*f]) + a*(-e^3 + 3*d*e*f + e^2*Sqrt[e^2 -
 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[-e + Sqrt[e^2 - 4*d*f] - 2*
f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f])]) + (Sqrt[2]*(c*d^2*(e + Sqrt[e^2 - 4*d*f]) + a*(e^3
 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[
e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 +
 c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (Sqrt[2]*(c*d^2*(-e +
Sqrt[e^2 - 4*d*f]) + a*(-e^3 + 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] -
d*f*Sqrt[e^2 - 4*d*f]))*Log[2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x
+ Sqrt[2*c*e^2 - 4*c*d*f + 4*a*f^2 - 2*c*e*Sqrt[e^2 - 4*d*f]]*Sqr
t[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f -
 e*Sqrt[e^2 - 4*d*f])]) - (Sqrt[2]*(c*d^2*(e + Sqrt[e^2 - 4*d*f])
 + a*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*
f]))*Log[-2*a*f + c*e*x + c*Sqrt[e^2 - 4*d*f]*x - Sqrt[4*a*f^2 +
2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]])/(Sqrt[
e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
]))/(2*(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)))
```

Maple [B] time = 0.039, size = 6124, normalized size = 12.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a+cx^2)^{\frac{3}{2}}\,(d+ex+fx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**3/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.72 $\int \frac{x^2}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=410

$$-\frac{f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(2d(cd-af)+ae\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{x(cd-af)+ae}{\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

```
[Out] -((a*e + (c*d - a*f)*x)/((a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2
])) - (f*(2*d*(c*d - a*f) + a*e*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[
(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*S
qrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2
- 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(2*d*(c*d - a*f) + a*e*(e +
 Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x
)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*
Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a
*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 1.7067, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148

$$-\frac{f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(2d(cd-af)+ae\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{x(cd-af)+ae}{\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```


```
[Out] -((a*e + (c*d - a*f)*x)/((a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2
])) - (f*(2*d*(c*d - a*f) + a*e*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[
(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*S
qrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2
- 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(2*d*(c*d - a*f) + a*e*(e +
 Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x
)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*
Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a
*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi in Sympy [A] time = 154.803, size = 398, normalized size = 0.97

$$\frac{\sqrt{2}f\left(-ae\left(e-\sqrt{-4df+e^2}\right)+2d\left(af-cd\right)\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(2af-cx\left(e-\sqrt{-4df+e^2}\right)\right)}{2\sqrt{a+cx^2}\sqrt{2af^2-2cdf+ce^2-ce\sqrt{-4df+e^2}}}\right)}{2\sqrt{-4df+e^2}\left(ace^2+\left(af-cd\right)^2\right)\sqrt{2af^2-2cdf+ce^2-ce\sqrt{-4df+e^2}}} - \frac{\sqrt{2}f\left(-ae\left(e+\sqrt{-4df+e^2}\right)+2d\left(af-cd\right)\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(2af-cx\left(e+\sqrt{-4df+e^2}\right)\right)}{2\sqrt{a+cx^2}\sqrt{2af^2-2cdf+ce^2+ce\sqrt{-4df+e^2}}}\right)}{2\sqrt{-4df+e^2}\left(ace^2+\left(af-cd\right)^2\right)\sqrt{2af^2-2cdf+ce^2+ce\sqrt{-4df+e^2}}} - \frac{2ae-2x\left(af-cd\right)}{2\sqrt{a+cx^2}\left(ace^2+\left(af-cd\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] sqrt(2)*f*(-a*e*(e - sqrt(-4*d*f + e**2)) + 2*d*(a*f - c*d))*atan
h(sqrt(2)*(2*a*f - c*x*(e - sqrt(-4*d*f + e**2)))/(2*sqrt(a + c*x
**2)*sqrt(2*a*f**2 - 2*c*d*f + c*e**2 - c*e*sqrt(-4*d*f + e**2)))
)/(2*sqrt(-4*d*f + e**2)*(a*c*e**2 + (a*f - c*d)**2)*sqrt(2*a*f**
2 - 2*c*d*f + c*e**2 - c*e*sqrt(-4*d*f + e**2))) - sqrt(2)*f*(-a*
e*(e + sqrt(-4*d*f + e**2)) + 2*d*(a*f - c*d))*atanh(sqrt(2)*(2*a
*f - c*x*(e + sqrt(-4*d*f + e**2)))/(2*sqrt(a + c*x**2)*sqrt(2*a*
f**2 - 2*c*d*f + c*e**2 + c*e*sqrt(-4*d*f + e**2))))/(2*sqrt(-4*d
*f + e**2)*(a*c*e**2 + (a*f - c*d)**2)*sqrt(2*a*f**2 - 2*c*d*f +
c*e**2 + c*e*sqrt(-4*d*f + e**2))) - (2*a*e - 2*x*(a*f - c*d))/(2
*sqrt(a + c*x**2)*(a*c*e**2 + (a*f - c*d)**2))
```

Mathematica [A] time = 1.42726, size = 596, normalized size = 1.45

$$\frac{\sqrt{2}f\left(a\left(e\sqrt{e^2-4df}+2df-e^2\right)-2cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{4af^2-2ce\sqrt{e^2-4df}-4cdf+2ce^2}+2af+cx\left(\sqrt{e^2-4df}-e\right)\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{\sqrt{2}f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\log\left(-\sqrt{\ldots}\right.}{\sqrt{e^2-4df}\ldots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] ((-2*(c*d*x + a*(e - f*x)))/Sqrt[a + c*x^2] - (Sqrt[2]*f*(-2*c*d^
2 + a*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Log[-e + Sqrt[e^2 - 4
*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
- e*Sqrt[e^2 - 4*d*f])]) - (Sqrt[2]*f*(2*c*d^2 + a*(e^2 - 2*d*f +
 e*Sqrt[e^2 - 4*d*f]))*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[
e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
]) + (Sqrt[2]*f*(-2*c*d^2 + a*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]
))*Log[2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x + Sqrt[2*c*e^2 - 4*c*
d*f + 4*a*f^2 - 2*c*e*Sqrt[e^2 - 4*d*f]]*Sqrt[a + c*x^2]])/(Sqrt[
e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])
]) + (Sqrt[2]*f*(2*c*d^2 + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))
*Log[-2*a*f + c*e*x + c*Sqrt[e^2 - 4*d*f]*x - Sqrt[4*a*f^2 + 2*c*
(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]])/(Sqrt[e^2
- 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]))/
(2*(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)))
```

Maple [B] time = 0.023, size = 4752, normalized size = 11.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] 1/f*x/a/(c*x^2+a)^(1/2)-1/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+e^2*c)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)
^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*e-2*f/(-4*d*f+e^2)^(1/2
)/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/((x-1/2*(-e+(-4
*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4
*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
e^2*c)/f^2)^(1/2)*d+1/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+e^2*c)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*
(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*e^2+2/f*
(-4*d*f+e^2)^(1/2)*c^2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e
^2*c)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+e^2*c)/f^2)^(1/2)*x*e+4*c^2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^
2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^
(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*x*d-4/f*c^2/(-(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-
1/f^2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*
(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*x*e^2-4/
(-4*d*f+e^2)^(1/2)*c^2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e
^2*c)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+e^2*c)/f^2)^(1/2)*x*e*d+2/f/(-4*d*f+e^2)^(1/2)*c^2/(-(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f*d+c^2/f
^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*x
*e^3+1/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)*2^(1/2)/((
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2
)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(
-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e+2*f/(
-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)
*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1
/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e-
(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/
2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4
*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))
/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/
f))*d-1/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+e^2*c)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c
)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/
f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+
1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)
^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^
2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f))*e^2-1/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)
/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f
*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*e+2*f/(-4*d*f+e^2)^(1/2)/((-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/((x+1/2*(e+(-4*d*f+e^2)^(1/
2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/
2)*d-1/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+e^2*c)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(
1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*
```

```
c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*e^2-2/f*(-4*d*f+e^2)^(1/2)*
c^2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f
*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/
2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1
/2)*x*e+4*c^2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(4*a
*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*
c)/f^2)^(1/2)*x*d-4/f*c^2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+e^2*c)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+e^2*c)/f^2)^(1/2)*x*e^2+4/(-4*d*f+e^2)^(1/2)*c^2/((-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f*d+c^2/f^2*e
^2-1/f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-
c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*x*e*d-2/
f/(-4*d*f+e^2)^(1/2)*c^2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
e^2*c)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+e^2*c)/f^2)^(1/2)*x*e^3+1/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+e^2*c)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e
^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*
c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2
)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^
2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f))*e-2*f/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+e^2*c)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^
2*c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))
/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f
^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+
e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f))*d+1/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+e^2*c)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^
2*c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))
/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f
^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+
e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f))*e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a+cx^2)^{\frac{3}{2}}\,(d+ex+fx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**2/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.73 $\int \frac{x}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx$

Optimal. Leaf size=411

$$\frac{f\left(2cde-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(2cde-\left(\sqrt{e^2-4df}+e\right)(cd-af)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{-af+cd-cex}{\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

```
[Out] -((c*d - a*f - c*e*x)/((a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2])
) + (f*(2*c*d*e - (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2
*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^
2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqr
t[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 -
2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*c*d*e - (c*d - a*f)*(e + S
qrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/
(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sq
rt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f
)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 1.96015, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{f\left(2cde-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(2cde-\left(\sqrt{e^2-4df}+e\right)(cd-af)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{-af+cd-cex}{\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```


```
[Out] -((c*d - a*f - c*e*x)/((a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2])
) + (f*(2*c*d*e - (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2
*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^
2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqr
t[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 -
2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*c*d*e - (c*d - a*f)*(e + S
qrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/
(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sq
rt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f
)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi in Sympy [A] time = 158.234, size = 400, normalized size = 0.97

$$\frac{\sqrt{2}f\left(2cde+\left(e-\sqrt{-4df+e^2}\right)(af-cd)\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(2af-cx\left(e-\sqrt{-4df+e^2}\right)\right)}{2\sqrt{a+cx^2}\sqrt{2af^2-2cdf+ce^2-ce\sqrt{-4df+e^2}}}\right)}{2\sqrt{-4df+e^2}\left(ace^2+(af-cd)^2\right)\sqrt{2af^2-2cdf+ce^2-ce\sqrt{-4df+e^2}}} - \frac{\sqrt{2}f\left(2cde+\left(e+\sqrt{-4df+e^2}\right)(af-cd)\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(2af-cx\left(e+\sqrt{-4df+e^2}\right)\right)}{2\sqrt{a+cx^2}\sqrt{2af^2-2cdf+ce^2+ce\sqrt{-4df+e^2}}}\right)}{2\sqrt{-4df+e^2}\left(ace^2+(af-cd)^2\right)\sqrt{2af^2-2cdf+ce^2+ce\sqrt{-4df+e^2}}} + \frac{2af-2cd+2cex}{2\sqrt{a+cx^2}\left(ace^2+(af-cd)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] sqrt(2)*f*(2*c*d*e + (e - sqrt(-4*d*f + e**2))*(a*f - c*d))*atanh
(sqrt(2)*(2*a*f - c*x*(e - sqrt(-4*d*f + e**2)))/(2*sqrt(a + c*x*
*2)*sqrt(2*a*f**2 - 2*c*d*f + c*e**2 - c*e*sqrt(-4*d*f + e**2))))
/(2*sqrt(-4*d*f + e**2)*(a*c*e**2 + (a*f - c*d)**2)*sqrt(2*a*f**2
 - 2*c*d*f + c*e**2 - c*e*sqrt(-4*d*f + e**2))) - sqrt(2)*f*(2*c*
d*e + (e + sqrt(-4*d*f + e**2))*(a*f - c*d))*atanh(sqrt(2)*(2*a*f
 - c*x*(e + sqrt(-4*d*f + e**2)))/(2*sqrt(a + c*x**2)*sqrt(2*a*f*
*2 - 2*c*d*f + c*e**2 + c*e*sqrt(-4*d*f + e**2))))/(2*sqrt(-4*d*f
 + e**2)*(a*c*e**2 + (a*f - c*d)**2)*sqrt(2*a*f**2 - 2*c*d*f + c*
e**2 + c*e*sqrt(-4*d*f + e**2))) + (2*a*f - 2*c*d + 2*c*e*x)/(2*s
qrt(a + c*x**2)*(a*c*e**2 + (a*f - c*d)**2))
```

Mathematica [A] time = 1.75164, size = 617, normalized size = 1.5

$$-\frac{\sqrt{2}f\left(af\left(\sqrt{e^2-4df}-e\right)-cd\left(\sqrt{e^2-4df}+e\right)\right)\log\left(\sqrt{a+cx^2}\sqrt{4af^2-2ce\sqrt{e^2-4df}-4cdf+2ce^2}+2af+cx\left(\sqrt{e^2-4df}-e\right)\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} - \frac{\sqrt{2}f\left(af\left(\sqrt{e^2-4df}+e\right)+cd\left(e-\sqrt{e^2-\cdots}\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*(-(c*d) + a*f + c*e*x))/Sqrt[a + c*x^2] + (Sqrt[2]*f*(a*f*(-e
 + Sqrt[e^2 - 4*d*f]) - c*d*(e + Sqrt[e^2 - 4*d*f]))*Log[-e + Sqr
t[e^2 - 4*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2
 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*f*(c*d*(e - Sqrt[e^2
 - 4*d*f]) + a*f*(e + Sqrt[e^2 - 4*d*f]))*Log[e + Sqrt[e^2 - 4*d*
f] + 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e
*Sqrt[e^2 - 4*d*f])]) - (Sqrt[2]*f*(a*f*(-e + Sqrt[e^2 - 4*d*f])
- c*d*(e + Sqrt[e^2 - 4*d*f]))*Log[2*a*f + c*(-e + Sqrt[e^2 - 4*d
*f])*x + Sqrt[2*c*e^2 - 4*c*d*f + 4*a*f^2 - 2*c*e*Sqrt[e^2 - 4*d*
f]]*Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 -
2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (Sqrt[2]*f*(c*d*(e - Sqrt[e^2 -
4*d*f]) + a*f*(e + Sqrt[e^2 - 4*d*f]))*Log[-2*a*f + c*e*x + c*Sqr
t[e^2 - 4*d*f]*x - Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 -
 4*d*f])]*Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]))/(2*(c^2*d^2 + a^2*f^2 + a*c
*(e^2 - 2*d*f)))
```

Maple [B] time = 0.02, size = 3000, normalized size = 7.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] -1/(-4*d*f+e^2)^(1/2)*f*e/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+e^2*c)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)
^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)+4*e*c^2/(-(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f
^2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-
(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*x-2/(-4*d*f
+e^2)^(1/2)*c^2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(
4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+(
-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(
-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+e^2*c)/f^2)^(1/2)*x*e^2+1/(-4*d*f+e^2)^(1/2)*f*e/(-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/
f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)
)/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))+f/(-(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-2*c*d*f+e^2*c)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e
-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)-2*(-4*d*f+
e^2)^(1/2)*c^2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(4
*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+(-
4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-
4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+e^2*c)/f^2)^(1/2)*x-f/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e
^2*c)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^
2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-
c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*
2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/
2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(
1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1
/2))/f))+1/(-4*d*f+e^2)^(1/2)*f*e/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+e^2*c)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*
f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)+4*e*c^2/((-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-
1/f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(
e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*x+2/(-4*d*f
+e^2)^(1/2)*c^2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(4
*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*
d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^
2*c)/f^2)^(1/2)*x*e^2-1/(-4*d*f+e^2)^(1/2)*f*e/((-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+e^2*c)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+e^2*c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c
*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f))+f/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+e^2*c)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(
1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)+2*(-4*d*f+e^2)^(1/2)*c^2/((
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f*d+c^2
/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
```

```
)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*x-
f/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)*2^(1/2)/(((-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))
/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^
2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.74 $\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=416

$$-\frac{f\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{c(x(cd-af)+ae)}{a\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

```
[Out] (c*(a*e + (c*d - a*f)*x))/(a*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c
*x^2]) - (f*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Arc
Tanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2
+ c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt
[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*
(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(2*a*f^2 + c*(e^2 - 2*
d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*
d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*
d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (
c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]
)])
```

Rubi [A] time = 1.51217, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{f\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{c(x(cd-af)+ae)}{a\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```


```
[Out] (c*(a*e + (c*d - a*f)*x))/(a*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c
*x^2]) - (f*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Arc
Tanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2
+ c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt
[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*
(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(2*a*f^2 + c*(e^2 - 2*
d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*
d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*
d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (
c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]
```

```
)])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.40287, size = 598, normalized size = 1.44

$$-\frac{\sqrt{2}f\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\log\left(\sqrt{a+cx^2}\sqrt{4af^2-2ce\sqrt{e^2-4df}-4cdf+2ce^2}+2af+cx\left(\sqrt{e^2-4df}-e\right)\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{2}f\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\log\left(\right.}{\sqrt{e^2-4a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*c*(c*d*x + a*(e - f*x)))/(a*Sqrt[a + c*x^2]) + (Sqrt[2]*f*(2*
a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Log[-e + Sqrt[e^2
- 4*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d
*f - e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*f*(-2*a*f^2 + c*(-e^2 + 2*
d*f + e*Sqrt[e^2 - 4*d*f]))*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(
Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*
d*f])]) - (Sqrt[2]*f*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f]))*Log[2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x + Sqrt[2*c*e^2 - 4
*c*d*f + 4*a*f^2 - 2*c*e*Sqrt[e^2 - 4*d*f]]*Sqrt[a + c*x^2]])/(Sq
rt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*
f])]) + (Sqrt[2]*f*(2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f
]))*Log[-2*a*f + c*e*x + c*Sqrt[e^2 - 4*d*f]*x - Sqrt[4*a*f^2 + 2
*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]])/(Sqrt[e
^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]
))/(2*(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)))
```

Maple [B] time = 0.02, size = 1713, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] 2/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2
*c)*f^2/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^
(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)-4*c^2*f/(-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^
2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(
-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*x+4/(-4*d*f+
e^2)^(1/2)*c^2*f/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/
(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+
```

```
(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+e^2*c)/f^2)^(1/2)*x*e-2/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+e^2*c)*f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f
)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))
/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))
/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-2/(-4*d*f+e^2)^(1/2)/((-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)*f^2/((x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2
)^(1/2)-4*c^2*f/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(4
*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*
d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^
2*c)/f^2)^(1/2)*x-4/(-4*d*f+e^2)^(1/2)*c^2*f/((-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4
*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f
+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*x*e+2/(-4*d*f+e^2)^(
1/2)/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)*f^2*2^(1/2)/(
((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e+(-4*d*f+e^2)
^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^
2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.75 $\int \frac{1}{x(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=526

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}+\frac{f\left(2e\left(af^2+c\left(e^2-2df\right)\right)-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(2e\left(af^2+c\left(e^2-2df\right)\right)-\left(\sqrt{e^2-4df}+e\right)\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{1}{ad\sqrt{a+cx^2}}$$

```
[Out] 1/(a*d*Sqrt[a + c*x^2]) - (a*(a*f^2 + c*(e^2 - d*f)) + c^2*d*e*x)
/(a*d*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) + (f*(2*e*(a*f^2
 + c*(e^2 - 2*d*f)) - (e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d
*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]
)])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2
*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*e*(a*f^2
 + c*(e^2 - 2*d*f)) - (e + Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d
*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]
)])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2
*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - ArcTanh[Sqrt[a
 + c*x^2]/Sqrt[a]]/(a^(3/2)*d)
```

Rubi [A] time = 4.82784, antiderivative size = 526, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}+\frac{f\left(2e\left(af^2+c\left(e^2-2df\right)\right)-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(2e\left(af^2+c\left(e^2-2df\right)\right)-\left(\sqrt{e^2-4df}+e\right)\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{1}{ad\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] 1/(a*d*Sqrt[a + c*x^2]) - (a*(a*f^2 + c*(e^2 - d*f)) + c^2*d*e*x)
/(a*d*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) + (f*(2*e*(a*f^2
```

```
 + c*(e^2 - 2*d*f)) - (e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d
*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]
)])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2
*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*e*(a*f^2
 + c*(e^2 - 2*d*f)) - (e + Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d
*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]
)])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2
*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - ArcTanh[Sqrt[a
 + c*x^2]/Sqrt[a]]/(a^(3/2)*d)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.17868, size = 889, normalized size = 1.69

$$\frac{c(c(d-ex)-af)}{a\left(c^2d^2+a^2f^2+ac\left(e^2-2df\right)\right)\sqrt{cx^2+a}}+\frac{\log(x)}{a^{3/2}d}-\frac{f\left(a\left(e+\sqrt{e^2-4df}\right)f^2+c\left(e^3+\sqrt{e^2-4df}e^2-3dfe-df\sqrt{e^2-4df}\right)\right)\log\left(-e-2fx+\sqrt{e^2-4df}\right)}{\sqrt{2}d\sqrt{e^2-4df}\left(c^2d^2+a^2f^2+ac\left(e^2-2df\right)\right)\sqrt{2af^2+c\left(e^2-\sqrt{e^2-4df}e-2df\right)}}+\frac{f\left(a\left(e-\sqrt{e^2-4df}\right)f^2+c\left(e^3-\sqrt{e^2-4df}e^2-3dfe+df\sqrt{e^2-4df}\right)\right)\log\left(e+2fx+\sqrt{e^2-4df}\right)}{\sqrt{2}d\sqrt{e^2-4df}\left(c^2d^2+a^2f^2+ac\left(e^2-2df\right)\right)\sqrt{2af^2+c\left(e^2+\sqrt{e^2-4df}e-2df\right)}}-\frac{\log\left(a+\sqrt{cx^2+a}\sqrt{a}\right)}{a^{3/2}d}+\frac{f\left(a\left(e+\sqrt{e^2-4df}\right)f^2+c\left(e^3+\sqrt{e^2-4df}e^2-3dfe-df\sqrt{e^2-4df}\right)\right)\log\left(2af+c\left(\sqrt{e^2-4df}-e\right)x+\sqrt{2ce^2-2c}\right.}{\sqrt{2}d\sqrt{e^2-4df}\left(c^2d^2+a^2f^2+ac\left(e^2-2df\right)\right)\sqrt{2af^2+c\left(e^2-\sqrt{e^2-4df}e-2df\right)}}+\frac{f\left(a\left(\sqrt{e^2-4df}-e\right)f^2+c\left(-e^3+\sqrt{e^2-4df}e^2+3dfe-df\sqrt{e^2-4df}\right)\right)\log\left(-2af+cex+c\sqrt{e^2-4df}x-\sqrt{4af^2+}\right.}{\sqrt{2}d\sqrt{e^2-4df}\left(c^2d^2+a^2f^2+ac\left(e^2-2df\right)\right)\sqrt{2af^2+c\left(e^2+\sqrt{e^2-4df}e-2df\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (c*(-(a*f) + c*(d - e*x)))/(a*(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d
*f))*Sqrt[a + c*x^2]) + Log[x]/(a^(3/2)*d) - (f*(a*f^2*(e + Sqrt[
e^2 - 4*d*f]) + c*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sq
rt[e^2 - 4*d*f]))*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x])/(Sqrt[2]*d
*Sqrt[e^2 - 4*d*f]*(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))*Sqrt[2
*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(a*f^2*(e -
 Sqrt[e^2 - 4*d*f]) + c*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] +
d*f*Sqrt[e^2 - 4*d*f]))*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt
[2]*d*Sqrt[e^2 - 4*d*f]*(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))*S
```

```
qrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - Log[a + S
qrt[a]*Sqrt[a + c*x^2]]/(a^(3/2)*d) + (f*(a*f^2*(e + Sqrt[e^2 - 4
*d*f]) + c*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2
- 4*d*f]))*Log[2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x + Sqrt[2*c*e^
2 - 4*c*d*f + 4*a*f^2 - 2*c*e*Sqrt[e^2 - 4*d*f]]*Sqrt[a + c*x^2]]
)/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*
d*f))*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f
*(a*f^2*(-e + Sqrt[e^2 - 4*d*f]) + c*(-e^3 + 3*d*e*f + e^2*Sqrt[e
^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[-2*a*f + c*e*x + c*Sqrt
[e^2 - 4*d*f]*x - Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 -
4*d*f])]*Sqrt[a + c*x^2]])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*(c^2*d^2
+ a^2*f^2 + a*c*(e^2 - 2*d*f))*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*
Sqrt[e^2 - 4*d*f])])
```

Maple [B] time = 0.023, size = 1945, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] -4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/(c*x^2+a)^(
1/2)+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a^(3/2)*l
n((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)+4*f^3/(-e+(-4*d*f+e^2)^(1/2)
)/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2
*c)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2
))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)-8*f^2/(-e+(-4*d*f+e^2)^(1/2)
)*c^2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(4*a*c-4*c^
2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f
^2)^(1/2)*x+8*f^2/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*c^2/
(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f*d+
c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2
))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2
))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1
/2)*x*e-4*f^3/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(-(-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)*2^(1/2)/((-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(
1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)
^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))+4*f^3/(e+(-4*d*f+e^2)^
(1/2))/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+e^2*c)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(
1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)+8*f^2/(e+(-4*d*f+e^2)^(1/2)
)*c^2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(4*a*c-4*c^2
/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^
(1/2)*x+8*f^2/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*c^2/((-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f*d+c^2/f^
2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2
*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/
2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*x*e-4
*f^3/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+e^2*c)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+e^2*c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c
*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4
```

```
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2+a)^{\frac{3}{2}}(fx^2+ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/(x*(a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.76 $\int \frac{1}{x^2(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=618

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{cdx\left(af^2+c\left(e^2-df\right)\right)+ae\left(af^2+c\left(e^2-2df\right)\right)}{ad^2\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

$$+\frac{f\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-2df\right)\right)-2\left(af^2\left(e^2-df\right)+c\left(d^2f^2-3de^2f+e^4\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-\right.}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e\right.}}\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$-\frac{f\left(e\left(\sqrt{e^2-4df}+e\right)\left(af^2+c\left(e^2-2df\right)\right)-2\left(af^2\left(e^2-df\right)+c\left(d^2f^2-3de^2f+e^4\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4d\right.}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2\right.}}\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$-\frac{e}{ad^2\sqrt{a+cx^2}}-\frac{1}{adx\sqrt{a+cx^2}}$$

```
[Out] -(e/(a*d^2*Sqrt[a + c*x^2])) - 1/(a*d*x*Sqrt[a + c*x^2]) - (2*c*x
)/(a^2*d*Sqrt[a + c*x^2]) + (a*e*(a*f^2 + c*(e^2 - 2*d*f)) + c*d*
(a*f^2 + c*(e^2 - d*f))*x)/(a*d^2*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[
a + c*x^2]) + (f*(e*(e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - 2*d
*f)) - 2*(a*f^2*(e^2 - d*f) + c*(e^4 - 3*d*e^2*f + d^2*f^2)))*Arc
Tanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2
+ c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt
[2]*d^2*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2
+ c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(e*(e + Sqrt[e^2 -
 4*d*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*(a*f^2*(e^2 - d*f) + c*(e^
4 - 3*d*e^2*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d
*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*(a*c*e^2
+ (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f])]) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^2)
```

Rubi [A] time = 5.44189, antiderivative size = 618, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{cdx\left(af^2+c\left(e^2-df\right)\right)+ae\left(af^2+c\left(e^2-2df\right)\right)}{ad^2\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

$$+\frac{f\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-2df\right)\right)-2\left(af^2\left(e^2-df\right)+c\left(d^2f^2-3de^2f+e^4\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-\right.}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e\right.}}\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$-\frac{f\left(e\left(\sqrt{e^2-4df}+e\right)\left(af^2+c\left(e^2-2df\right)\right)-2\left(af^2\left(e^2-df\right)+c\left(d^2f^2-3de^2f+e^4\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4d\right.}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2\right.}}\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$-\frac{e}{ad^2\sqrt{a+cx^2}}-\frac{1}{adx\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -(e/(a*d^2*Sqrt[a + c*x^2])) - 1/(a*d*x*Sqrt[a + c*x^2]) - (2*c*x
)/(a^2*d*Sqrt[a + c*x^2]) + (a*e*(a*f^2 + c*(e^2 - 2*d*f)) + c*d*
(a*f^2 + c*(e^2 - d*f))*x)/(a*d^2*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[
a + c*x^2]) + (f*(e*(e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - 2*d
*f)) - 2*(a*f^2*(e^2 - d*f) + c*(e^4 - 3*d*e^2*f + d^2*f^2)))*Arc
Tanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2
+ c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt
[2]*d^2*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2
+ c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(e*(e + Sqrt[e^2 -
 4*d*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*(a*f^2*(e^2 - d*f) + c*(e^
4 - 3*d*e^2*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d
*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*(a*c*e^2
+ (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f])]) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 3.15975, size = 996, normalized size = 1.61

$$-\frac{\sqrt{cx^2+a}\left(\frac{(cdx+a(e-fx))c^2}{(c^2d^2+a^2f^2+ac(e^2-2df))(cx^2+a)}+\frac{1}{dx}\right)}{a^2}-\frac{e\log(x)}{a^{3/2}d^2}$$
$$-\frac{f\left(af^2\left(-e^2-\sqrt{e^2-4df}e+2df\right)-c\left(e^4+\sqrt{e^2-4df}e^3-4dfe^2-2df\sqrt{e^2-4df}e+2d^2f^2\right)\right)\log\left(-e-2fx+\sqrt{e^2-\cdots}\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\left(c^2d^2+a^2f^2+ac\left(e^2-2df\right)\right)\sqrt{2af^2+c\left(e^2-\sqrt{e^2-4df}e-2df\right)}}$$
$$+\frac{f\left(a\left(-e^2+\sqrt{e^2-4df}e+2df\right)f^2+c\left(-e^4+\sqrt{e^2-4df}e^3+4dfe^2-2df\sqrt{e^2-4df}e-2d^2f^2\right)\right)\log\left(e+2fx+\sqrt{e^2-\cdots}\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\left(c^2d^2+a^2f^2+ac\left(e^2-2df\right)\right)\sqrt{2af^2+c\left(e^2+\sqrt{e^2-4df}e-2df\right)}}$$
$$+\frac{e\log\left(a+\sqrt{cx^2+a}\sqrt{a}\right)}{a^{3/2}d^2}$$
$$+\frac{f\left(af^2\left(-e^2-\sqrt{e^2-4df}e+2df\right)-c\left(e^4+\sqrt{e^2-4df}e^3-4dfe^2-2df\sqrt{e^2-4df}e+2d^2f^2\right)\right)\log\left(2af+c\left(\sqrt{e^2-4d\cdots}\right.\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\left(c^2d^2+a^2f^2+ac\left(e^2-2df\right)\right)\sqrt{2af^2+c\left(e^2-\sqrt{e^2-4d\cdots}\right.}}$$
$$+\frac{f\left(a\left(e^2-\sqrt{e^2-4df}e-2df\right)f^2+c\left(e^4-\sqrt{e^2-4df}e^3-4dfe^2+2df\sqrt{e^2-4df}e+2d^2f^2\right)\right)\log\left(-2af+cex+c\sqrt{e^2\cdots}\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\left(c^2d^2+a^2f^2+ac\left(e^2-2df\right)\right)\sqrt{2af^2+c\left(e^2+\sqrt{e^2-4d\cdots}\right.}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[a + c*x^2]*(1/(d*x) + (c^2*(c*d*x + a*(e - f*x)))/((c^2*d
^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))*(a + c*x^2))))/a^2) - (e*Log[x]
)/(a^(3/2)*d^2) - (f*(a*f^2*(-e^2 + 2*d*f - e*Sqrt[e^2 - 4*d*f])
- c*(e^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d*f] - 2*d*e*
f*Sqrt[e^2 - 4*d*f]))*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x])/(Sqrt[
```

```
2]*d^2*Sqrt[e^2 - 4*d*f]*(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))*
Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(a*f^
2*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]) + c*(-e^4 + 4*d*e^2*f - 2*
d^2*f^2 + e^3*Sqrt[e^2 - 4*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]))*Log
[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*(
c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))*Sqrt[2*a*f^2 + c*(e^2 - 2*
d*f + e*Sqrt[e^2 - 4*d*f])]) + (e*Log[a + Sqrt[a]*Sqrt[a + c*x^2]
])/(a^(3/2)*d^2) + (f*(a*f^2*(-e^2 + 2*d*f - e*Sqrt[e^2 - 4*d*f])
 - c*(e^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d*f] - 2*d*e
*f*Sqrt[e^2 - 4*d*f]))*Log[2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x +
 Sqrt[2*c*e^2 - 4*c*d*f + 4*a*f^2 - 2*c*e*Sqrt[e^2 - 4*d*f]]*Sqrt
[a + c*x^2]])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*(c^2*d^2 + a^2*f^2 +
 a*c*(e^2 - 2*d*f))*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 -
4*d*f])]) + (f*(a*f^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + c*(e^
4 - 4*d*e^2*f + 2*d^2*f^2 - e^3*Sqrt[e^2 - 4*d*f] + 2*d*e*f*Sqrt[
e^2 - 4*d*f]))*Log[-2*a*f + c*e*x + c*Sqrt[e^2 - 4*d*f]*x - Sqrt[
4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2
]])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*(c^2*d^2 + a^2*f^2 + a*c*(e^2
- 2*d*f))*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Maple [B] time = 0.025, size = 2046, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] 4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x/(c*x^2+a)^
(1/2)+8*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c/a^2*x/
(c*x^2+a)^(1/2)+8*f^4/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2
)/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/((x-1/2*(-e+(-4
*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4
*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
e^2*c)/f^2)^(1/2)-16*f^3/(-e+(-4*d*f+e^2)^(1/2))^2*c^2/(-(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f*d+c^2/f^2*e^
2-1/f^2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-
c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*
(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*x+16*f
^3/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*c^2/(-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1
/f^2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(
e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*x*e-8*f^4
/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+e^2*c)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+e^2*c)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*
d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*
c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+
2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-
1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-8*f^4/(e+(-4*d*f+e^2)^(1/2))^2/(-
4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-2*c*d*f+e^2*c)/f^2)^(1/2)-16*f^3/(e+(-4*d*f+e^2)^(1/2))^2*c^2/
((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f*d+c
^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))
/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*
x-16*f^3/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*c^2/((-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f*d+c^2/f^2*e
^2-1/f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-
c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*x*e+8*f^
4/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)
```

```
*c*e+2*a*f^2-2*c*d*f+e^2*c)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+e^2*c)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*
(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(
-4*d*f+e^2)^(1/2))/f))-16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*
d*f+e^2)^(1/2))^2/a/(c*x^2+a)^(1/2)+16*f^2*e/(-e+(-4*d*f+e^2)^(1/
2))^2/(e+(-4*d*f+e^2)^(1/2))^2/a^(3/2)*ln((2*a+2*a^(1/2)*(c*x^2+a
)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2+a)^{\frac{3}{2}}(fx^2+ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/(x**2*(a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.77 $\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

Optimal. Leaf size=392

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}f}+\frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f}-\frac{d\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{5/2}}+\frac{d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{5/2}}-\frac{d\sqrt{a+bx+cx^2}}{f^2}-\frac{bd\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3cf}$$

```
[Out] -((d*Sqrt[a + b*x + c*x^2])/f^2) + (b*(b + 2*c*x)*Sqrt[a + b*x +
c*x^2])/(8*c^2*f) - (a + b*x + c*x^2)^(3/2)/(3*c*f) - (b*d*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2)
 - (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2])])/(16*c^(5/2)*f) - (d*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f
]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)
/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/
(2*f^(5/2)) + (d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*S
qrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d +
 b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(5/2))
```

Rubi [A] time = 1.87466, antiderivative size = 392, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}f}+\frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f}-\frac{d\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{5/2}}+\frac{d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{5/2}}-\frac{d\sqrt{a+bx+cx^2}}{f^2}-\frac{bd\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3cf}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -((d*Sqrt[a + b*x + c*x^2])/f^2) + (b*(b + 2*c*x)*Sqrt[a + b*x +
c*x^2])/(8*c^2*f) - (a + b*x + c*x^2)^(3/2)/(3*c*f) - (b*d*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2)
 - (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2])])/(16*c^(5/2)*f) - (d*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f
]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)
/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/
(2*f^(5/2)) + (d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*S
qrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d +
```

```
 b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(5/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.74233, size = 419, normalized size = 1.07

$$\frac{-\frac{2\sqrt{f}\sqrt{a+x(b+cx)}\left(2cf(4a+bx)-3b^2f+8c^2(3d+fx^2)\right)}{c^2}-\frac{3b\sqrt{f}\left(-4acf+b^2f+8c^2d\right)\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)}{c^{5/2}}-24d\log\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{af+}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] ((-2*Sqrt[f]*Sqrt[a + x*(b + c*x)]*(-3*b^2*f + 2*c*f*(4*a + b*x)
+ 8*c^2*(3*d + f*x^2)))/c^2 - 24*d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] +
 a*f]*Log[Sqrt[d]*Sqrt[f] - f*x] - 24*d*Sqrt[c*d - b*Sqrt[d]*Sqrt
[f] + a*f]*Log[Sqrt[d]*Sqrt[f] + f*x] - (3*b*Sqrt[f]*(8*c^2*d + b
^2*f - 4*a*c*f)*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])
/c^(5/2) + 24*d*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Log[Sqrt[d]*(
-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x + 2*Sqrt
[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + 24*d*Sq
rt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Log[Sqrt[d]*(b*(Sqrt[d] + Sqrt[
f]*x) + 2*(a*Sqrt[f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt[f]
 + a*f]*Sqrt[a + x*(b + c*x)]))])/(48*f^(5/2))
```

Maple [B] time = 0.025, size = 1817, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/3*(c*x^2+b*x+a)^(3/2)/c/f+1/4/f*b/c*(c*x^2+b*x+a)^(1/2)*x+1/8/
f*b^2/c^2*(c*x^2+b*x+a)^(1/2)+1/4/f*b/c^(3/2)*ln((1/2*b+c*x)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))*a-1/16/f*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))-1/2/f^2*d*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d
*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/
2)-1/2/f^3*d*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))
/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^
(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/
4/f^2*d*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1
/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)
/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/c^(1/2)*b+1/2/f^3*d/((b*(d*
f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(
d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)
^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1
/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*(d*
```

```
f)^(1/2)+1/2/f^2*d/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*
f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*(
(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)
)/(x-(d*f)^(1/2)/f))*a+1/2/f^3*d^2/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1
/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d
*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/
f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f
*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c-1/2/f^2*d*((x+(d*f)^(1/2)/
f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)
^(1/2)+f*a+c*d))^(1/2)+1/2/f^3*d*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)
+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d
*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1
/2))*c^(1/2)*(d*f)^(1/2)-1/4/f^2*d*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*
f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*
(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^
(1/2))/c^(1/2)*b-1/2/f^3*d/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*l
n((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*
f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2
)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*
f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*(d*f)^(1/2)+1/2/f^
2*d/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+
f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*
(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f
)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2
))/(x+(d*f)^(1/2)/f))*a+1/2/f^3*d^2/(1/f*(-b*(d*f)^(1/2)+f*a+c*d)
)^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*
f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(
d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/
f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(c*x^2 + b*x + a)*x^3/(f*x^2 - d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(c*x^2 + b*x + a)*x^3/(f*x^2 - d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**3*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(c*x^2 + b*x + a)*x^3/(f*x^2 - d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.78 $\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

Optimal. Leaf size=316

$$-\frac{\left(4acf+b^2(-f)+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^2}+\frac{\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^2}+\frac{\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^2}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf}$$

```
[Out] -((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c*f) - ((8*c^2*d - b^2*f
+ 4*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(8*c^(3/2)*f^2) + (Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*
ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(
2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2
*f^2) + (Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*S
qrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d +
 b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^2)
```

Rubi [A] time = 1.10572, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$-\frac{\left(4acf+b^2(-f)+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^2}+\frac{\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^2}+\frac{\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^2}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c*f) - ((8*c^2*d - b^2*f
+ 4*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(8*c^(3/2)*f^2) + (Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*
ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(
2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2
*f^2) + (Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*S
qrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d +
 b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.39483, size = 397, normalized size = 1.26

$$-\frac{\frac{(4acf+b^2(-f)+8c^2d)\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)}{c^{3/2}}+4\sqrt{d}\log\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{af+b\sqrt{d}\sqrt{f}+cd}-4\sqrt{d}\log\left(\sqrt{d}\sqrt{f}+fx\right)\sqrt{af+\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -((2*f*(b + 2*c*x)*Sqrt[a + x*(b + c*x)])/c + 4*Sqrt[d]*Sqrt[c*d
+ b*Sqrt[d]*Sqrt[f] + a*f]*Log[Sqrt[d]*Sqrt[f] - f*x] - 4*Sqrt[d]
*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Log[Sqrt[d]*Sqrt[f] + f*x] +
 ((8*c^2*d - b^2*f + 4*a*c*f)*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a +
x*(b + c*x)]])/c^(3/2) + 4*Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] +
 a*f]*Log[Sqrt[d]*(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b
*Sqrt[f]*x + 2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b
+ c*x)])] - 4*Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Log[Sqr
t[d]*(b*(Sqrt[d] + Sqrt[f]*x) + 2*(a*Sqrt[f] + c*Sqrt[d]*x + Sqrt
[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]))])/(8*f^2)
```

Maple [B] time = 0.021, size = 1810, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/f*x*(c*x^2+b*x+a)^(1/2)-1/4/f/c*(c*x^2+b*x+a)^(1/2)*b-1/2/f/
c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/8/f/c^(3/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2-1/2*d/(d*f)^(1
/2)/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/
2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-1/2*d/f^2*ln((1/2*(2*c*(d*
f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2
*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c
*d)/f)^(1/2))*c^(1/2)-1/4*d/(d*f)^(1/2)/f*ln((1/2*(2*c*(d*f)^(1/2
)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c
*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^
(1/2))/c^(1/2)*b+1/2*d/f^2/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((
2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2
)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(
2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/
f)^(1/2))/(x-(d*f)^(1/2)/f))*b+1/2*d/(d*f)^(1/2)/f/((b*(d*f)^(1/2
)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1
/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*
((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+
(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a+1/2*d^2/(d
*f)^(1/2)/f^2/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1
/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d
*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-
(d*f)^(1/2)/f))*c+1/2*d/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+1/f*
(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+
c*d))^(1/2)-1/2*d/f^2*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)
^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b
*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))*c^(1/2
```

```
)+1/4*d/(d*f)^(1/2)/f*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)
^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b
*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/c^(1/2
)*b+1/2*d/f^2/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d
*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2
*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*
(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+
c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b-1/2*d/(d*f)^(1/2)/f/(1/f*(-b*(d
*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-
2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a
+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x
+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/
2)/f))*a-1/2*d^2/(d*f)^(1/2)/f^2/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(
1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*
(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f
)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(
-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(c*x^2 + b*x + a)*x^2/(f*x^2 - d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(c*x^2 + b*x + a)*x^2/(f*x^2 - d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**2*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(c*x^2 + b*x + a)*x^2/(f*x^2 - d),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.79 $\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

Optimal. Leaf size=282

$$-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{3/2}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{3/2}}-\frac{\sqrt{a+bx+cx^2}}{f}-\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f}$$

```
[Out] -(Sqrt[a + b*x + c*x^2]/f) - (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sq
rt[a + b*x + c*x^2])])/(2*Sqrt[c]*f) - (Sqrt[c*d - b*Sqrt[d]*Sqrt
[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sq
rt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c
*x^2])])/(2*f^(3/2)) + (Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTa
nh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqr
t[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(3
/2))
```

Rubi [A] time = 0.734932, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{3/2}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{3/2}}-\frac{\sqrt{a+bx+cx^2}}{f}-\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/f) - (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sq
rt[a + b*x + c*x^2])])/(2*Sqrt[c]*f) - (Sqrt[c*d - b*Sqrt[d]*Sqrt
[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sq
rt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c
*x^2])])/(2*f^(3/2)) + (Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTa
nh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqr
t[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(3
/2))
```

Rubi in Sympy [A] time = 138.653, size = 257, normalized size = 0.91

$$-\frac{b\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f}-\frac{\sqrt{a+bx+cx^2}}{f}-\frac{\sqrt{af-b\sqrt{d}\sqrt{f}+cd}\operatorname{atanh}\left(\frac{-2a\sqrt{f}+b\sqrt{d}+x\left(-b\sqrt{f}+2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{\frac{3}{2}}}-\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\operatorname{atanh}\left(\frac{-2a\sqrt{f}-b\sqrt{d}+x\left(-b\sqrt{f}-2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -b*atanh((b + 2*c*x)/(2*sqrt(c)*sqrt(a + b*x + c*x**2)))/(2*sqrt(
c)*f) - sqrt(a + b*x + c*x**2)/f - sqrt(a*f - b*sqrt(d)*sqrt(f) +
 c*d)*atanh((-2*a*sqrt(f) + b*sqrt(d) + x*(-b*sqrt(f) + 2*c*sqrt(
d)))/(2*sqrt(a + b*x + c*x**2)*sqrt(a*f - b*sqrt(d)*sqrt(f) + c*d
)))/(2*f**(3/2)) - sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)*atanh((-2*
a*sqrt(f) - b*sqrt(d) + x*(-b*sqrt(f) - 2*c*sqrt(d)))/(2*sqrt(a +
 b*x + c*x**2)*sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)))/(2*f**(3/2))
```

Mathematica [A] time = 2.17196, size = 448, normalized size = 1.59

$$-\frac{\frac{\log\left(\sqrt{d}\sqrt{f}-fx\right)\left(af^{3/2}+b\sqrt{d}f+cd\sqrt{f}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}+\frac{\log\left(\sqrt{d}\sqrt{f}+fx\right)\left(af^{3/2}-b\sqrt{d}f+cd\sqrt{f}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}-\frac{\left(af^{3/2}-b\sqrt{d}f+cd\sqrt{f}\right)\log\left(\sqrt{d}\left(2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd+\cdots}\right.\right.}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -(2*f*Sqrt[a + x*(b + c*x)] + ((c*d*Sqrt[f] + b*Sqrt[d]*f + a*f^(
3/2))*Log[Sqrt[d]*Sqrt[f] - f*x])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] +
a*f] + ((c*d*Sqrt[f] - b*Sqrt[d]*f + a*f^(3/2))*Log[Sqrt[d]*Sqrt[
f] + f*x])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] + (b*f*Log[b + 2*c
*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/Sqrt[c] - ((c*d*Sqrt[f] -
b*Sqrt[d]*f + a*f^(3/2))*Log[Sqrt[d]*(-(b*Sqrt[d]) + 2*a*Sqrt[f]
- 2*c*Sqrt[d]*x + b*Sqrt[f]*x + 2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] +
a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]
 - ((c*d*Sqrt[f] + b*Sqrt[d]*f + a*f^(3/2))*Log[Sqrt[d]*(b*(Sqrt[
d] + Sqrt[f]*x) + 2*(a*Sqrt[f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]))])/Sqrt[c*d + b*Sqrt[d]*
Sqrt[f] + a*f])/(2*f^2)
```

Maple [B] time = 0.019, size = 1667, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1
/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-1/2/f^2*ln((1/2*(2*c*(d*f
)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*
c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*
```

```
d)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/f*ln((1/2*(2*c*(d*f)^(1/2)+b
*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d
*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/
2))/c^(1/2)*b+1/2/f^2/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*
(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+
2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(
d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1
/2))/(x-(d*f)^(1/2)/f))*b*(d*f)^(1/2)+1/2/f/((b*(d*f)^(1/2)+f*a+c
*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f
)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*
f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f
)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a+1/2/f^2/((b*(d*f)
^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*
f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(
1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2
)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c*d-1/2
/f*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/
2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)+1/2/f^2*ln((1/2/f*(-2*c
*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)
^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(
1/2)+f*a+c*d))^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/f*ln((1/2/f*(-2*c*(
d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2
*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/
2)+f*a+c*d))^(1/2))/c^(1/2)*b-1/2/f^2/(1/f*(-b*(d*f)^(1/2)+f*a+c*
d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+
b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x
+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+
1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*(d*f)^(
1/2)+1/2/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)
^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1
/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2
*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d
))^(1/2))/(x+(d*f)^(1/2)/f))*a+1/2/f^2/(1/f*(-b*(d*f)^(1/2)+f*a+c
*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)
+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((
x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)
+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(c*x^2 + b*x + a)*x/(f*x^2 - d),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(c*x^2 + b*x + a)*x/(f*x^2 - d),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(c*x^2 + b*x + a)*x/(f*x^2 - d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.80 $\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

Optimal. Leaf size=266

$$\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}f}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}f}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}$$

```
[Out] -((Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/f) + (Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] -
2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d
]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*f) + (Sqrt[c
*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] +
(2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*
f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*f)
```

Rubi [A] time = 0.531991, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}f}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}f}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x + c*x^2]/(d - f*x^2),x]
```

```
[Out] -((Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/f) + (Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] -
2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d
]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*f) + (Sqrt[c
*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] +
(2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*
f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*f)
```

Rubi in Sympy [A] time = 89.7376, size = 240, normalized size = 0.9

$$-\frac{\sqrt{c}\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}+\frac{\sqrt{af-b\sqrt{d}\sqrt{f}+cd}\operatorname{atanh}\left(\frac{-2a\sqrt{f}+b\sqrt{d}+x\left(-b\sqrt{f}+2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}f}-\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\operatorname{atanh}\left(\frac{-2a\sqrt{f}-b\sqrt{d}+x\left(-b\sqrt{f}-2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -sqrt(c)*atanh((b + 2*c*x)/(2*sqrt(c)*sqrt(a + b*x + c*x**2)))/f
+ sqrt(a*f - b*sqrt(d)*sqrt(f) + c*d)*atanh((-2*a*sqrt(f) + b*sqr
t(d) + x*(-b*sqrt(f) + 2*c*sqrt(d)))/(2*sqrt(a + b*x + c*x**2)*sq
rt(a*f - b*sqrt(d)*sqrt(f) + c*d)))/(2*sqrt(d)*f) - sqrt(a*f + b*
sqrt(d)*sqrt(f) + c*d)*atanh((-2*a*sqrt(f) - b*sqrt(d) + x*(-b*sq
rt(f) - 2*c*sqrt(d)))/(2*sqrt(a + b*x + c*x**2)*sqrt(a*f + b*sqrt
(d)*sqrt(f) + c*d)))/(2*sqrt(d)*f)
```

Mathematica [A] time = 0.717376, size = 343, normalized size = 1.29

$$\log\left(\sqrt{d}\sqrt{f}-fx\right)\left(-\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\right)+\log\left(\sqrt{d}\sqrt{f}+fx\right)\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}-\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\log$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x + c*x^2]/(d - f*x^2),x]
```

```
[Out] (-(Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Log[Sqrt[d]*Sqrt[f] - f*x]
) + Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Log[Sqrt[d]*Sqrt[f] + f*x
] - 2*Sqrt[c]*Sqrt[d]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c
*x)]] - Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Log[Sqrt[d]*(-(b*Sqrt
[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x + 2*Sqrt[c*d - b
*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + Sqrt[c*d + b*Sq
rt[d]*Sqrt[f] + a*f]*Log[Sqrt[d]*(b*(Sqrt[d] + Sqrt[f]*x) + 2*(a*
Sqrt[f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[
a + x*(b + c*x)]))])/(2*Sqrt[d]*f)
```

Maple [B] time = 0.019, size = 1669, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(
x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-1/2/f*ln((1/2*(
2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/
2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2
)+f*a+c*d)/f)^(1/2))*c^(1/2)-1/4/(d*f)^(1/2)*ln((1/2*(2*c*(d*f)^(
1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(
2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/
f)^(1/2))/c^(1/2)*b+1/2/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2
*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)
/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2
*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f
)^(1/2))/(x-(d*f)^(1/2)/f))*b+1/2/(d*f)^(1/2)/((b*(d*f)^(1/2)+f*a
+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b
*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(
d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d
*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a+1/2/(d*f)^(1/2)
/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d
)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f
*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(
x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)
/f))*c*d+1/2/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(
1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-1
/2/f*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2
```

```
)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2
)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))*c^(1/2)+1/4/(d*f)^(1/2)
*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((
x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)
+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/c^(1/2)*b+1/2/f/(1/f*(-b*(d
*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-
2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a
+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x
+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/
2)/f))*b-1/2/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln(
(2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/
f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)
^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a-1/2/(d*f)^(1/2)/f/(1
/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*
d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^
(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2
)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+
(d*f)^(1/2)/f))*c*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(c*x^2 + b*x + a)/(f*x^2 - d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 100.791, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(c*x^2 + b*x + a)/(f*x^2 - d),x, algorithm="fricas")
```

```
[Out] [1/4*(f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2))*log((
2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt((d*f^2*sqrt(b^2/(d*f^3
)) + c*d + a*f)/(d*f^2)) + b^2 + (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*
f^3)))/x) - f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2))
*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt((d*f^2*sqrt(b^2/
(d*f^3)) + c*d + a*f)/(d*f^2)) + b^2 + (b*f^2*x + 2*a*f^2)*sqrt(b
^2/(d*f^3)))/x) + f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a*f)/(
d*f^2))*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt(-(d*f^2*s
qrt(b^2/(d*f^3)) - c*d - a*f)/(d*f^2)) + b^2 - (b*f^2*x + 2*a*f^2
)*sqrt(b^2/(d*f^3)))/x) - f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3)) - c*d
- a*f)/(d*f^2))*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt(-
(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a*f)/(d*f^2)) + b^2 - (b*f^2*x +
 2*a*f^2)*sqrt(b^2/(d*f^3)))/x) + 2*sqrt(c)*log(-8*c^2*x^2 - 8*b*
c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c))
/f, 1/4*(f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2))*lo
g((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt((d*f^2*sqrt(b^2/(d*
f^3)) + c*d + a*f)/(d*f^2)) + b^2 + (b*f^2*x + 2*a*f^2)*sqrt(b^2/
(d*f^3)))/x) - f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^
2))*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt((d*f^2*sqrt(b
^2/(d*f^3)) + c*d + a*f)/(d*f^2)) + b^2 + (b*f^2*x + 2*a*f^2)*sqr
t(b^2/(d*f^3)))/x) + f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a*f
)/(d*f^2))*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt(-(d*f^
2*sqrt(b^2/(d*f^3)) - c*d - a*f)/(d*f^2)) + b^2 - (b*f^2*x + 2*a*
f^2)*sqrt(b^2/(d*f^3)))/x) - f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3)) - c
*d - a*f)/(d*f^2))*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*f*sqr
```

```
t(-(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a*f)/(d*f^2)) + b^2 - (b*f^2*
x + 2*a*f^2)*sqrt(b^2/(d*f^3)))/x) - 4*sqrt(-c)*arctan(1/2*(2*c*x
 + b)/(sqrt(c*x^2 + b*x + a)*sqrt(-c))))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{a + bx + cx^2}}{-d + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(c*x^2 + b*x + a)/(f*x^2 - d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.81 $\int \frac{\sqrt{a+bx+cx^2}}{x(d-fx^2)}\,dx$

Optimal. Leaf size=267

$$-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\sqrt{f}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{f}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$

```
[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]
)/d) - (Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] -
2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d
]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*Sqrt[f]) + (Sqrt[c
*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] +
(2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*
f]*Sqrt[a + b*x + c*x^2])])/(2*d*Sqrt[f])
```

Rubi [A] time = 1.69338, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321

$$-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\sqrt{f}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{f}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x + c*x^2]/(x*(d - f*x^2)),x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]
)/d) - (Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] -
2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d
]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*Sqrt[f]) + (Sqrt[c
*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] +
(2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*
f]*Sqrt[a + b*x + c*x^2])])/(2*d*Sqrt[f])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)**(1/2)/x/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.524783, size = 473, normalized size = 1.77

$$-\frac{\log\left(\sqrt{d}\sqrt{f}-fx\right)\left(a\sqrt{d}f+bd\sqrt{f}+cd^{3/2}\right)}{\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\log\left(\sqrt{d}\sqrt{f}+fx\right)\left(a\sqrt{d}f-bd\sqrt{f}+cd^{3/2}\right)}{\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\left(a\sqrt{d}f-bd\sqrt{f}+cd^{3/2}\right)\log\left(\sqrt{d}\left(2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd+}\right.\right.}{\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x + c*x^2]/(x*(d - f*x^2)),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[d]*Log[x] - ((c*d^(3/2) + b*d*Sqrt[f] + a*Sqrt[d]
*f)*Log[Sqrt[d]*Sqrt[f] - f*x])/(Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqr
t[f] + a*f]) - ((c*d^(3/2) - b*d*Sqrt[f] + a*Sqrt[d]*f)*Log[Sqrt[
d]*Sqrt[f] + f*x])/(Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f])
- 2*Sqrt[a]*Sqrt[d]*Log[2*a + b*x + 2*Sqrt[a]*Sqrt[a + x*(b + c*x
)]] + ((c*d^(3/2) - b*d*Sqrt[f] + a*Sqrt[d]*f)*Log[Sqrt[d]*(-(b*S
qrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x + 2*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(Sqrt[f]*Sqrt
[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + ((c*d^(3/2) + b*d*Sqrt[f] + a*
Sqrt[d]*f)*Log[Sqrt[d]*(b*(Sqrt[d] + Sqrt[f]*x) + 2*(a*Sqrt[f] +
c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b +
 c*x)]))])/(Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))/(2*d^(3
/2))
```

Maple [B] time = 0.023, size = 1764, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x)
```

```
[Out] 1/d*(c*x^2+b*x+a)^(1/2)+1/2/d*b*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))/c^(1/2)-1/d*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a
)^(1/2))/x)-1/2/d*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*
(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-1/2/d/f*ln((1/
2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^
(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(
1/2)+f*a+c*d)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/d*ln((1/2*(2*c*(d
*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^
2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+
c*d)/f)^(1/2))/c^(1/2)*b+1/2/d/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2
)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f
)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)
^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a
+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*(d*f)^(1/2)+1/2/d/((b*(d*f)^
(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1
/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)
/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a+1/2/f/
((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f
+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+
c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(
d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)
)*c-1/2/d*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d
*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)+1/2/d/f*ln((1/2/
f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(
1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*
(d*f)^(1/2)+f*a+c*d))^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/d*ln((1/2/f*
(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/
2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d
*f)^(1/2)+f*a+c*d))^(1/2))/c^(1/2)*b-1/2/d/f/(1/f*(-b*(d*f)^(1/2)
+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)
^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1
```

```
/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1
/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*
(d*f)^(1/2)+1/2/d/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-
b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/
f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+
1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+
f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a+1/2/f/(1/f*(-b*(d*f)^(1/2)+
f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^
(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/
2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/
2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 - d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x), x)
```

Fricas [A] time = 17.4678, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x),x, algorithm="fricas")
```

```
[Out] [1/4*(d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f))*log((
2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2*f*sqrt(
b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) + 2*b*c*x + b^2 + (b*d*f*x + 2
*a*d*f)*sqrt(b^2/(d^3*f)))/x) - d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) +
 c*d + a*f)/(d^2*f))*log(-(2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2
/(d^3*f))*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) - 2
*b*c*x - b^2 - (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) - d*sqrt
(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f))*log((2*sqrt(c*x^
2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sqrt(b^2/(d^3*f
)) - c*d - a*f)/(d^2*f)) + 2*b*c*x + b^2 - (b*d*f*x + 2*a*d*f)*sq
rt(b^2/(d^3*f)))/x) + d*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*
f)/(d^2*f))*log(-(2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))
*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f)) - 2*b*c*x -
 b^2 + (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) + 2*sqrt(a)*log(
-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*
a)*sqrt(a) + 8*a^2)/x^2))/d, 1/4*(d*sqrt((d^2*f*sqrt(b^2/(d^3*f))
 + c*d + a*f)/(d^2*f))*log((2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^
2/(d^3*f))*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) +
2*b*c*x + b^2 + (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) - d*sqr
t((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f))*log(-(2*sqrt(c*x
^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2*f*sqrt(b^2/(d^3*f
)) + c*d + a*f)/(d^2*f)) - 2*b*c*x - b^2 - (b*d*f*x + 2*a*d*f)*sq
rt(b^2/(d^3*f)))/x) - d*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*
f)/(d^2*f))*log((2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*
sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f)) + 2*b*c*x +
b^2 - (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) + d*sqrt(-(d^2*f*
sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f))*log(-(2*sqrt(c*x^2 + b*x
+ a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d
 - a*f)/(d^2*f)) - 2*b*c*x - b^2 + (b*d*f*x + 2*a*d*f)*sqrt(b^2/(
d^3*f)))/x) - 4*sqrt(-a)*arctan(1/2*(b*x + 2*a)/(sqrt(c*x^2 + b*x
 + a)*sqrt(-a))))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{a + bx + cx^2}}{-dx + fx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)**(1/2)/x/(-f*x**2+d),x)
```

```
[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x)
```

GIAC/XCAS [A] time = 0.292513, size = 1, normalized size = 0.

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x),x, algorithm="giac")
```

```
[Out] sage2
```

3.82 $\int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)}\,dx$

Optimal. Leaf size=286

$$\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}}-\frac{\sqrt{a+bx+cx^2}}{dx}-\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d}$$

```
[Out] -(Sqrt[a + b*x + c*x^2]/(d*x)) - (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a
]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a]*d) + (Sqrt[c*d - b*Sqrt[d]*
Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] -
b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
 + c*x^2])])/(2*d^(3/2)) + (Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*A
rcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2
*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*
d^(3/2))
```

Rubi [A] time = 1.54478, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}}-\frac{\sqrt{a+bx+cx^2}}{dx}-\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x + c*x^2]/(x^2*(d - f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(d*x)) - (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a
]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a]*d) + (Sqrt[c*d - b*Sqrt[d]*
Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] -
b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
 + c*x^2])])/(2*d^(3/2)) + (Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*A
rcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2
*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*
d^(3/2))
```

Rubi in Sympy [A] time = 176.368, size = 257, normalized size = 0.9

$$-\frac{\sqrt{a+bx+cx^2}}{dx}+\frac{\sqrt{af-b\sqrt{d}\sqrt{f}+cd}\,\operatorname{atanh}\left(\frac{-2a\sqrt{f}+b\sqrt{d}+x\left(-b\sqrt{f}+2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{\frac{3}{2}}} - \frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\operatorname{atanh}\left(\frac{-2a\sqrt{f}-b\sqrt{d}+x\left(-b\sqrt{f}-2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{\frac{3}{2}}} - \frac{b\,\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)**(1/2)/x**2/(-f*x**2+d),x)
```

```
[Out] -sqrt(a + b*x + c*x**2)/(d*x) + sqrt(a*f - b*sqrt(d)*sqrt(f) + c*
d)*atanh((-2*a*sqrt(f) + b*sqrt(d) + x*(-b*sqrt(f) + 2*c*sqrt(d))
)/(2*sqrt(a + b*x + c*x**2)*sqrt(a*f - b*sqrt(d)*sqrt(f) + c*d)))
/(2*d**(3/2)) - sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)*atanh((-2*a*s
qrt(f) - b*sqrt(d) + x*(-b*sqrt(f) - 2*c*sqrt(d)))/(2*sqrt(a + b*
x + c*x**2)*sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)))/(2*d**(3/2)) -
b*atanh((2*a + b*x)/(2*sqrt(a)*sqrt(a + b*x + c*x**2)))/(2*sqrt(a
)*d)
```

Mathematica [A] time = 1.39548, size = 379, normalized size = 1.33

$$-\log\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{af+b\sqrt{d}\sqrt{f}+cd}+\log\left(\sqrt{d}\sqrt{f}+fx\right)\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}-\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\log\left(\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x + c*x^2]/(x^2*(d - f*x^2)),x]
```

```
[Out] ((-2*Sqrt[d]*Sqrt[a + x*(b + c*x)])/x + (b*Sqrt[d]*Log[x])/Sqrt[a
] - Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Log[Sqrt[d]*Sqrt[f] - f*x
] + Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Log[Sqrt[d]*Sqrt[f] + f*x
] - (b*Sqrt[d]*Log[2*a + b*x + 2*Sqrt[a]*Sqrt[a + x*(b + c*x)]])/
Sqrt[a] - Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Log[Sqrt[d]*(-(b*Sq
rt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x + 2*Sqrt[c*d -
 b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + Sqrt[c*d + b*
Sqrt[d]*Sqrt[f] + a*f]*Log[Sqrt[d]*(b*(Sqrt[d] + Sqrt[f]*x) + 2*(
a*Sqrt[f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqr
t[a + x*(b + c*x)]))])/(2*d^(3/2))
```

Maple [B] time = 0.032, size = 1819, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x)
```

```
[Out] -1/d/a/x*(c*x^2+b*x+a)^(3/2)+1/d*b/a*(c*x^2+b*x+a)^(1/2)-1/2/d*b/
a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+1/d*c/a*(c*
x^2+b*x+a)^(1/2)*x+1/d*c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))-1/2*f/d/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(
1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-1/
2/d*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+
((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+
```

```
(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))*c^(1/2)-1/4*f/d/(d*f)^(1/2)*ln(
(1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*
f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f
)^(1/2)+f*a+c*d)/f)^(1/2))/c^(1/2)*b+1/2/d/((b*(d*f)^(1/2)+f*a+c*
d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)
/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f
)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)
^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b+1/2*f/d/(d*f)^(1/2
)/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)
/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*
a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x
-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/
f))*a+1/2/(d*f)^(1/2)/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*
(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+
2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(
d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1
/2))/(x-(d*f)^(1/2)/f))*c+1/2*f/d/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^
2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1
/2)+f*a+c*d))^(1/2)-1/2/d*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(
d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/
2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))*c^
(1/2)+1/4*f/d/(d*f)^(1/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(
d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/
2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/c^
(1/2)*b+1/2/d/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d
*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2
*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*
(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+
c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b-1/2*f/d/(d*f)^(1/2)/(1/f*(-b*(d
*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-
2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a
+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x
+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/
2)/f))*a-1/2/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln(
(2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/
f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)
^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 - d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^2), x)
```

Fricas [A] time = 25.1813, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*d*x*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log
((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt(b^2*f/d^5)
 + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x
) - sqrt(a)*d*x*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log((
2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt(b^2*f/d^5) +
```

```
 c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x)
+ sqrt(a)*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)*log((2
*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*sqrt(b^2*f/d^5) -
 c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x)
- sqrt(a)*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)*log((2
*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*sqrt(b^2*f/d^5) -
 c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x)
+ b*x*log((4*(a*b*x + 2*a^2)*sqrt(c*x^2 + b*x + a) - (8*a*b*x + (
b^2 + 4*a*c)*x^2 + 8*a^2)*sqrt(a))/x^2) - 4*sqrt(c*x^2 + b*x + a)
*sqrt(a))/(sqrt(a)*d*x), 1/4*(sqrt(-a)*d*x*sqrt((d^3*sqrt(b^2*f/d
^5) + c*d + a*f)/d^3)*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*d*
sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*
a*d^2)*sqrt(b^2*f/d^5))/x) - sqrt(-a)*d*x*sqrt((d^3*sqrt(b^2*f/d^
5) + c*d + a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*s
qrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a
*d^2)*sqrt(b^2*f/d^5))/x) + sqrt(-a)*d*x*sqrt(-(d^3*sqrt(b^2*f/d^
5) - c*d - a*f)/d^3)*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*d*s
qrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2*
a*d^2)*sqrt(b^2*f/d^5))/x) - sqrt(-a)*d*x*sqrt(-(d^3*sqrt(b^2*f/d
^5) - c*d - a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*
sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2
*a*d^2)*sqrt(b^2*f/d^5))/x) - 2*b*x*arctan(1/2*(b*x + 2*a)*sqrt(-
a)/(sqrt(c*x^2 + b*x + a)*a)) - 4*sqrt(c*x^2 + b*x + a)*sqrt(-a))
/(sqrt(-a)*d*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)**(1/2)/x**2/(-f*x**2+d),x)

[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^2),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.83 $\int \frac{\sqrt{a+bx+cx^2}}{x^3(d-fx^2)}\,dx$

Optimal. Leaf size=353

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d}-\frac{\sqrt{a}f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^2}+\frac{\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^2}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2}$$

```
[Out] -((2*a + b*x)*Sqrt[a + b*x + c*x^2])/(4*a*d*x^2) + ((b^2 - 4*a*c)
*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(3/
2)*d) - (Sqrt[a]*f*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x +
c*x^2])])/d^2 - (Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcT
anh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2
) + (Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[
d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*S
qrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2)
```

Rubi [A] time = 1.93866, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d}-\frac{\sqrt{a}f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^2}+\frac{\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^2}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x + c*x^2]/(x^3*(d - f*x^2)),x]
```


```
[Out] -((2*a + b*x)*Sqrt[a + b*x + c*x^2])/(4*a*d*x^2) + ((b^2 - 4*a*c)
*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(3/
2)*d) - (Sqrt[a]*f*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x +
c*x^2])])/d^2 - (Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcT
anh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2
) + (Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[
d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*S
qrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)**(1/2)/x**3/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.789699, size = 505, normalized size = 1.43

$$\frac{\log(x)(4a(2af+cd)-b^2d)}{a^{3/2}}+\frac{(b^2d-4a(2af+cd))\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+bx\right)}{a^{3/2}}-\frac{4\log\left(\sqrt{d}\sqrt{f}-fx\right)\left(af^{3/2}+b\sqrt{d}f+cd\sqrt{f}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{4\log\left(\sqrt{d}\sqrt{f}+fx\right)\left(af^{3/2}-b\sqrt{\cdots}\right.}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+\cdots}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x + c*x^2]/(x^3*(d - f*x^2)),x]
```

```
[Out] ((-2*d*(2*a + b*x)*Sqrt[a + x*(b + c*x)])/(a*x^2) + ((-(b^2*d) +
4*a*(c*d + 2*a*f))*Log[x])/a^(3/2) - (4*(c*d*Sqrt[f] + b*Sqrt[d]*
f + a*f^(3/2))*Log[Sqrt[d]*Sqrt[f] - f*x])/Sqrt[c*d + b*Sqrt[d]*S
qrt[f] + a*f] - (4*(c*d*Sqrt[f] - b*Sqrt[d]*f + a*f^(3/2))*Log[Sq
rt[d]*Sqrt[f] + f*x])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] + ((b^2
*d - 4*a*(c*d + 2*a*f))*Log[2*a + b*x + 2*Sqrt[a]*Sqrt[a + x*(b +
 c*x)]])/a^(3/2) + (4*(c*d*Sqrt[f] - b*Sqrt[d]*f + a*f^(3/2))*Log
[Sqrt[d]*(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*
x + 2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]
)/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] + (4*(c*d*Sqrt[f] + b*Sqrt[
d]*f + a*f^(3/2))*Log[Sqrt[d]*(b*(Sqrt[d] + Sqrt[f]*x) + 2*(a*Sqr
t[f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
 x*(b + c*x)]))])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])/(8*d^2)
```

Maple [B] time = 0.026, size = 1953, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x)
```

```
[Out] -1/2/d/a/x^2*(c*x^2+b*x+a)^(3/2)+1/4/d*b/a^2/x*(c*x^2+b*x+a)^(3/2
)-1/4/d*b^2/a^2*(c*x^2+b*x+a)^(1/2)+1/8/d*b^2/a^(3/2)*ln((2*a+b*x
+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-1/4/d*b/a^2*c*(c*x^2+b*x+a)^(1
/2)*x+1/2/d*c/a*(c*x^2+b*x+a)^(1/2)-1/2/d*c/a^(1/2)*ln((2*a+b*x+2
*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+f/d^2*(c*x^2+b*x+a)^(1/2)+1/2*f/
d^2*b*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-f/d^2*a
^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-1/2*f/d^2*((
x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b
*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-1/2/d^2*ln((1/2*(2*c*(d*f)^(1/2)+b
*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d
*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/
2))*c^(1/2)*(d*f)^(1/2)-1/4*f/d^2*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f
+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(
1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/c
^(1/2)*b+1/2/d^2/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)
^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b
*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^
(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/
(x-(d*f)^(1/2)/f))*b*(d*f)^(1/2)+1/2*f/d^2/((b*(d*f)^(1/2)+f*a+c*
d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)
/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f
)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)
^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a+1/2/d/((b*(d*f)^(1
/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^
```

```
(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2
)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f
)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c-1/2*f/d^
2*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2
)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)+1/2/d^2*ln((1/2/f*(-2*c*
(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^
2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1
/2)+f*a+c*d))^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4*f/d^2*ln((1/2/f*(-2*
c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f
)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^
(1/2)+f*a+c*d))^(1/2))/c^(1/2)*b-1/2/d^2/(1/f*(-b*(d*f)^(1/2)+f*a
+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/
2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*
((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/
f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*(d*f
)^(1/2)+1/2*f/d^2/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-
b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/
f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+
1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+
f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a+1/2/d/(1/f*(-b*(d*f)^(1/2)+
f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^
(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/
2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/
2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{cx^2+bx+a}}{(fx^2-d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^3),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^3), x)
```

Fricas [A] time = 136.041, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^3),x, algorithm="fricas")
```

```
[Out] [1/16*(4*a^(3/2)*d^2*x^2*sqrt((d^4*sqrt(b^2*f^3/d^7) + c*d*f + a*
f^2)/d^4)*log((2*sqrt(c*x^2 + b*x + a)*d^5*sqrt(b^2*f^3/d^7)*sqrt
((d^4*sqrt(b^2*f^3/d^7) + c*d*f + a*f^2)/d^4) + 2*b*c*f^2*x + b^2
*f^2 + (b*d^3*f*x + 2*a*d^3*f)*sqrt(b^2*f^3/d^7))/x) - 4*a^(3/2)*
d^2*x^2*sqrt((d^4*sqrt(b^2*f^3/d^7) + c*d*f + a*f^2)/d^4)*log(-(2
*sqrt(c*x^2 + b*x + a)*d^5*sqrt(b^2*f^3/d^7)*sqrt((d^4*sqrt(b^2*f
^3/d^7) + c*d*f + a*f^2)/d^4) - 2*b*c*f^2*x - b^2*f^2 - (b*d^3*f*
x + 2*a*d^3*f)*sqrt(b^2*f^3/d^7))/x) - 4*a^(3/2)*d^2*x^2*sqrt(-(d
^4*sqrt(b^2*f^3/d^7) - c*d*f - a*f^2)/d^4)*log((2*sqrt(c*x^2 + b*
x + a)*d^5*sqrt(b^2*f^3/d^7)*sqrt(-(d^4*sqrt(b^2*f^3/d^7) - c*d*f
 - a*f^2)/d^4) + 2*b*c*f^2*x + b^2*f^2 - (b*d^3*f*x + 2*a*d^3*f)*
sqrt(b^2*f^3/d^7))/x) + 4*a^(3/2)*d^2*x^2*sqrt(-(d^4*sqrt(b^2*f^3
/d^7) - c*d*f - a*f^2)/d^4)*log(-(2*sqrt(c*x^2 + b*x + a)*d^5*sqr
t(b^2*f^3/d^7)*sqrt(-(d^4*sqrt(b^2*f^3/d^7) - c*d*f - a*f^2)/d^4)
 - 2*b*c*f^2*x - b^2*f^2 + (b*d^3*f*x + 2*a*d^3*f)*sqrt(b^2*f^3/d
^7))/x) + (8*a^2*f - (b^2 - 4*a*c)*d)*x^2*log((4*(a*b*x + 2*a^2)*
sqrt(c*x^2 + b*x + a) - (8*a*b*x + (b^2 + 4*a*c)*x^2 + 8*a^2)*sqr
t(a))/x^2) - 4*(b*d*x + 2*a*d)*sqrt(c*x^2 + b*x + a)*sqrt(a))/(a^
(3/2)*d^2*x^2), 1/8*(2*sqrt(-a)*a*d^2*x^2*sqrt((d^4*sqrt(b^2*f^3/
```

```
d^7) + c*d*f + a*f^2)/d^4)*log((2*sqrt(c*x^2 + b*x + a)*d^5*sqrt(
b^2*f^3/d^7)*sqrt((d^4*sqrt(b^2*f^3/d^7) + c*d*f + a*f^2)/d^4) +
2*b*c*f^2*x + b^2*f^2 + (b*d^3*f*x + 2*a*d^3*f)*sqrt(b^2*f^3/d^7)
)/x) - 2*sqrt(-a)*a*d^2*x^2*sqrt((d^4*sqrt(b^2*f^3/d^7) + c*d*f +
 a*f^2)/d^4)*log(-(2*sqrt(c*x^2 + b*x + a)*d^5*sqrt(b^2*f^3/d^7)*
sqrt((d^4*sqrt(b^2*f^3/d^7) + c*d*f + a*f^2)/d^4) - 2*b*c*f^2*x -
 b^2*f^2 - (b*d^3*f*x + 2*a*d^3*f)*sqrt(b^2*f^3/d^7))/x) - 2*sqrt
(-a)*a*d^2*x^2*sqrt(-(d^4*sqrt(b^2*f^3/d^7) - c*d*f - a*f^2)/d^4)
*log((2*sqrt(c*x^2 + b*x + a)*d^5*sqrt(b^2*f^3/d^7)*sqrt(-(d^4*sq
rt(b^2*f^3/d^7) - c*d*f - a*f^2)/d^4) + 2*b*c*f^2*x + b^2*f^2 - (
b*d^3*f*x + 2*a*d^3*f)*sqrt(b^2*f^3/d^7))/x) + 2*sqrt(-a)*a*d^2*x
^2*sqrt(-(d^4*sqrt(b^2*f^3/d^7) - c*d*f - a*f^2)/d^4)*log(-(2*sqr
t(c*x^2 + b*x + a)*d^5*sqrt(b^2*f^3/d^7)*sqrt(-(d^4*sqrt(b^2*f^3/
d^7) - c*d*f - a*f^2)/d^4) - 2*b*c*f^2*x - b^2*f^2 + (b*d^3*f*x +
 2*a*d^3*f)*sqrt(b^2*f^3/d^7))/x) - (8*a^2*f - (b^2 - 4*a*c)*d)*x
^2*arctan(1/2*(b*x + 2*a)*sqrt(-a)/(sqrt(c*x^2 + b*x + a)*a)) - 2
*(b*d*x + 2*a*d)*sqrt(c*x^2 + b*x + a)*sqrt(-a))/(sqrt(-a)*a*d^2*
x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{a + bx + cx^2}}{-dx^3 + fx^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)**(1/2)/x**3/(-f*x**2+d),x)
```

```
[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^3),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.84 $\int \frac{x^3\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx$

Optimal. Leaf size=501

$$\frac{3b\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}f}-\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f}-\frac{d\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cf^3}-\frac{bd\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^3}+\frac{b(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{16c^2f}-\frac{d\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{7/2}}+\frac{d\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{7/2}}-\frac{d\left(a+bx+cx^2\right)^{3/2}}{3f^2}-\frac{\left(a+bx+cx^2\right)^{5/2}}{5cf}$$

```
[Out] (-3*b*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(128*c^3*f
) - (d*(8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x
^2])/(8*c*f^3) - (d*(a + b*x + c*x^2)^(3/2))/(3*f^2) + (b*(b + 2*
c*x)*(a + b*x + c*x^2)^(3/2))/(16*c^2*f) - (a + b*x + c*x^2)^(5/2
)/(5*c*f) + (3*b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(256*c^(7/2)*f) - (b*d*(24*c^2*d - b^2*f
+ 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])
])/(16*c^(3/2)*f^3) - (d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Ar
cTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*
Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f
^(7/2)) + (d*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqr
t[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b
*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(7/2))
```

Rubi [A] time = 2.85712, antiderivative size = 501, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\frac{3b\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}f}-\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f}-\frac{d\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cf^3}-\frac{bd\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^3}+\frac{b(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{16c^2f}-\frac{d\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{7/2}}+\frac{d\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{7/2}}-\frac{d\left(a+bx+cx^2\right)^{3/2}}{3f^2}-\frac{\left(a+bx+cx^2\right)^{5/2}}{5cf}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] (-3*b*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(128*c^3*f
) - (d*(8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x
^2])/(8*c*f^3) - (d*(a + b*x + c*x^2)^(3/2))/(3*f^2) + (b*(b + 2*
c*x)*(a + b*x + c*x^2)^(3/2))/(16*c^2*f) - (a + b*x + c*x^2)^(5/2
)/(5*c*f) + (3*b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(256*c^(7/2)*f) - (b*d*(24*c^2*d - b^2*f
+ 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])
])/(16*c^(3/2)*f^3) - (d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Ar
cTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*
Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f
^(7/2)) + (d*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqr
t[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b
*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(7/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.57451, size = 536, normalized size = 1.07

$$-\frac{2\sqrt{f}\sqrt{a+x(b+cx)}\left(24c^2f\left(16a^2f+7abfx+b^2\left(10d+fx^2\right)\right)-30b^2cf^2(10a+bx)+16c^3f\left(160ad+48afx^2+70bdx+33bfx^3\right)+45b^4f^2+128c^4\left(15d^2+5dfx^2+3f^2x^4\right)\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] ((-2*Sqrt[f]*Sqrt[a + x*(b + c*x)]*(45*b^4*f^2 - 30*b^2*c*f^2*(10
*a + b*x) + 16*c^3*f*(160*a*d + 70*b*d*x + 48*a*f*x^2 + 33*b*f*x^
3) + 128*c^4*(15*d^2 + 5*d*f*x^2 + 3*f^2*x^4) + 24*c^2*f*(16*a^2*
f + 7*a*b*f*x + b^2*(10*d + f*x^2))))/c^3 - 1920*d*(c*d + b*Sqrt[
d]*Sqrt[f] + a*f)^(3/2)*Log[Sqrt[d]*Sqrt[f] - f*x] - 1920*d*(c*d
- b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Log[Sqrt[d]*Sqrt[f] + f*x] + (15
*b*Sqrt[f]*(-384*c^4*d^2 - 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c
*f^2 + 16*c^2*f*(b^2*d + 3*a^2*f))*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt
[a + x*(b + c*x)]])/c^(7/2) + 1920*d*(c*d - b*Sqrt[d]*Sqrt[f] + a
*f)^(3/2)*Log[Sqrt[d]*(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x
 + b*Sqrt[f]*x + 2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x
*(b + c*x)])] + 1920*d*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Log[
Sqrt[d]*(b*(Sqrt[d] + Sqrt[f]*x) + 2*(a*Sqrt[f] + c*Sqrt[d]*x + S
qrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]))])/(384
0*f^(7/2))
```

Maple [B] time = 0.04, size = 4884, normalized size = 9.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)

[Out] -1/2/f^2*d*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(
d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*a-1/2/f^3*d^2*(
(x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f
)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*c-1/2/f^2*d*((x-(d*f)^(1/2)
/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+
f*a+c*d)/f)^(1/2)*a-1/2/f^3*d^2*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)
^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*
c+1/16/f*b^2/c^2*(c*x^2+b*x+a)^(3/2)-3/128/f*b^4/c^3*(c*x^2+b*x+a
)^(1/2)+3/256/f*b^5/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))+1/8/f*b/c*(c*x^2+b*x+a)^(3/2)*x+3/16/f*b/c*(c*x^2+b*x+a)^(
1/2)*x*a-1/4/f^3*d*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f
*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*x*c*(d*f)^(1/
2)-3/4/f^3*d*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))
/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^
(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)*a-
3/8/f^2*d/c^(1/2)*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2
)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(
d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))*a*b-3/16/f^3*d*ln
((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d
*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*
f)^(1/2)+f*a+c*d)/f)^(1/2))/c^(1/2)*b^2*(d*f)^(1/2)+1/f^3*d^2/((b
*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2
*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d
)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f
)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a
*c+3/16/f^3*d*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f
))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(
d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/c^(1/2)*b^2*(d
*f)^(1/2)+1/f^3*d^2/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*
(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2
)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*
c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2
)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a*c+1/4/f^3*d*((x+(d*f)^(1/
2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d
*f)^(1/2)+f*a+c*d))^(1/2)*x*c*(d*f)^(1/2)+3/4/f^3*d*ln((1/2/f*(-2
*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/
f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)
^(1/2)+f*a+c*d))^(1/2))*c^(1/2)*(d*f)^(1/2)*a-3/8/f^2*d/c^(1/2)*l
n((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+
(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1
/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))*a*b+3/32/f*b^2/c^2*(c*x^2+b*x
+a)^(1/2)*a+3/16/f*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))*a^2-3/32/f*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))*a-1/8/f^2*d*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f
)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*x*b-5/8/f^
3*d*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)
/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*b*(d*f)^(1/2)-1/16/f^2*d/c*(
(x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(
b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*b^2+1/32/f^2*d/c^(3/2)*ln((1/2*(2
*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2
)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)
+f*a+c*d)/f)^(1/2))*b^3-3/4/f^3*d^2*ln((1/2*(2*c*(d*f)^(1/2)+b*f)
/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)
^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))
*c^(1/2)*b-1/2/f^4*d^2*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)
^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f
*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))*c^(3/2)*(d*f
)^(1/2)+1/2/f^3*d^2/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d
*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*
((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*
f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2
))/(x-(d*f)^(1/2)/f))*b^2+1/2/f^2*d/((b*(d*f)^(1/2)+f*a+c*d)/f)^(
1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(
d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)
/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+
f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a^2+1/2/f^4*d^3/((b*(d*f)^(
1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)
^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/
2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/
```

```
f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c^2-1/8/f
^2*d*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(
1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*x*b+5/8/f^3*d*((x+(d*
f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*
(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*b*(d*f)^(1/2)-1/16/f^2*d/c*((x+(d
*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f
*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*b^2+1/32/f^2*d/c^(3/2)*ln((1/2/f
*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1
/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(
d*f)^(1/2)+f*a+c*d))^(1/2))*b^3-3/4/f^3*d^2*ln((1/2/f*(-2*c*(d*f)
^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1
/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f
*a+c*d))^(1/2))*c^(1/2)*b+1/2/f^4*d^2*ln((1/2/f*(-2*c*(d*f)^(1/2)
+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2
*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d
))^(1/2))*c^(3/2)*(d*f)^(1/2)-1/6/f^2*d*((x+(d*f)^(1/2)/f)^2*c+1/
f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*
a+c*d))^(3/2)-1/6/f^2*d*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b
*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(3/2)-1/f^4*d^
2/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*
a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d
*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^
(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))
/(x+(d*f)^(1/2)/f))*b*(d*f)^(1/2)*c-1/f^3*d/(1/f*(-b*(d*f)^(1/2)+
f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^
(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/
2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/
2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*(
d*f)^(1/2)*a+1/f^3*d/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(
d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2
*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d
*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/
2))/(x-(d*f)^(1/2)/f))*b*(d*f)^(1/2)*a+1/f^4*d^2/((b*(d*f)^(1/2)+
f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2
)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((
x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b
*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*(d*f)^(1/2)*
c-3/64/f*b^3/c^2*(c*x^2+b*x+a)^(1/2)*x+1/2/f^3*d^2/(1/f*(-b*(d*f)
^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c
*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*
d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d
*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/
f))*b^2+1/2/f^2*d/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-
b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/
f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+
1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+
f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a^2+1/2/f^4*d^3/(1/f*(-b*(d*f
)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*
c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c
*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(
d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)
/f))*c^2-1/5*(c*x^2+b*x+a)^(5/2)/c/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^2 + b*x + a)^(3/2)*x^3/(f*x^2 - d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^2 + b*x + a)^(3/2)*x^3/(f*x^2 - d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^2 + b*x + a)^(3/2)*x^3/(f*x^2 - d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.85 $\int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx$

Optimal. Leaf size=417

$$-\frac{\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}f^3}-\frac{\sqrt{a+bx+cx^2}\left(2cx\left(12acf-3b^2f+16c^2d\right)+b\left(12acf-3b^2f+80c^2d\right)\right)}{64c^2f^2}+\frac{\sqrt{d}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^3}+\frac{\sqrt{d}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^3}-\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf}$$

```
[Out] -((b*(80*c^2*d - 3*b^2*f + 12*a*c*f) + 2*c*(16*c^2*d - 3*b^2*f +
12*a*c*f)*x)*Sqrt[a + b*x + c*x^2])/(64*c^2*f^2) - ((b + 2*c*x)*(
a + b*x + c*x^2)^(3/2))/(8*c*f) - ((128*c^4*d^2 + 192*a*c^3*d*f +
 3*b^4*f^2 - 24*a*b^2*c*f^2 + 48*c^2*f*(b^2*d + a^2*f))*ArcTanh[(
b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(5/2)*f^3)
+ (Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[
d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*S
qrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^3) + (Sqrt[d]
*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*S
qrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqr
t[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^3)
```

Rubi [A] time = 2.21485, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}f^3}-\frac{\sqrt{a+bx+cx^2}\left(2cx\left(12acf-3b^2f+16c^2d\right)+b\left(12acf-3b^2f+80c^2d\right)\right)}{64c^2f^2}+\frac{\sqrt{d}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^3}+\frac{\sqrt{d}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^3}-\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] -((b*(80*c^2*d - 3*b^2*f + 12*a*c*f) + 2*c*(16*c^2*d - 3*b^2*f +
12*a*c*f)*x)*Sqrt[a + b*x + c*x^2])/(64*c^2*f^2) - ((b + 2*c*x)*(
a + b*x + c*x^2)^(3/2))/(8*c*f) - ((128*c^4*d^2 + 192*a*c^3*d*f +
 3*b^4*f^2 - 24*a*b^2*c*f^2 + 48*c^2*f*(b^2*d + a^2*f))*ArcTanh[(
b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(5/2)*f^3)
+ (Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[
d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*S
qrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^3) + (Sqrt[d]
*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*S
qrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqr
```

```
t[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.26291, size = 482, normalized size = 1.16

$$-\frac{\frac{\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)}{c^{5/2}}+\frac{2f\sqrt{a+x(b+cx)}\left(4bc\left(5af+20cd+6cfx^2\right)+8c^2x\left(5af+4cd+2cfx\right.\right.}{c^2}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] -((2*f*Sqrt[a + x*(b + c*x)]*(-3*b^3*f + 2*b^2*c*f*x + 8*c^2*x*(4
*c*d + 5*a*f + 2*c*f*x^2) + 4*b*c*(20*c*d + 5*a*f + 6*c*f*x^2)))/
c^2 + 64*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Log[Sqrt[d
]*Sqrt[f] - f*x] - 64*Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/
2)*Log[Sqrt[d]*Sqrt[f] + f*x] + ((128*c^4*d^2 + 192*a*c^3*d*f + 3
*b^4*f^2 - 24*a*b^2*c*f^2 + 48*c^2*f*(b^2*d + a^2*f))*Log[b + 2*c
*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/c^(5/2) + 64*Sqrt[d]*(c*d
- b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Log[Sqrt[d]*(-(b*Sqrt[d]) + 2*a*
Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x + 2*Sqrt[c*d - b*Sqrt[d]*Sq
rt[f] + a*f]*Sqrt[a + x*(b + c*x)])] - 64*Sqrt[d]*(c*d + b*Sqrt[d
]*Sqrt[f] + a*f)^(3/2)*Log[Sqrt[d]*(b*(Sqrt[d] + Sqrt[f]*x) + 2*(
a*Sqrt[f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqr
t[a + x*(b + c*x)]))])/(128*f^3)
```

Maple [B] time = 0.025, size = 4900, normalized size = 11.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/6*d/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+
b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(3/2)-5/8*d/
f^2*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1
/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*b-1/2*d^2/f^3*ln((1/2/
f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(
1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*
(d*f)^(1/2)+f*a+c*d))^(1/2))*c^(3/2)-5/8*d/f^2*((x-(d*f)^(1/2)/f)
^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a
+c*d)/f)^(1/2)*b-1/2*d^2/f^3*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x
-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+
b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))*c^(3/2
)-1/6*d/(d*f)^(1/2)/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f
)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(3/2)-1/8/f/c*(c
```

```
*x^2+b*x+a)^(3/2)*b-3/8/f*(c*x^2+b*x+a)^(1/2)*x*a+3/64/f/c^2*(c*x
^2+b*x+a)^(1/2)*b^3-3/8/f/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))*a^2-3/128/f/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))*b^4+3/16/f/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))*b^2*a-1/4*d/f^2*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)
+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*x*c-3/
4*d/f^2*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1
/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)
/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))*c^(1/2)*a-3/16*d/f^2*ln((1/
2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^
(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(
1/2)+f*a+c*d)/f)^(1/2))/c^(1/2)*b^2-1/4*d/f^2*((x+(d*f)^(1/2)/f)^
2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1
/2)+f*a+c*d))^(1/2)*x*c-3/4*d/f^2*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f
)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(
d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(
1/2))*c^(1/2)*a-3/16*d/f^2*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+
(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1
/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/c
^(1/2)*b^2+1/2*d/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(
d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(
1/2)*a+1/2*d^2/(d*f)^(1/2)/f^2*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(
d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(
1/2)*c-1/2*d/(d*f)^(1/2)/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2
)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*a-1/2
*d^2/(d*f)^(1/2)/f^2*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)
/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*c-1/4/f*x*(
c*x^2+b*x+a)^(3/2)-d^2/(d*f)^(1/2)/f^2/(1/f*(-b*(d*f)^(1/2)+f*a+c
*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)
+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((
x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)
+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a*c-3/8*
d/(d*f)^(1/2)/f/c^(1/2)*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f
)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/
f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))*a*b+d^2/(d*
f)^(1/2)/f^2/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/
2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*
f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2
)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(
d*f)^(1/2)/f))*a*c+3/8*d/(d*f)^(1/2)/f/c^(1/2)*ln((1/2/f*(-2*c*(d
*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*
c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2
)+f*a+c*d))^(1/2))*a*b+3/32/f/c*(c*x^2+b*x+a)^(1/2)*x*b^2-3/16/f/
c*(c*x^2+b*x+a)^(1/2)*b*a+1/32*d/(d*f)^(1/2)/f/c^(3/2)*ln((1/2*(2
*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2
)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)
+f*a+c*d)/f)^(1/2))*b^3-3/4*d^2/(d*f)^(1/2)/f^2*ln((1/2*(2*c*(d*f
)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*
c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*
d)/f)^(1/2))*c^(1/2)*b+1/2*d^2/(d*f)^(1/2)/f^2/((b*(d*f)^(1/2)+f*
a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+
b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-
(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(
d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b^2+d/f^2/((b*(d
*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*
(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f
)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(
1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*a+
d^2/f^3/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*
a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1
/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f
)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^
(1/2)/f))*b*c+1/2*d/(d*f)^(1/2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/
2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*
f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f
)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*
a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a^2+1/2*d^3/(d*f)^(1/2)/f^3/(
(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+
(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c
*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d
*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))
*c^2-1/16*d/(d*f)^(1/2)/f/c*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/
```

```
2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*b^2+
1/8*d/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+
b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*x*b+1/
16*d/(d*f)^(1/2)/f/c*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)
+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*b^2-1
/32*d/(d*f)^(1/2)/f/c^(3/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x
+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(
1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))*
b^3+3/4*d^2/(d*f)^(1/2)/f^2*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x
+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(
1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))*
c^(1/2)*b-1/2*d^2/(d*f)^(1/2)/f^2/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^
(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)
*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*
f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*
(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b^2+d/f^2/(1/
f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d
)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(
1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)
+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(
d*f)^(1/2)/f))*b*a+d^2/f^3/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*l
n((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*
f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2
)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*
f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*c-1/2*d/(d*f)^(1/2
)/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+
f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*
(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f
)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2
))/(x+(d*f)^(1/2)/f))*a^2-1/2*d^3/(d*f)^(1/2)/f^3/(1/f*(-b*(d*f)^
(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*
(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d
))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*
f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f
))*c^2-1/8*d/(d*f)^(1/2)/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2
)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*x*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^2 + b*x + a)^(3/2)*x^2/(f*x^2 - d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^2 + b*x + a)^(3/2)*x^2/(f*x^2 - d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{bx^3\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{cx^4\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(a*x**2*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integ
ral(b*x**3*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*
x**4*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c*x^2 + b*x + a)^(3/2)*x^2/(f*x^2 - d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.86 $\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2} dx$

Optimal. Leaf size=349

$$-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cf^2}-\frac{b\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^2}-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{5/2}}+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{5/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}$$

```
[Out] -((8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/
(8*c*f^2) - (a + b*x + c*x^2)^(3/2)/(3*f) - (b*(24*c^2*d - b^2*f
+ 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])
])/(16*c^(3/2)*f^2) - ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcT
anh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(
5/2)) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d]
 + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(5/2))
```

Rubi [A] time = 1.34543, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$

$$-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cf^2}-\frac{b\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^2}-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{5/2}}+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{5/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```


```
[Out] -((8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/
(8*c*f^2) - (a + b*x + c*x^2)^(3/2)/(3*f) - (b*(24*c^2*d - b^2*f
+ 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])
])/(16*c^(3/2)*f^2) - ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcT
anh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(
5/2)) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d]
 + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(5/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.969773, size = 416, normalized size = 1.19

$$-\frac{2\sqrt{f}\sqrt{a+x(b+cx)}\left(2cf(16a+7bx)+3b^2f+8c^2\left(3d+fx^2\right)\right)}{c}+\frac{3b\sqrt{f}\left(-12acf+b^2f-24c^2d\right)\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)}{c^{3/2}}-24\log\left(\sqrt{d}\sqrt{f}+fx\right)\left(af\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] ((-2*Sqrt[f]*Sqrt[a + x*(b + c*x)]*(3*b^2*f + 2*c*f*(16*a + 7*b*x
) + 8*c^2*(3*d + f*x^2)))/c - 24*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^
(3/2)*Log[Sqrt[d]*Sqrt[f] - f*x] - 24*(c*d - b*Sqrt[d]*Sqrt[f] +
a*f)^(3/2)*Log[Sqrt[d]*Sqrt[f] + f*x] + (3*b*Sqrt[f]*(-24*c^2*d +
 b^2*f - 12*a*c*f)*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)
]])/c^(3/2) + 24*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Log[Sqrt[d
]*(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x + 2*S
qrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + 24*(
c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Log[Sqrt[d]*(b*(Sqrt[d] + Sq
rt[f]*x) + 2*(a*Sqrt[f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + x*(b + c*x)]))])/(48*f^(5/2))
```

Maple [B] time = 0.022, size = 4567, normalized size = 13.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/32/f/c^(3/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/
f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+
(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))*b^3-1/2/f^2*(
(x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f
)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*c*d+1/2/f/(1/f*(-b*(d*f)^(1
/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d
*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))
^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))
*a^2-1/16/f/c*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(
x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*b^2-1/8/f*((
x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)
+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*x*b+5/8/f^2*((x+(d*f)^(1/2)/
f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)
^(1/2)+f*a+c*d))^(1/2)*b*(d*f)^(1/2)-1/2/f^2*((x-(d*f)^(1/2)/f)^2
*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c
*d)/f)^(1/2)*c*d+1/2/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b
*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)
+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*
(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(
```

```
1/2))/(x-(d*f)^(1/2)/f))*a^2-5/8/f^2*((x-(d*f)^(1/2)/f)^2*c+(2*c*
(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(
1/2)*b*(d*f)^(1/2)-1/16/f/c*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*b^2+
1/32/f/c^(3/2)*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f
))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f
)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))*b^3-1/8/f*((x-(d*f)^
(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(
1/2)+f*a+c*d)/f)^(1/2)*x*b-1/6/f*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c
*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))
^(3/2)-1/6/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d
*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(3/2)-3/16/f^2*ln((1/2*(2
*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2
)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)
+f*a+c*d)/f)^(1/2))/c^(1/2)*b^2*(d*f)^(1/2)-3/4/f^2*ln((1/2*(2*c*
(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f
)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*
a+c*d)/f)^(1/2))*c^(1/2)*d*b-1/2/f^3*ln((1/2*(2*c*(d*f)^(1/2)+b*f
)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)
)*c^(3/2)*(d*f)^(1/2)*d+1/2/f^2/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)
*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)
^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^
2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+
c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b^2*d+1/2/f^3/((b*(d*f)^(1/2)+f
*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)
+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x
-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*
(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c^2*d^2+1/4/f^2
*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)
/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*x*c*(d*f)^(1/2)+3/4/f^2*l
n((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+
(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1
/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))*c^(1/2)*(d*f)^(1/2)*a-3/8/f/c
^(1/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1
/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1
/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))*a*b+3/16/f^2*ln((1/2/
f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(
1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*
(d*f)^(1/2)+f*a+c*d))^(1/2))/c^(1/2)*b^2*(d*f)^(1/2)-3/4/f^2*ln((
1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*
f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*
(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))*c^(1/2)*d*b+1/2/f^3*ln((1/2/f*(-
2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)
/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f
)^(1/2)+f*a+c*d))^(1/2))*c^(3/2)*(d*f)^(1/2)*d+1/2/f^2/(1/f*(-b*(
d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(
-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*
a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(
x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1
/2)/f))*b^2*d+1/2/f^3/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/
f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1
/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^
2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1
/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c^2*d^2+1/f^2/((b*(d*f)^(
1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)
^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/
2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/
f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a*c*d+1/f
^2/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d
)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f
*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(
x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)
/f))*b*(d*f)^(1/2)*a-1/f^2/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*l
n((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*
f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2
)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*
f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*(d*f)^(1/2)*a+1/f^
2/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*
a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d
*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^
(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))
```

```
/(x+(d*f)^(1/2)/f))*a*c*d-1/2/f*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*
(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^
(1/2)*a-1/2/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(
d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*a-3/8/f/c^(1/2)*ln
((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d
*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*
f)^(1/2)+f*a+c*d)/f)^(1/2))*a*b-1/4/f^2*((x-(d*f)^(1/2)/f)^2*c+(2
*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f
)^(1/2)*x*c*(d*f)^(1/2)-3/4/f^2*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c
*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))*c^(
1/2)*(d*f)^(1/2)*a-1/f^3/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln(
(2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/
f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)
^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*(d*f)^(1/2)*c*d+1/f^
3/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)
/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*
a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x
-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/
f))*b*(d*f)^(1/2)*c*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^2 + b*x + a)^(3/2)*x/(f*x^2 - d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^2 + b*x + a)^(3/2)*x/(f*x^2 - d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx - \int \frac{bx^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx - \int \frac{cx^3\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(a*x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral
(b*x**2*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**
3*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^2 + b*x + a)^(3/2)*x/(f*x^2 - d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.87 $\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx$

Optimal. Leaf size=315

$$-\frac{\left(12acf+3b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}f^2}+\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}f^2}+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}f^2}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f}$$

```
[Out] -((5*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*f) - ((8*c^2*d + 3*b^2*
f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])])/(8*Sqrt[c]*f^2) + ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Arc
Tanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*S
qrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sq
rt[d]*f^2) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sq
rt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d +
b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*f^2)
```

Rubi [A] time = 1.15618, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$-\frac{\left(12acf+3b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}f^2}+\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}f^2}+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}f^2}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + c*x^2)^(3/2)/(d - f*x^2),x]
```


```
[Out] -((5*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*f) - ((8*c^2*d + 3*b^2*
f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])])/(8*Sqrt[c]*f^2) + ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Arc
Tanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*S
qrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sq
rt[d]*f^2) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sq
rt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d +
b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*f^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Mathematica [A]   time = 1.10664, size = 396, normalized size = 1.26

$$-\frac{\frac{(12acf+3b^2f+8c^2d)\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)}{\sqrt{c}}-\frac{4\log\left(\sqrt{d}\sqrt{f}+fx\right)\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}{\sqrt{d}}+\frac{4\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\log\left(\sqrt{d}\left(2\sqrt{a+x(b+cx)}\sqrt{a\ldots}\right.\right.}{\sqrt{d}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x + c*x^2)^(3/2)/(d - f*x^2),x]
```

```
[Out] -(2*f*(5*b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (4*(c*d + b*Sqrt[d]*S
qrt[f] + a*f)^(3/2)*Log[Sqrt[d]*Sqrt[f] - f*x])/Sqrt[d] - (4*(c*d
 - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Log[Sqrt[d]*Sqrt[f] + f*x])/Sqr
t[d] + ((8*c^2*d + 3*b^2*f + 12*a*c*f)*Log[b + 2*c*x + 2*Sqrt[c]*
Sqrt[a + x*(b + c*x)]])/Sqrt[c] + (4*(c*d - b*Sqrt[d]*Sqrt[f] + a
*f)^(3/2)*Log[Sqrt[d]*(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x
 + b*Sqrt[f]*x + 2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x
*(b + c*x)])])/Sqrt[d] - (4*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)
*Log[Sqrt[d]*(b*(Sqrt[d] + Sqrt[f]*x) + 2*(a*Sqrt[f] + c*Sqrt[d]*
x + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]))])
/Sqrt[d])/(8*f^2)
```

Maple [B]   time = 0.022, size = 4574, normalized size = 14.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/f^2/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+
c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2
)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/
f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1
/2)/f))*b*c*d+1/2/(d*f)^(1/2)/f^2/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/
2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*
f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f
)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*
a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c^2*d^2-3/4/(d*f)^(1/2)/f*ln(
(1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*
f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f
)^(1/2)+f*a+c*d)/f)^(1/2))*c^(1/2)*d*b+1/2/(d*f)^(1/2)/f/((b*(d*f
)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d
*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^
(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/
2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b^2*d+
3/4/(d*f)^(1/2)/f*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/
2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*
(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))*c^(1/2)*d*
b-1/2/(d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*
(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2
)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*
c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2
)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b^2*d+1/f^2/(1/f*(-b*(d*f)^
(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*
(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d
))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*
```

f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f
))*b*c*d-1/2/(d*f)^(1/2)/f^2/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)
ln((2/f(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(
d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1
/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(
d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c^2*d^2-3/4/f*ln((
1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*
f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*
(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))*c^(1/2)*a-3/16/f*ln((1/2/f*(-2*c
*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)
^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(
1/2)+f*a+c*d))^(1/2))/c^(1/2)*b^2-1/32/(d*f)^(1/2)/c^(3/2)*ln((1/
2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)
^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-
b*(d*f)^(1/2)+f*a+c*d))^(1/2))*b^3-1/4/f*((x+(d*f)^(1/2)/f)^2*c+1
/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f
*a+c*d))^(1/2)*x*c-1/2/f^2*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(
d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*
f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))*c^(3/2)*
d+1/2/(d*f)^(1/2)/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f
)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((
b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)
^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))
/(x-(d*f)^(1/2)/f))*a^2-1/4/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(
1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*x*
c-1/8/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*
(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*x*b-1/16/(d*f)
^(1/2)/c*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^
(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*b^2-3/4/f*ln((1/2*(2*c*
(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f
)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*
a+c*d)/f)^(1/2))*c^(1/2)*a-3/16/f*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f
+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(
1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/c
^(1/2)*b^2+1/32/(d*f)^(1/2)/c^(3/2)*ln((1/2*(2*c*(d*f)^(1/2)+b*f)
/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)
^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))
*b^3-1/2/f^2*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f)
)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d
f)^(1/2)/f)+1/f(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))*c^(3/2)*d-1/2/(
d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)
^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1
/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2
c(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d
))^(1/2))/(x+(d*f)^(1/2)/f))*a^2+1/8/(d*f)^(1/2)*((x+(d*f)^(1/2)/
f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)
^(1/2)+f*a+c*d))^(1/2)*x*b+1/16/(d*f)^(1/2)/c*((x+(d*f)^(1/2)/f)^
2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1
/2)+f*a+c*d))^(1/2)*b^2+1/6/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/
f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*
a+c*d))^(3/2)-1/6/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(
1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(3/2)+3/
8/(d*f)^(1/2)/c^(1/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)
^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b
f)(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))*a*b+1/
2/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)
*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*c*d+1/f/(1
/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*
d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^
(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2
)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+
(d*f)^(1/2)/f))*b*a+1/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(
b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f
)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c
*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^
(1/2))/(x-(d*f)^(1/2)/f))*b*a+1/2/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^
2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1
/2)+f*a+c*d))^(1/2)*a-1/2/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c
*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^
(1/2)*a-5/8/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(
d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*b-5/8/f*((x+(d*f)^
(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b

```
*(d*f)^(1/2)+f*a+c*d))^(1/2)*b-3/8/(d*f)^(1/2)/c^(1/2)*ln((1/2*(2
*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2
)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)
+f*a+c*d)/f)^(1/2))*a*b-1/2/(d*f)^(1/2)/f*((x-(d*f)^(1/2)/f)^2*c+
(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)
/f)^(1/2)*c*d-1/(d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2
)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+
(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(
1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*
(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a*c*d+1/(d*f)^(1/
2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c
*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)
+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f
*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/
2)/f))*a*c*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^2 + b*x + a)^(3/2)/(f*x^2 - d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^2 + b*x + a)^(3/2)/(f*x^2 - d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{bx\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{cx^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b
*x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**2*sqr
t(a + b*x + c*x**2)/(-d + f*x**2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c*x^2 + b*x + a)^(3/2)/(f*x^2 - d),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.88 $\int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)}\,dx$

Optimal. Leaf size=469

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d}-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cdf}-\frac{b\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}df}+\frac{\left(8ac+b^2+2bcx\right)\sqrt{a+bx+cx^2}}{8cd}-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2df^{3/2}}+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2df^{3/2}}$$

```
[Out] ((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a + b*x + c*x^2])/(8*c*d) - ((8*c^2
*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*d*f
) - (a^(3/2)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2]
)])/d - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])])/(16*c^(3/2)*d) - (b*(24*c^2*d - b^2*f + 12*a*c*f
)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(
3/2)*d*f) - ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqr
t[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b
*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*f^(3/2)) +
((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*S
qrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqr
t[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*f^(3/2))
```

Rubi [A] time = 2.77797, antiderivative size = 469, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d}-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cdf}-\frac{b\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}df}+\frac{\left(8ac+b^2+2bcx\right)\sqrt{a+bx+cx^2}}{8cd}-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2df^{3/2}}+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2df^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)),x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a + b*x + c*x^2])/(8*c*d) - ((8*c^2
*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*d*f
) - (a^(3/2)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2]
```

```
)])/d - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])])/(16*c^(3/2)*d) - (b*(24*c^2*d - b^2*f + 12*a*c*f
)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(
3/2)*d*f) - ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqr
t[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b
*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*f^(3/2)) +
((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*S
qrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqr
t[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*f^(3/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)**(3/2)/x/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.865798, size = 443, normalized size = 0.94

$$-\frac{x\left(fx^2-d\right)\left(2a^{3/2}f^{3/2}\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+bx\right)-2a^{3/2}f^{3/2}\log(x)+2cd\sqrt{f}\sqrt{a+x(b+cx)}+3b\sqrt{c}d\sqrt{f}\log\left(2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)),x]
```

```
[Out] -(x*(-d + f*x^2)*(2*c*d*Sqrt[f]*Sqrt[a + x*(b + c*x)] - 2*a^(3/2)
*f^(3/2)*Log[x] + (c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Log[Sqrt[
d]*Sqrt[f] - f*x] + (c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Log[Sqr
t[d]*Sqrt[f] + f*x] + 2*a^(3/2)*f^(3/2)*Log[2*a + b*x + 2*Sqrt[a]
*Sqrt[a + x*(b + c*x)]] + 3*b*Sqrt[c]*d*Sqrt[f]*Log[b + 2*c*x + 2
*Sqrt[c]*Sqrt[a + x*(b + c*x)]] - (c*d - b*Sqrt[d]*Sqrt[f] + a*f)
^(3/2)*Log[Sqrt[d]*(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x +
b*Sqrt[f]*x + 2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b
 + c*x)])] - (c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Log[Sqrt[d]*(b
*(Sqrt[d] + Sqrt[f]*x) + 2*(a*Sqrt[f] + c*Sqrt[d]*x + Sqrt[c*d +
b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]))]))/(2*d*f^(3/2)*
(-(d*x) + f*x^3))
```

Maple [B] time = 0.026, size = 4765, normalized size = 10.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x)
```

```
[Out] -1/8/d*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*x*b-1/16/d/c*((x+(d
*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f
*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*b^2+1/32/d/c^(3/2)*ln((1/2/f*(-2
*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/
f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)
```

```
^(1/2)+f*a+c*d))^(1/2))*b^3-3/4/f*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f
)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(
d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(
1/2))*c^(1/2)*b+1/2/f^2*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*
f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)
+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))*c^(3
/2)*(d*f)^(1/2)+1/2/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/
f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1
/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^
2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1
/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b^2+1/2/d/(1/f*(-b*(d*f)^
(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*
(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d
))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*
f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f
))*a^2-1/8/d*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d
*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*x*b-1/16/d/c*((x-(d
*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*
f)^(1/2)+f*a+c*d)/f)^(1/2)*b^2+1/32/d/c^(3/2)*ln((1/2*(2*c*(d*f)^
(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+
(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)
/f)^(1/2))*b^3-3/4/f*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(
1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(
x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))*c^(1/2)*b-1/2/
f^2*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+
((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+
(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))*c^(3/2)*(d*f)^(1/2)+1/2/f/((b*(
d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c
*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/
f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^
(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b^2
+1/2/d/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a
+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/
2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)
/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(
1/2)/f))*a^2-1/16/d/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))*b^3+1/4/d*b*(c*x^2+b*x+a)^(1/2)*x+1/8/d/c*(c*x^2+b*x+a)^(1
/2)*b^2-1/6/d*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(
x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(3/2)-1/6/d*((x-(d
*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*
f)^(1/2)+f*a+c*d)/f)^(3/2)+1/3/d*(c*x^2+b*x+a)^(3/2)-3/8/d/c^(1/2
)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+(
(x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f
)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))*a*b+1/4/d/f*((x+(d*f)^(1/2
)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*
f)^(1/2)+f*a+c*d))^(1/2)*x*c*(d*f)^(1/2)+3/4/d/f*ln((1/2/f*(-2*c*
(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^
2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1
/2)+f*a+c*d))^(1/2))*c^(1/2)*(d*f)^(1/2)*a+3/16/d/f*ln((1/2/f*(-2
*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/
f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)
^(1/2)+f*a+c*d))^(1/2))/c^(1/2)*b^2*(d*f)^(1/2)-1/f^2/(1/f*(-b*(d
*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-
2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a
+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x
+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/
2)/f))*b*(d*f)^(1/2)*c-1/4/d/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^
(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*x
*c*(d*f)^(1/2)-3/4/d/f*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)
^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f
*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))*c^(1/2)*(d*f
)^(1/2)*a-3/16/d/f*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/
2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-
(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/c^(1/2)*b^2*(d*f
)^(1/2)+1/f^2/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1
/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d
*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-
(d*f)^(1/2)/f))*b*(d*f)^(1/2)*c-1/2/f*((x-(d*f)^(1/2)/f)^2*c+(2*c
*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^
(1/2)*c+1/d*a*(c*x^2+b*x+a)^(1/2)-1/d*a^(3/2)*ln((2*a+b*x+2*a^(1/
2)*(c*x^2+b*x+a)^(1/2))/x)-1/2/d*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f
```

```
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)
*a-1/2/d*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*
f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*a-1/2/f*((x+(d*f)
^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-
b*(d*f)^(1/2)+f*a+c*d))^(1/2)*c+3/4/d*b/c^(1/2)*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(
1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*
(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f
)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(
-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a*c+1/2*d/f^2/
(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+
c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f
)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1
/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(
x+(d*f)^(1/2)/f))*c^2+1/2*d/f^2/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)
*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)
^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^
2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+
c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c^2-5/8/d/f*((x-(d*f)^(1/2)/f)^
2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+
c*d)/f)^(1/2)*b*(d*f)^(1/2)-3/8/d/c^(1/2)*ln((1/2*(2*c*(d*f)^(1/2
)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c
*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^
(1/2))*a*b+1/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(
1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(
d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1
/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x
-(d*f)^(1/2)/f))*a*c+5/8/d/f*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*
f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/
2)*b*(d*f)^(1/2)-1/d/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2
/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(
1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)
^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(
1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*(d*f)^(1/2)*a+1/d/f/((
b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(
2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*
d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*
f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*
b*(d*f)^(1/2)*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{(fx^2 - d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a\sqrt{a+bx+cx^2}}{-dx+fx^3}\,dx-\int \frac{bx\sqrt{a+bx+cx^2}}{-dx+fx^3}\,dx-\int \frac{cx^2\sqrt{a+bx+cx^2}}{-dx+fx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)**(3/2)/x/(-f*x**2+d),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x) - Integral
(b*x*sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x) - Integral(c*x**2
*sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x),x, algorithm="giac")
```

```
[Out] Timed out
```

3.89 $\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)}\,dx$

Optimal. Leaf size=463

$$-\frac{(12acf+3b^2f+8c^2d)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}df}+\frac{3(4ac+b^2)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}d}+\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}f}+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}f}-\frac{(a+bx+cx^2)^{3/2}}{dx}+\frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{3\sqrt{a}b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2d}$$

```
[Out] (3*(3*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d) - ((5*b + 2*c*x)*Sq
rt[a + b*x + c*x^2])/(4*d) - (a + b*x + c*x^2)^(3/2)/(d*x) - (3*S
qrt[a]*b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/
(2*d) + (3*(b^2 + 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
b*x + c*x^2])])/(8*Sqrt[c]*d) - ((8*c^2*d + 3*b^2*f + 12*a*c*f)*A
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]
*d*f) + ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d]
 - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*f) + ((c*
d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[
f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f]
 + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*f)
```

Rubi [A] time = 2.60011, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$-\frac{(12acf+3b^2f+8c^2d)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}df}+\frac{3(4ac+b^2)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}d}+\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}f}+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}f}-\frac{(a+bx+cx^2)^{3/2}}{dx}+\frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{3\sqrt{a}b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)),x]
```


```
[Out] (3*(3*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d) - ((5*b + 2*c*x)*Sq
rt[a + b*x + c*x^2])/(4*d) - (a + b*x + c*x^2)^(3/2)/(d*x) - (3*S
qrt[a]*b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/
(2*d) + (3*(b^2 + 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
b*x + c*x^2])])/(8*Sqrt[c]*d) - ((8*c^2*d + 3*b^2*f + 12*a*c*f)*A
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]
*d*f) + ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d]
 - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqr
```

```
t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*f) + ((c*
d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[
f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f]
 + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*f)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)**(3/2)/x**2/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.779554, size = 433, normalized size = 0.94

$$-\frac{2c^{3/2}d^{3/2}x\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)+2a\sqrt{d}f\sqrt{a+x(b+cx)}+3\sqrt{a}b\sqrt{d}fx\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+bx\right)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)),x]
```

```
[Out] -(2*a*Sqrt[d]*f*Sqrt[a + x*(b + c*x)] - 3*Sqrt[a]*b*Sqrt[d]*f*x*L
og[x] + (c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*x*Log[Sqrt[d]*Sqrt[
f] - f*x] - (c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*x*Log[Sqrt[d]*S
qrt[f] + f*x] + 3*Sqrt[a]*b*Sqrt[d]*f*x*Log[2*a + b*x + 2*Sqrt[a]
*Sqrt[a + x*(b + c*x)]] + 2*c^(3/2)*d^(3/2)*x*Log[b + 2*c*x + 2*S
qrt[c]*Sqrt[a + x*(b + c*x)]] + (c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(
3/2)*x*Log[Sqrt[d]*(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x +
b*Sqrt[f]*x + 2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b
 + c*x)])] - (c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*x*Log[Sqrt[d]*
(b*(Sqrt[d] + Sqrt[f]*x) + 2*(a*Sqrt[f] + c*Sqrt[d]*x + Sqrt[c*d
+ b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]))])/(2*d^(3/2)*f
*x)
```

Maple [B] time = 0.027, size = 4799, normalized size = 10.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x)
```

```
[Out] 1/6*f/d/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+
b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(3/2)-1/4/d*
((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/
f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*x*c-3/4/d*ln((1/2/f*(-2*c*
(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^
2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1
/2)+f*a+c*d))^(1/2))*c^(1/2)*a-3/16/d*ln((1/2/f*(-2*c*(d*f)^(1/2)
+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2
*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d
))^(1/2))/c^(1/2)*b^2+3/4/(d*f)^(1/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)
+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2
```

```
*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d
))^(1/2))*c^(1/2)*b-1/2/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d)
)^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*
f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(
d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/
f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b^2-1/6*f/d
/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d
*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(3/2)-1/4/d*((x-(d*f)^(1/
2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2
)+f*a+c*d)/f)^(1/2)*x*c-3/4/d*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(
x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)
+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))*c^(1/
2)*a-3/16/d*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/
c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(
1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/c^(1/2)*b^2-3/4/(d*f)^(
1/2)*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)
+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)
+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))*c^(1/2)*b+1/2/(d*f)^(1/2)/((b*
(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*
c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)
/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)
^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b^
2+3/2/d*c^(1/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/d
/a/x*(c*x^2+b*x+a)^(5/2)+1/d*b/a*(c*x^2+b*x+a)^(3/2)+3/8/d*b^2/c^
(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3/2/d*b*a^(1/2)
*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+3/2/d*c*(c*x^2+b*x
+a)^(1/2)*x+1/2*f/d/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*
(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^
(1/2)*a-5/8/d*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(
x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*b+1/2/(d*f)^
(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^
(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*c-1/2/f*ln((1/2/f*(-
2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)
/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f
)^(1/2)+f*a+c*d))^(1/2))*c^(3/2)-1/2/(d*f)^(1/2)*((x-(d*f)^(1/2)/
f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f
*a+c*d)/f)^(1/2)*c-1/2/f*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*
f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)
/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))*c^(3/2)+1/
d/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)
/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*
a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x
-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/
f))*b*a+1/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2
)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f
)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)
+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d
*f)^(1/2)/f))*b*c+1/(d*f)^(1/2)/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)
*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)
^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^
2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+
c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a*c+1/d/(1/f*(-b*(d*f)^(1/2)+f*
a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1
/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)
*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)
/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*a+1
/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f
*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(
d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)
^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)
)/(x+(d*f)^(1/2)/f))*b*c-1/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+f*a+c
*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)
+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((
x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)
+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a*c+1/d*
c/a*(c*x^2+b*x+a)^(3/2)*x-3/8*f/d/(d*f)^(1/2)/c^(1/2)*ln((1/2*(2*
c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)
/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+
f*a+c*d)/f)^(1/2))*a*b+3/8*f/d/(d*f)^(1/2)/c^(1/2)*ln((1/2/f*(-2*
c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f
)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^
(1/2)+f*a+c*d))^(1/2))*a*b-5/8/d*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f
```

```
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)
*b+9/4/d*b*(c*x^2+b*x+a)^(1/2)-1/2*f/d/(d*f)^(1/2)*((x-(d*f)^(1/2
)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)
+f*a+c*d)/f)^(1/2)*a+1/2/f*d/(d*f)^(1/2)/((b*(d*f)^(1/2)+f*a+c*d)
/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f
*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^
(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(
1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c^2-1/8*f/d/(d*f)^(1/2
)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f
)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*x*b+1/8*f/d/(d*f)^(1/2)*((x+(d
*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f
*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*x*b+1/16*f/d/(d*f)^(1/2)/c*((x+(
d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/
f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*b^2-1/32*f/d/(d*f)^(1/2)/c^(3/2
)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+(
(x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f
)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))*b^3-1/2*f/d/(d*f)^(1/2)/(1
/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*
d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^
(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2
)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+
(d*f)^(1/2)/f))*a^2-1/2/f*d/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+f*a+
c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2
)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*(
(x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f
)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c^2+1/2
*f/d/(d*f)^(1/2)/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)
^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b
*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^
(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/
(x-(d*f)^(1/2)/f))*a^2-1/16*f/d/(d*f)^(1/2)/c*((x-(d*f)^(1/2)/f)^
2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+
c*d)/f)^(1/2)*b^2+1/32*f/d/(d*f)^(1/2)/c^(3/2)*ln((1/2*(2*c*(d*f)
^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c
+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d
)/f)^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(cx^2+bx+a\right)^{\frac{3}{2}}}{(fx^2-d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^2),x, algorithm="maxima")

[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx-\int \frac{bx\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx-\int \frac{cx^2\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)**(3/2)/x**2/(-f*x**2+d),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x) - Integ
ral(b*x*sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x) - Integral(
c*x**2*sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.90 $\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)}\,dx$

Optimal. Leaf size=614

$$-\frac{a^{3/2}f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{bf\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2}-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cd^2}-\frac{b\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2}+\frac{f\left(8ac+b^2+2bcx\right)\sqrt{a+bx+cx^2}}{8cd^2}-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}d}-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^2\sqrt{f}}+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^2\sqrt{f}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{2dx^2}-\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx}+\frac{3b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2d}$$

```
[Out] (-3*(b - 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d*x) + (f*(b^2 + 8*a*c
+ 2*b*c*x)*Sqrt[a + b*x + c*x^2])/(8*c*d^2) - ((8*c^2*d + b^2*f +
 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*d^2) - (a + b*x
 + c*x^2)^(3/2)/(2*d*x^2) - (3*(b^2 + 4*a*c)*ArcTanh[(2*a + b*x)/
(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[a]*d) - (a^(3/2)*f*Ar
cTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d^2 + (3*b*
Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
2*d) - (b*(b^2 - 12*a*c)*f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ b*x + c*x^2])])/(16*c^(3/2)*d^2) - (b*(24*c^2*d - b^2*f + 12*a*
c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*
c^(3/2)*d^2) - ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*
Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[f
]) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] +
2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[f])
```

Rubi [A] time = 3.19505, antiderivative size = 614, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464

$$-\frac{a^{3/2} f \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{bf\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2}-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cd^2}-\frac{b\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2}+\frac{f\left(8ac+b^2+2bcx\right)\sqrt{a+bx+cx^2}}{8cd^2}-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}d}-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^2\sqrt{f}}+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^2\sqrt{f}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{2dx^2}-\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx}+\frac{3b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)),x]
```

```
[Out] (-3*(b - 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d*x) + (f*(b^2 + 8*a*c
+ 2*b*c*x)*Sqrt[a + b*x + c*x^2])/(8*c*d^2) - ((8*c^2*d + b^2*f +
 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*d^2) - (a + b*x
 + c*x^2)^(3/2)/(2*d*x^2) - (3*(b^2 + 4*a*c)*ArcTanh[(2*a + b*x)/
(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[a]*d) - (a^(3/2)*f*Ar
cTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d^2 + (3*b*
Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
2*d) - (b*(b^2 - 12*a*c)*f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ b*x + c*x^2])])/(16*c^(3/2)*d^2) - (b*(24*c^2*d - b^2*f + 12*a*
c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*
c^(3/2)*d^2) - ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*
Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[f
]) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] +
2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[f])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)**(3/2)/x**3/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.97355, size = 429, normalized size = 0.7

$$\frac{\frac{\log(x)\left(4a(2af+3cd)+3b^2d\right)}{\sqrt{a}}-\frac{\left(4a(2af+3cd)+3b^2d\right)\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+bx\right)}{\sqrt{a}}-\frac{4\log\left(\sqrt{d}\sqrt{f}+fx\right)\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}{\sqrt{f}}+4\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)),x]
```

```
[Out] ((-2*d*(2*a + 5*b*x)*Sqrt[a + x*(b + c*x)])/x^2 + ((3*b^2*d + 4*a
*(3*c*d + 2*a*f))*Log[x])/Sqrt[a] - (4*(c*d + b*Sqrt[d]*Sqrt[f] +
 a*f)^(3/2)*Log[Sqrt[d]*Sqrt[f] - f*x])/Sqrt[f] - (4*(c*d - b*Sqr
t[d]*Sqrt[f] + a*f)^(3/2)*Log[Sqrt[d]*Sqrt[f] + f*x])/Sqrt[f] - (
(3*b^2*d + 4*a*(3*c*d + 2*a*f))*Log[2*a + b*x + 2*Sqrt[a]*Sqrt[a
+ x*(b + c*x)]])/Sqrt[a] + (4*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/
2)*Log[Sqrt[d]*(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sq
rt[f]*x + 2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c
*x)])])/Sqrt[f] + (4*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Log[Sq
rt[d]*(b*(Sqrt[d] + Sqrt[f]*x) + 2*(a*Sqrt[f] + c*Sqrt[d]*x + Sqr
t[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]))])/Sqrt[f
])/(8*d^2)
```

Maple [B] time = 0.029, size = 5056, normalized size = 8.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{(fx^2 - d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^3),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a\sqrt{a + bx + cx^2}}{-dx^3 + fx^5}\,dx - \int \frac{bx\sqrt{a + bx + cx^2}}{-dx^3 + fx^5}\,dx - \int \frac{cx^2\sqrt{a + bx + cx^2}}{-dx^3 + fx^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)**(3/2)/x**3/(-f*x**2+d),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x) - Integ
ral(b*x*sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x) - Integral(
c*x**2*sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^3),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.91 $\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2}\,dx$

Optimal. Leaf size=189

$$-\frac{(12ac+3b^2+8c^2)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}}-\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}-\frac{1}{2}(a-b+c)^{3/2}\tanh^{-1}\left(\frac{2a+x(b-2c)-b}{2\sqrt{a-b+c}\sqrt{a+bx+cx^2}}\right)+\frac{1}{2}(a+b+c)^{3/2}\tanh^{-1}\left(\frac{2a+x(b+2c)+b}{2\sqrt{a+b+c}\sqrt{a+bx+cx^2}}\right)$$

```
[Out] -((5*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/4 - ((a - b + c)^(3/2)*Arc
Tanh[(2*a - b + (b - 2*c)*x)/(2*Sqrt[a - b + c]*Sqrt[a + b*x + c*
x^2])])/2 - ((3*b^2 + 12*a*c + 8*c^2)*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]) + ((a + b + c)^(3/2)*Arc
Tanh[(2*a + b + (b + 2*c)*x)/(2*Sqrt[a + b + c]*Sqrt[a + b*x + c*
x^2])])/2
```

Rubi [A] time = 0.669511, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{(12ac+3b^2+8c^2)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}}-\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}-\frac{1}{2}(a-b+c)^{3/2}\tanh^{-1}\left(\frac{2a+x(b-2c)-b}{2\sqrt{a-b+c}\sqrt{a+bx+cx^2}}\right)+\frac{1}{2}(a+b+c)^{3/2}\tanh^{-1}\left(\frac{2a+x(b+2c)+b}{2\sqrt{a+b+c}\sqrt{a+bx+cx^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + c*x^2)^(3/2)/(1 - x^2),x]
```

```
[Out] -((5*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/4 - ((a - b + c)^(3/2)*Arc
Tanh[(2*a - b + (b - 2*c)*x)/(2*Sqrt[a - b + c]*Sqrt[a + b*x + c*
x^2])])/2 - ((3*b^2 + 12*a*c + 8*c^2)*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]) + ((a + b + c)^(3/2)*Arc
Tanh[(2*a + b + (b + 2*c)*x)/(2*Sqrt[a + b + c]*Sqrt[a + b*x + c*
x^2])])/2
```

Rubi in Sympy [A] time = 98.2925, size = 170, normalized size = 0.9

$$-\frac{\left(\frac{5b}{2}+cx\right)\sqrt{a+bx+cx^2}}{2}+\frac{(a-b+c)^{\frac{3}{2}}\operatorname{atanh}\left(\frac{-2a+b+x(-b+2c)}{2\sqrt{a-b+c}\sqrt{a+bx+cx^2}}\right)}{2}-\frac{(a+b+c)^{\frac{3}{2}}\operatorname{atanh}\left(\frac{-2a-b+x(-b-2c)}{2\sqrt{a+b+c}\sqrt{a+bx+cx^2}}\right)}{2}-\frac{\left(\frac{3b^2}{4}+c(3a+2c)\right)\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)**(3/2)/(-x**2+1),x)
```

```
[Out] -(5*b/2 + c*x)*sqrt(a + b*x + c*x**2)/2 + (a - b + c)**(3/2)*atan
h((-2*a + b + x*(-b + 2*c))/(2*sqrt(a - b + c)*sqrt(a + b*x + c*x
**2)))/2 - (a + b + c)**(3/2)*atanh((-2*a - b + x*(-b - 2*c))/(2*
sqrt(a + b + c)*sqrt(a + b*x + c*x**2)))/2 - (3*b**2/4 + c*(3*a +
 2*c))*atanh((b + 2*c*x)/(2*sqrt(c)*sqrt(a + b*x + c*x**2)))/(2*s
qrt(c))
```

Mathematica [A] time = 0.694451, size = 207, normalized size = 1.1

$$\frac{1}{8}\left(-\frac{(4c(3a+2c)+3b^2)\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)}{\sqrt{c}}\right.$$

$$-2(5b+2cx)\sqrt{a+x(b+cx)}+4\log(x+1)(a-b+c)^{3/2}$$

$$-4(a-b+c)^{3/2}\log\left(2\sqrt{a-b+c}\sqrt{a+x(b+cx)}+2a+b(x-1)-2cx\right)-4\log(1-x)(a+b+c)^{3/2}+4(a+b+c)^{3/2}\log\left(2\sqrt{a+b+c}\sqrt{\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x + c*x^2)^(3/2)/(1 - x^2),x]
```

```
[Out] (-2*(5*b + 2*c*x)*Sqrt[a + x*(b + c*x)] - 4*(a + b + c)^(3/2)*Log
[1 - x] + 4*(a - b + c)^(3/2)*Log[1 + x] - ((3*b^2 + 4*c*(3*a + 2
*c))*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/Sqrt[c] -
4*(a - b + c)^(3/2)*Log[2*a + b*(-1 + x) - 2*c*x + 2*Sqrt[a - b +
 c]*Sqrt[a + x*(b + c*x)]] + 4*(a + b + c)^(3/2)*Log[2*a + b + b*
x + 2*c*x + 2*Sqrt[a + b + c]*Sqrt[a + x*(b + c*x)]])/8
```

Maple [B] time = 0.027, size = 1346, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)^(3/2)/(-x^2+1),x)
```

```
[Out] -3/8*b/c^(1/2)*ln((1/2*b+c+c*(-1+x))/c^(1/2)+((-1+x)^2*c+(b+2*c)*
(-1+x)+a+b+c)^(1/2))*a+3/8*b/c^(1/2)*ln((1/2*b-c+c*(1+x))/c^(1/2)
+((1+x)^2*c+(b-2*c)*(1+x)+a-b+c)^(1/2))*a-1/2*a*((-1+x)^2*c+(b+2*
c)*(-1+x)+a+b+c)^(1/2)-1/2*c*((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(1
/2)-5/8*((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(1/2)*b+1/2*a*((1+x)^2*
c+(b-2*c)*(1+x)+a-b+c)^(1/2)+1/2*c*((1+x)^2*c+(b-2*c)*(1+x)+a-b+c
)^(1/2)-5/8*((1+x)^2*c+(b-2*c)*(1+x)+a-b+c)^(1/2)*b-1/2*c^(3/2)*l
n((1/2*b+c+c*(-1+x))/c^(1/2)+((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(1
/2))-1/2*c^(3/2)*ln((1/2*b-c+c*(1+x))/c^(1/2)+((1+x)^2*c+(b-2*c)*
(1+x)+a-b+c)^(1/2))-1/6*((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(3/2)+1
/6*((1+x)^2*c+(b-2*c)*(1+x)+a-b+c)^(3/2)-1/4*c*((-1+x)^2*c+(b+2*c
)*(-1+x)+a+b+c)^(1/2)*x-1/8*b*((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(
1/2)*x+3/4*b*ln((1/2*b-c+c*(1+x))/c^(1/2)+((1+x)^2*c+(b-2*c)*(1+x
)+a-b+c)^(1/2))*c^(1/2)+1/2*b*(a-b+c)^(1/2)*ln((2*a-2*b+2*c+(b-2*
c)*(1+x)+2*(a-b+c)^(1/2)*((1+x)^2*c+(b-2*c)*(1+x)+a-b+c)^(1/2))/(
1+x))-1/2*c*(a-b+c)^(1/2)*ln((2*a-2*b+2*c+(b-2*c)*(1+x)+2*(a-b+c)
^(1/2)*((1+x)^2*c+(b-2*c)*(1+x)+a-b+c)^(1/2))/(1+x))-1/2*a*(a-b+c
)^(1/2)*ln((2*a-2*b+2*c+(b-2*c)*(1+x)+2*(a-b+c)^(1/2)*((1+x)^2*c+
(b-2*c)*(1+x)+a-b+c)^(1/2))/(1+x))+1/8*b*((1+x)^2*c+(b-2*c)*(1+x)
+a-b+c)^(1/2)*x+1/16/c*((1+x)^2*c+(b-2*c)*(1+x)+a-b+c)^(1/2)*b^2-
1/32/c^(3/2)*ln((1/2*b-c+c*(1+x))/c^(1/2)+((1+x)^2*c+(b-2*c)*(1+x
)+a-b+c)^(1/2))*b^3-1/4*c*((1+x)^2*c+(b-2*c)*(1+x)+a-b+c)^(1/2)*x
-1/16/c*((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(1/2)*b^2+1/32/c^(3/2)*
ln((1/2*b+c+c*(-1+x))/c^(1/2)+((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(
1/2))*b^3+1/2*a*(a+b+c)^(1/2)*ln((2*a+2*b+2*c+(b+2*c)*(-1+x)+2*(a
+b+c)^(1/2)*((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(1/2))/(-1+x))-3/4*
b*ln((1/2*b+c+c*(-1+x))/c^(1/2)+((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)
^(1/2))*c^(1/2)+1/2*b*(a+b+c)^(1/2)*ln((2*a+2*b+2*c+(b+2*c)*(-1+x
)+2*(a+b+c)^(1/2)*((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(1/2))/(-1+x)
)+1/2*c*(a+b+c)^(1/2)*ln((2*a+2*b+2*c+(b+2*c)*(-1+x)+2*(a+b+c)^(1
/2)*((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(1/2))/(-1+x))-3/4*c^(1/2)*
ln((1/2*b-c+c*(1+x))/c^(1/2)+((1+x)^2*c+(b-2*c)*(1+x)+a-b+c)^(1/2
))*a-3/16/c^(1/2)*ln((1/2*b-c+c*(1+x))/c^(1/2)+((1+x)^2*c+(b-2*c)
*(1+x)+a-b+c)^(1/2))*b^2-3/4*c^(1/2)*ln((1/2*b+c+c*(-1+x))/c^(1/2
```

```
)+((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(1/2))*a-3/16/c^(1/2)*ln((1/2
*b+c+c*(-1+x))/c^(1/2)+((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(1/2))*b
^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c*x^2 + b*x + a)^(3/2)/(x^2 - 1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 166.762, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c*x^2 + b*x + a)^(3/2)/(x^2 - 1),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(a - b + c)^(3/2)*sqrt(c)*log(-((b^2 + 4*(a - 2*b)*c + 8
*c^2)*x^2 - 4*sqrt(c*x^2 + b*x + a)*((b - 2*c)*x + 2*a - b)*sqrt(
a - b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c + 2*(4*a*b - 3*b^2 - 4*(
a - b)*c)*x)/(x^2 + 2*x + 1)) + 4*(a + b + c)^(3/2)*sqrt(c)*log(-
((b^2 + 4*(a + 2*b)*c + 8*c^2)*x^2 + 4*sqrt(c*x^2 + b*x + a)*((b
+ 2*c)*x + 2*a + b)*sqrt(a + b + c) + 8*a^2 + 8*a*b + b^2 + 4*a*c
 + 2*(4*a*b + 3*b^2 + 4*(a + b)*c)*x)/(x^2 - 2*x + 1)) - 4*sqrt(c
*x^2 + b*x + a)*(2*c*x + 5*b)*sqrt(c) + (3*b^2 + 12*a*c + 8*c^2)*
log(4*(2*c^2*x + b*c)*sqrt(c*x^2 + b*x + a) - (8*c^2*x^2 + 8*b*c*
x + b^2 + 4*a*c)*sqrt(c)))/sqrt(c), 1/16*(8*(a - b + c)*sqrt(-a +
 b - c)*sqrt(c)*arctan(-1/2*((b - 2*c)*x + 2*a - b)/(sqrt(c*x^2 +
 b*x + a)*sqrt(-a + b - c))) + 4*(a + b + c)^(3/2)*sqrt(c)*log(-(
(b^2 + 4*(a + 2*b)*c + 8*c^2)*x^2 + 4*sqrt(c*x^2 + b*x + a)*((b +
 2*c)*x + 2*a + b)*sqrt(a + b + c) + 8*a^2 + 8*a*b + b^2 + 4*a*c
+ 2*(4*a*b + 3*b^2 + 4*(a + b)*c)*x)/(x^2 - 2*x + 1)) - 4*sqrt(c*
x^2 + b*x + a)*(2*c*x + 5*b)*sqrt(c) + (3*b^2 + 12*a*c + 8*c^2)*l
og(4*(2*c^2*x + b*c)*sqrt(c*x^2 + b*x + a) - (8*c^2*x^2 + 8*b*c*x
 + b^2 + 4*a*c)*sqrt(c)))/sqrt(c), 1/16*(8*(a + b + c)*sqrt(-a -
b - c)*sqrt(c)*arctan(1/2*((b + 2*c)*x + 2*a + b)/(sqrt(c*x^2 + b
*x + a)*sqrt(-a - b - c))) + 4*(a - b + c)^(3/2)*sqrt(c)*log(-((b
^2 + 4*(a - 2*b)*c + 8*c^2)*x^2 - 4*sqrt(c*x^2 + b*x + a)*((b - 2
*c)*x + 2*a - b)*sqrt(a - b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c +
2*(4*a*b - 3*b^2 - 4*(a - b)*c)*x)/(x^2 + 2*x + 1)) - 4*sqrt(c*x^
2 + b*x + a)*(2*c*x + 5*b)*sqrt(c) + (3*b^2 + 12*a*c + 8*c^2)*log
(4*(2*c^2*x + b*c)*sqrt(c*x^2 + b*x + a) - (8*c^2*x^2 + 8*b*c*x +
 b^2 + 4*a*c)*sqrt(c)))/sqrt(c), 1/16*(8*(a - b + c)*sqrt(-a + b
- c)*sqrt(c)*arctan(-1/2*((b - 2*c)*x + 2*a - b)/(sqrt(c*x^2 + b*
x + a)*sqrt(-a + b - c))) + 8*(a + b + c)*sqrt(-a - b - c)*sqrt(c
)*arctan(1/2*((b + 2*c)*x + 2*a + b)/(sqrt(c*x^2 + b*x + a)*sqrt(
-a - b - c))) - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + 5*b)*sqrt(c) + (
3*b^2 + 12*a*c + 8*c^2)*log(4*(2*c^2*x + b*c)*sqrt(c*x^2 + b*x +
a) - (8*c^2*x^2 + 8*b*c*x + b^2 + 4*a*c)*sqrt(c)))/sqrt(c), 1/8*(
2*(a - b + c)^(3/2)*sqrt(-c)*log(-((b^2 + 4*(a - 2*b)*c + 8*c^2)*
x^2 - 4*sqrt(c*x^2 + b*x + a)*((b - 2*c)*x + 2*a - b)*sqrt(a - b
+ c) + 8*a^2 - 8*a*b + b^2 + 4*a*c + 2*(4*a*b - 3*b^2 - 4*(a - b)
*c)*x)/(x^2 + 2*x + 1)) + 2*(a + b + c)^(3/2)*sqrt(-c)*log(-((b^2
 + 4*(a + 2*b)*c + 8*c^2)*x^2 + 4*sqrt(c*x^2 + b*x + a)*((b + 2*c
)*x + 2*a + b)*sqrt(a + b + c) + 8*a^2 + 8*a*b + b^2 + 4*a*c + 2*
(4*a*b + 3*b^2 + 4*(a + b)*c)*x)/(x^2 - 2*x + 1)) - 2*sqrt(c*x^2
+ b*x + a)*(2*c*x + 5*b)*sqrt(-c) - (3*b^2 + 12*a*c + 8*c^2)*arct
an(1/2*(2*c*x + b)*sqrt(-c)/(sqrt(c*x^2 + b*x + a)*c)))/sqrt(-c),
```

```
 1/8*(4*(a - b + c)*sqrt(-a + b - c)*sqrt(-c)*arctan(-1/2*((b - 2
*c)*x + 2*a - b)/(sqrt(c*x^2 + b*x + a)*sqrt(-a + b - c))) + 2*(a
 + b + c)^(3/2)*sqrt(-c)*log(-((b^2 + 4*(a + 2*b)*c + 8*c^2)*x^2
+ 4*sqrt(c*x^2 + b*x + a)*((b + 2*c)*x + 2*a + b)*sqrt(a + b + c)
 + 8*a^2 + 8*a*b + b^2 + 4*a*c + 2*(4*a*b + 3*b^2 + 4*(a + b)*c)*
x)/(x^2 - 2*x + 1)) - 2*sqrt(c*x^2 + b*x + a)*(2*c*x + 5*b)*sqrt(
-c) - (3*b^2 + 12*a*c + 8*c^2)*arctan(1/2*(2*c*x + b)*sqrt(-c)/(s
qrt(c*x^2 + b*x + a)*c)))/sqrt(-c), 1/8*(4*(a + b + c)*sqrt(-a -
b - c)*sqrt(-c)*arctan(1/2*((b + 2*c)*x + 2*a + b)/(sqrt(c*x^2 +
b*x + a)*sqrt(-a - b - c))) + 2*(a - b + c)^(3/2)*sqrt(-c)*log(-(
(b^2 + 4*(a - 2*b)*c + 8*c^2)*x^2 - 4*sqrt(c*x^2 + b*x + a)*((b -
 2*c)*x + 2*a - b)*sqrt(a - b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c
+ 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*x)/(x^2 + 2*x + 1)) - 2*sqrt(c*
x^2 + b*x + a)*(2*c*x + 5*b)*sqrt(-c) - (3*b^2 + 12*a*c + 8*c^2)*
arctan(1/2*(2*c*x + b)*sqrt(-c)/(sqrt(c*x^2 + b*x + a)*c)))/sqrt(
-c), 1/8*(4*(a - b + c)*sqrt(-a + b - c)*sqrt(-c)*arctan(-1/2*((b
 - 2*c)*x + 2*a - b)/(sqrt(c*x^2 + b*x + a)*sqrt(-a + b - c))) +
4*(a + b + c)*sqrt(-a - b - c)*sqrt(-c)*arctan(1/2*((b + 2*c)*x +
 2*a + b)/(sqrt(c*x^2 + b*x + a)*sqrt(-a - b - c))) - 2*sqrt(c*x^
2 + b*x + a)*(2*c*x + 5*b)*sqrt(-c) - (3*b^2 + 12*a*c + 8*c^2)*ar
ctan(1/2*(2*c*x + b)*sqrt(-c)/(sqrt(c*x^2 + b*x + a)*c)))/sqrt(-c
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a\sqrt{a + bx + cx^2}}{x^2 - 1}\, dx - \int \frac{bx\sqrt{a + bx + cx^2}}{x^2 - 1}\, dx - \int \frac{cx^2\sqrt{a + bx + cx^2}}{x^2 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)**(3/2)/(-x**2+1),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(x**2 - 1), x) - Integral(b*x*
sqrt(a + b*x + c*x**2)/(x**2 - 1), x) - Integral(c*x**2*sqrt(a +
b*x + c*x**2)/(x**2 - 1), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c*x^2 + b*x + a)^(3/2)/(x^2 - 1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.92 $\int \frac{\sqrt{-1-x+x^2}}{1-x^2} \, dx$

Optimal. Leaf size=75

$$-\frac{1}{2}\tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right)+\tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

```
[Out] -ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^2])]/2 + ArcTanh[(1 - 2*x)/(2*
Sqrt[-1 - x + x^2])] + ArcTanh[(1 + 3*x)/(2*Sqrt[-1 - x + x^2])]/
2
```

Rubi [A] time = 0.142708, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{1}{2}\tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right)+\tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-1 - x + x^2]/(1 - x^2),x]
```

```
[Out] -ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^2])]/2 + ArcTanh[(1 - 2*x)/(2*
Sqrt[-1 - x + x^2])] + ArcTanh[(1 + 3*x)/(2*Sqrt[-1 - x + x^2])]/
2
```

Rubi in Sympy [A] time = 37.4938, size = 56, normalized size = 0.75

$$-\frac{\operatorname{atan}\left(\frac{-x+3}{2\sqrt{x^2-x-1}}\right)}{2}-\operatorname{atanh}\left(\frac{2x-1}{2\sqrt{x^2-x-1}}\right)+\frac{\operatorname{atanh}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-x-1)**(1/2)/(-x**2+1),x)
```

```
[Out] -atan((-x + 3)/(2*sqrt(x**2 - x - 1)))/2 - atanh((2*x - 1)/(2*sqr
t(x**2 - x - 1))) + atanh((3*x + 1)/(2*sqrt(x**2 - x - 1)))/2
```

Mathematica [A] time = 0.0214565, size = 79, normalized size = 1.05

$$-\log\left(-2\sqrt{x^2-x-1}-2x+1\right)-\frac{1}{2}\log\left(-2\sqrt{x^2-x-1}+3x+1\right)+\frac{1}{2}\tan^{-1}\left(\frac{x-3}{2\sqrt{x^2-x-1}}\right)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-1 - x + x^2]/(1 - x^2),x]
```

```
[Out] ArcTan[(-3 + x)/(2*Sqrt[-1 - x + x^2])]/2 + Log[1 + x]/2 - Log[1
- 2*x - 2*Sqrt[-1 - x + x^2]] - Log[1 + 3*x - 2*Sqrt[-1 - x + x^2
]]/2
```

Maple [A] time = 0.02, size = 102, normalized size = 1.4

$$\frac{1}{2}\sqrt{(1+x)^2-2-3x}-\frac{3}{4}\ln\left(-\frac{1}{2}+x+\sqrt{(1+x)^2-2-3x}\right)-\frac{1}{2}Artanh\left(\frac{-1-3x}{2}\frac{1}{\sqrt{(1+x)^2-2-3x}}\right)-\frac{1}{2}\sqrt{(-1+x)^2+x-2}-\frac{1}{4}\ln\left(-\frac{1}{2}+x+\sqrt{(-1+x)^2+x-2}\right)+\frac{1}{2}\arctan\left(\frac{-3+x}{2}\frac{1}{\sqrt{(-1+x)^2+x-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-x-1)^(1/2)/(-x^2+1),x)
```

```
[Out] 1/2*((1+x)^2-2-3*x)^(1/2)-3/4*ln(-1/2+x+((1+x)^2-2-3*x)^(1/2))-1/
2*arctanh(1/2*(-1-3*x)/((1+x)^2-2-3*x)^(1/2))-1/2*((-1+x)^2+x-2)^
(1/2)-1/4*ln(-1/2+x+((-1+x)^2+x-2)^(1/2))+1/2*arctan(1/2*(-3+x)/(
(-1+x)^2+x-2)^(1/2))
```

Maxima [A] time = 0.79461, size = 112, normalized size = 1.49

$$\frac{1}{2}\arcsin\left(\frac{2\sqrt{5}x}{5|2x-2|}-\frac{6\sqrt{5}}{5|2x-2|}\right)-\log\left(x+\sqrt{x^2-x-1}-\frac{1}{2}\right)-\frac{1}{2}\log\left(\frac{2\sqrt{x^2-x-1}}{|2x+2|}+\frac{2}{|2x+2|}-\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(x^2 - x - 1)/(x^2 - 1),x, algorithm="maxima")
```

```
[Out] 1/2*arcsin(2/5*sqrt(5)*x/abs(2*x - 2) - 6/5*sqrt(5)/abs(2*x - 2))
 - log(x + sqrt(x^2 - x - 1) - 1/2) - 1/2*log(2*sqrt(x^2 - x - 1)
/abs(2*x + 2) + 2/abs(2*x + 2) - 3/2)
```

Fricas [A] time = 0.299299, size = 95, normalized size = 1.27

$$\arctan\left(-x+\sqrt{x^2-x-1}+1\right)-\frac{1}{2}\log\left(-x+\sqrt{x^2-x-1}\right)+\frac{1}{2}\log\left(-x+\sqrt{x^2-x-1}-2\right)+\log\left(-2x+2\sqrt{x^2-x-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(x^2 - x - 1)/(x^2 - 1),x, algorithm="fricas")
```

```
[Out] arctan(-x + sqrt(x^2 - x - 1) + 1) - 1/2*log(-x + sqrt(x^2 - x -
1)) + 1/2*log(-x + sqrt(x^2 - x - 1) - 2) + log(-2*x + 2*sqrt(x^2
 - x - 1) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{x^2-x-1}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-x-1)**(1/2)/(-x**2+1),x)
```

```
[Out] -Integral(sqrt(x**2 - x - 1)/(x**2 - 1), x)
```

GIAC/XCAS [A] time = 0.275068, size = 99, normalized size = 1.32

$$\arctan\left(-x+\sqrt{x^2-x-1}+1\right)-\frac{1}{2}\ln\left(\left|-x+\sqrt{x^2-x-1}\right|\right)+\frac{1}{2}\ln\left(\left|-x+\sqrt{x^2-x-1}-2\right|\right)+\ln\left(\left|-2x+2\sqrt{x^2-x-1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(x^2 - x - 1)/(x^2 - 1),x, algorithm="giac")
```

```
[Out] arctan(-x + sqrt(x^2 - x - 1) + 1) - 1/2*ln(abs(-x + sqrt(x^2 - x
 - 1))) + 1/2*ln(abs(-x + sqrt(x^2 - x - 1) - 2)) + ln(abs(-2*x +
 2*sqrt(x^2 - x - 1) + 1))
```

3.93 $\int \frac{(x+x^2)^{3/2}}{1+x^2} dx$

Optimal. Leaf size=130

$$\frac{1}{4}\sqrt{x^2+x}(2x+5)+\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{-x+\sqrt{2}+1}{\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x^2+x}}\right) -\sqrt{\sqrt{2}-1}\tanh^{-1}\left(\frac{-x-\sqrt{2}+1}{\sqrt{2\left(\sqrt{2}-1\right)}\sqrt{x^2+x}}\right)-\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

```
[Out] ((5 + 2*x)*Sqrt[x + x^2])/4 + Sqrt[1 + Sqrt[2]]*ArcTan[(1 + Sqrt[
2] - x)/(Sqrt[2*(1 + Sqrt[2])]*Sqrt[x + x^2])] - Sqrt[-1 + Sqrt[2
]]*ArcTanh[(1 - Sqrt[2] - x)/(Sqrt[2*(-1 + Sqrt[2])]*Sqrt[x + x^2
])] - (5*ArcTanh[x/Sqrt[x + x^2]])/4
```

Rubi [A] time = 0.38194, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$

$$\frac{1}{4}\sqrt{x^2+x}(2x+5)+\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{-x+\sqrt{2}+1}{\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x^2+x}}\right) -\sqrt{\sqrt{2}-1}\tanh^{-1}\left(\frac{-x-\sqrt{2}+1}{\sqrt{2\left(\sqrt{2}-1\right)}\sqrt{x^2+x}}\right)-\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x + x^2)^(3/2)/(1 + x^2),x]
```


```
[Out] ((5 + 2*x)*Sqrt[x + x^2])/4 + Sqrt[1 + Sqrt[2]]*ArcTan[(1 + Sqrt[
2] - x)/(Sqrt[2*(1 + Sqrt[2])]*Sqrt[x + x^2])] - Sqrt[-1 + Sqrt[2
]]*ArcTanh[(1 - Sqrt[2] - x)/(Sqrt[2*(-1 + Sqrt[2])]*Sqrt[x + x^2
])] - (5*ArcTanh[x/Sqrt[x + x^2]])/4
```

Rubi in Sympy [A] time = 39.4762, size = 141, normalized size = 1.08

$$\frac{\left(x+\frac{5}{2}\right)\sqrt{x^2+x}}{2}-\frac{\left(2+2\sqrt{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x-\sqrt{2}-1\right)}{2\sqrt{1+\sqrt{2}}\sqrt{x^2+x}}\right)}{2\sqrt{1+\sqrt{2}}}-\frac{5\operatorname{atanh}\left(\frac{x}{\sqrt{x^2+x}}\right)}{4}-\frac{\left(-2\sqrt{2}+2\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(x-1+\sqrt{2}\right)}{2\sqrt{-1+\sqrt{2}}\sqrt{x^2+x}}\right)}{2\sqrt{-1+\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+x)**(3/2)/(x**2+1),x)
```

```
[Out] (x + 5/2)*sqrt(x**2 + x)/2 - (2 + 2*sqrt(2))*atan(sqrt(2)*(x - sq
rt(2) - 1)/(2*sqrt(1 + sqrt(2))*sqrt(x**2 + x)))/(2*sqrt(1 + sqrt
(2))) - 5*atanh(x/sqrt(x**2 + x))/4 - (-2*sqrt(2) + 2)*atanh(sqrt
(2)*(x - 1 + sqrt(2))/(2*sqrt(-1 + sqrt(2))*sqrt(x**2 + x)))/(2*s
qrt(-1 + sqrt(2)))
```

Mathematica [C] time = 0.199655, size = 124, normalized size = 0.95

$$\frac{\sqrt{x}\sqrt{x+1}\left(2\sqrt{x+1}x^{3/2}+5\sqrt{x+1}\sqrt{x}-4\sqrt{2-2i}\tan^{-1}\left((1-i)^{3/2}\sqrt{\frac{x}{2x+2}}\right)-4\sqrt{2+2i}\tan^{-1}\left((1+i)^{3/2}\sqrt{\frac{x}{2x+2}}\right)-5\sinh^{-1}\left(\sqrt{x}\right)\right)}{4\sqrt{x(x+1)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + x^2)^(3/2)/(1 + x^2),x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(5*Sqrt[x]*Sqrt[1 + x] + 2*x^(3/2)*Sqrt[1 +
x] - 5*ArcSinh[Sqrt[x]] - 4*Sqrt[2 - 2*I]*ArcTan[(1 - I)^(3/2)*Sq
rt[x/(2 + 2*x)]] - 4*Sqrt[2 + 2*I]*ArcTan[(1 + I)^(3/2)*Sqrt[x/(2
 + 2*x)]]))/(4*Sqrt[x*(1 + x)])
```

Maple [B] time = 0.08, size = 789, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+x)^(3/2)/(x^2+1),x)
```

```
[Out] 1/2*x*(x^2+x)^(1/2)+5/4*(x^2+x)^(1/2)-5/8*ln(1/2+x+(x^2+x)^(1/2))
+1/2*(4*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2-3*2^(1/2)*(-2^(1/2)-1+x)
^2/(1-x-2^(1/2))^2+4+3*2^(1/2))^(1/2)*2^(1/2)*((-2+2*2^(1/2))^(1/
2)*arctan(1/2*(-2^(1/2)-1+x)/(1-x-2^(1/2))*(-4+3*2^(1/2))*(24*(-2
^(1/2)-1+x)^2/(1-x-2^(1/2))^2+17*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^
(1/2))^2-2^(1/2))*(-2+2*2^(1/2))^(1/2)*((-4+3*2^(1/2))*(-(-2^(1/2
)-1+x)^2/(1-x-2^(1/2))^2+12*2^(1/2)+17))^(1/2)/((-2^(1/2)-1+x)^4/
(1-x-2^(1/2))^4-34*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+1))*(1+2^(1/2
))^(1/2)*2^(1/2)-2*(-2+2*2^(1/2))^(1/2)*arctan(1/2*(-2^(1/2)-1+x)
/(1-x-2^(1/2))*(-4+3*2^(1/2))*(24*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^
2+17*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2-2^(1/2))*(-2+2*2^(1
/2))^(1/2)*((-4+3*2^(1/2))*(-(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+12*
2^(1/2)+17))^(1/2)/((-2^(1/2)-1+x)^4/(1-x-2^(1/2))^4-34*(-2^(1/2)
-1+x)^2/(1-x-2^(1/2))^2+1))*(1+2^(1/2))^(1/2)-4*arctanh(1/2*(4*(-
2^(1/2)-1+x)^2/(1-x-2^(1/2))^2-3*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^
(1/2))^2+4+3*2^(1/2))^(1/2)/(1+2^(1/2))^(1/2))*2^(1/2)+6*arctanh(
1/2*(4*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2-3*2^(1/2)*(-2^(1/2)-1+x)^
2/(1-x-2^(1/2))^2+4+3*2^(1/2))^(1/2)/(1+2^(1/2))^(1/2)))/(-(3*2^(
1/2)*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2-4*(-2^(1/2)-1+x)^2/(1-x-2^(
1/2))^2-3*2^(1/2)-4)/(1+(-2^(1/2)-1+x)/(1-x-2^(1/2)))^2)^(1/2)/(1
+(-2^(1/2)-1+x)/(1-x-2^(1/2)))/(-4+3*2^(1/2))/(1+2^(1/2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+x\right)^{\frac{3}{2}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x)^(3/2)/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + x)^(3/2)/(x^2 + 1), x)
```

Fricas [A] time = 0.336094, size = 1806, normalized size = 13.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x)^(3/2)/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/32*(4*sqrt(2)*(32*x^3 + 112*x^2 - sqrt(2)*(32*x^3 + 112*x^2 + 6
6*x + 1) + 66*x + 1)*sqrt(x^2 + x)*sqrt((sqrt(2) - 2)/(2*sqrt(2)
- 3)) - sqrt(2)*(128*x^4 + 512*x^3 + 472*x^2 - sqrt(2)*(128*x^4 +
 512*x^3 + 472*x^2 + 88*x - 9) + 88*x - 9)*sqrt((sqrt(2) - 2)/(2*
sqrt(2) - 3)) + 64*(4*8^(1/4)*sqrt(x^2 + x)*(2*x + 1) - 8^(1/4)*(
8*x^2 + 8*x + 1))*arctan((2*sqrt(2)*(sqrt(2) - 1)*sqrt((sqrt(2) -
 2)/(2*sqrt(2) - 3)) + 8^(1/4)*sqrt(2))/(2*sqrt(2)*sqrt(x^2 + x)*
(sqrt(2) - 1)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) - 2*sqrt(2)*(sq
rt(2)*x - x)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) + 2*(sqrt(2) - 1
)*sqrt((8^(1/4)*sqrt(2)*(sqrt(2)*(17*x + 7) - 24*x - 10)*sqrt((sq
rt(2) - 2)/(2*sqrt(2) - 3)) - 40*x^2 + 14*sqrt(2)*(2*x^2 + x + 1)
 - (8^(1/4)*sqrt(2)*(17*sqrt(2) - 24)*sqrt((sqrt(2) - 2)/(2*sqrt(
2) - 3)) + 28*sqrt(2)*x - 40*x)*sqrt(x^2 + x) + 2*sqrt(2)*(7*sqrt
(2) - 10) - 20*x - 20)/(7*sqrt(2) - 10))*sqrt((sqrt(2) - 2)/(2*sq
rt(2) - 3)) - 8^(1/4)*(sqrt(2) - 2))) - 64*(4*8^(1/4)*sqrt(x^2 +
x)*(2*x + 1) - 8^(1/4)*(8*x^2 + 8*x + 1))*arctan((2*sqrt(2)*(sqrt
(2) - 1)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) - 8^(1/4)*sqrt(2))/(
2*sqrt(2)*sqrt(x^2 + x)*(sqrt(2) - 1)*sqrt((sqrt(2) - 2)/(2*sqrt(
2) - 3)) - 2*sqrt(2)*(sqrt(2)*x - x)*sqrt((sqrt(2) - 2)/(2*sqrt(2
) - 3)) + 2*(sqrt(2) - 1)*sqrt(-(8^(1/4)*sqrt(2)*(sqrt(2)*(17*x +
 7) - 24*x - 10)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) + 40*x^2 - 1
4*sqrt(2)*(2*x^2 + x + 1) - (8^(1/4)*sqrt(2)*(17*sqrt(2) - 24)*sq
rt((sqrt(2) - 2)/(2*sqrt(2) - 3)) - 28*sqrt(2)*x + 40*x)*sqrt(x^2
 + x) - 2*sqrt(2)*(7*sqrt(2) - 10) + 20*x + 20)/(7*sqrt(2) - 10))
*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) + 8^(1/4)*(sqrt(2) - 2))) +
20*(4*sqrt(2)*sqrt(x^2 + x)*(sqrt(2)*(2*x + 1) - 2*x - 1)*sqrt((s
qrt(2) - 2)/(2*sqrt(2) - 3)) + sqrt(2)*(8*x^2 - sqrt(2)*(8*x^2 +
8*x + 1) + 8*x + 1)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)))*log(-2*x
 + 2*sqrt(x^2 + x) - 1) - 16*(4*8^(1/4)*sqrt(x^2 + x)*(sqrt(2)*(2
*x + 1) - 2*x - 1) + 8^(1/4)*(8*x^2 - sqrt(2)*(8*x^2 + 8*x + 1) +
 8*x + 1))*log(-2*(8^(1/4)*sqrt(2)*(sqrt(2)*(17*x + 7) - 24*x - 1
0)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) + 40*x^2 - 14*sqrt(2)*(2*x
^2 + x + 1) - (8^(1/4)*sqrt(2)*(17*sqrt(2) - 24)*sqrt((sqrt(2) -
2)/(2*sqrt(2) - 3)) - 28*sqrt(2)*x + 40*x)*sqrt(x^2 + x) - 2*sqrt
(2)*(7*sqrt(2) - 10) + 20*x + 20)/(7*sqrt(2) - 10)) + 16*(4*8^(1/
4)*sqrt(x^2 + x)*(sqrt(2)*(2*x + 1) - 2*x - 1) + 8^(1/4)*(8*x^2 -
 sqrt(2)*(8*x^2 + 8*x + 1) + 8*x + 1))*log(2*(8^(1/4)*sqrt(2)*(sq
rt(2)*(17*x + 7) - 24*x - 10)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3))
 - 40*x^2 + 14*sqrt(2)*(2*x^2 + x + 1) - (8^(1/4)*sqrt(2)*(17*sqr
t(2) - 24)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) + 28*sqrt(2)*x - 4
0*x)*sqrt(x^2 + x) + 2*sqrt(2)*(7*sqrt(2) - 10) - 20*x - 20)/(7*s
qrt(2) - 10)))/(4*sqrt(2)*sqrt(x^2 + x)*(sqrt(2)*(2*x + 1) - 2*x
- 1)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) + sqrt(2)*(8*x^2 - sqrt(
2)*(8*x^2 + 8*x + 1) + 8*x + 1)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3
)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(x+1))^{\frac{3}{2}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+x)**(3/2)/(x**2+1),x)
```

```
[Out] Integral((x*(x + 1))**(3/2)/(x**2 + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+x\right)^{\frac{3}{2}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x)^(3/2)/(x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate((x^2 + x)^(3/2)/(x^2 + 1), x)
```

3.94 $\int \frac{x^4}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal. Leaf size=369

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}f}+\frac{3b\sqrt{a+bx+cx^2}}{4c^2f}+\frac{d^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{d^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{d\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2}-\frac{x\sqrt{a+bx+cx^2}}{2cf}$$

```
[Out] (3*b*Sqrt[a + b*x + c*x^2])/(4*c^2*f) - (x*Sqrt[a + b*x + c*x^2])
/(2*c*f) - (d*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])])/(Sqrt[c]*f^2) - ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*f) + (d^(3/2)*ArcTanh[(b*
Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^2*Sqrt[c
*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (d^(3/2)*ArcTanh[(b*Sqrt[d] + 2*
a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*
Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^2*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f])
```

Rubi [A] time = 1.57568, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}f}+\frac{3b\sqrt{a+bx+cx^2}}{4c^2f}+\frac{d^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{d^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{d\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2}-\frac{x\sqrt{a+bx+cx^2}}{2cf}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] (3*b*Sqrt[a + b*x + c*x^2])/(4*c^2*f) - (x*Sqrt[a + b*x + c*x^2])
/(2*c*f) - (d*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])])/(Sqrt[c]*f^2) - ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*f) + (d^(3/2)*ArcTanh[(b*
Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^2*Sqrt[c
*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (d^(3/2)*ArcTanh[(b*Sqrt[d] + 2*
a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*
Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^2*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f])
```

Rubi in Sympy [A] time = 144.552, size = 338, normalized size = 0.92

$$\frac{3b\sqrt{a+bx+cx^2}}{4c^2f} - \frac{d^{\frac{3}{2}}\operatorname{atanh}\left(\frac{-2a\sqrt{f}-b\sqrt{d}+x\left(-b\sqrt{f}-2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{d^{\frac{3}{2}}\operatorname{atanh}\left(\frac{2a\sqrt{f}-b\sqrt{d}+x\left(b\sqrt{f}-2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^2\sqrt{af-b\sqrt{d}\sqrt{f}+cd}} - \frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{d\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} - \frac{\left(-4ac+3b^2\right)\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{\frac{5}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] 3*b*sqrt(a + b*x + c*x**2)/(4*c**2*f) - d**(3/2)*atanh((-2*a*sqrt
(f) - b*sqrt(d) + x*(-b*sqrt(f) - 2*c*sqrt(d)))/(2*sqrt(a + b*x +
 c*x**2)*sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)))/(2*f**2*sqrt(a*f +
 b*sqrt(d)*sqrt(f) + c*d)) - d**(3/2)*atanh((2*a*sqrt(f) - b*sqrt
(d) + x*(b*sqrt(f) - 2*c*sqrt(d)))/(2*sqrt(a + b*x + c*x**2)*sqrt
(a*f - b*sqrt(d)*sqrt(f) + c*d)))/(2*f**2*sqrt(a*f - b*sqrt(d)*sq
rt(f) + c*d)) - x*sqrt(a + b*x + c*x**2)/(2*c*f) - d*atanh((b + 2
*c*x)/(2*sqrt(c)*sqrt(a + b*x + c*x**2)))/(sqrt(c)*f**2) - (-4*a*
c + 3*b**2)*atanh((b + 2*c*x)/(2*sqrt(c)*sqrt(a + b*x + c*x**2)))
/(8*c**(5/2)*f)
```

Mathematica [A] time = 4.7619, size = 399, normalized size = 1.08

$$-\frac{\frac{\left(-4acf+3b^2f+8c^2d\right)\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)}{c^{5/2}} + \frac{2f(2cx-3b)\sqrt{a+x(b+cx)}}{c^2} + \frac{4d^{3/2}\log\left(\sqrt{d}\sqrt{f}-fx\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{4d^{3/2}\log\left(\sqrt{d}\sqrt{f}+fx\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}} + \frac{4d^{3/2}\log\left(\sqrt{d}\left(2\sqrt{a}\right.\right.}{}}{8f^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -((2*f*(-3*b + 2*c*x)*Sqrt[a + x*(b + c*x)])/c^2 + (4*d^(3/2)*Log
[Sqrt[d]*Sqrt[f] - f*x])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f] - (4
*d^(3/2)*Log[Sqrt[d]*Sqrt[f] + f*x])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f]
 + a*f] + ((8*c^2*d + 3*b^2*f - 4*a*c*f)*Log[b + 2*c*x + 2*Sqrt[c
]*Sqrt[a + x*(b + c*x)]])/c^(5/2) + (4*d^(3/2)*Log[Sqrt[d]*(-(b*S
qrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x + 2*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d - b*
Sqrt[d]*Sqrt[f] + a*f] - (4*d^(3/2)*Log[Sqrt[d]*(b*(Sqrt[d] + Sqr
t[f]*x) + 2*(a*Sqrt[f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt[
f] + a*f]*Sqrt[a + x*(b + c*x)]))])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f]
+ a*f])/(8*f^2)
```

Maple [A] time = 0.025, size = 516, normalized size = 1.4

$$-\frac{d}{f^2}\ln\left(1\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)\frac{1}{\sqrt{c}}-\frac{x}{2cf}\sqrt{cx^2+bx+a}+\frac{3b}{4c^2f}\sqrt{cx^2+bx+a}$$
$$-\frac{3b^2}{8f}\ln\left(1\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)c^{-\frac{5}{2}}+\frac{a}{2f}\ln\left(1\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)c^{-\frac{3}{2}}$$
$$+\frac{d^2}{2f^2}\ln\left(1\left(2\frac{b\sqrt{df}+fa+cd}{f}+\frac{1}{f}\left(2c\sqrt{df}+bf\right)\left(x-\frac{1}{f}\sqrt{df}\right)+2\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{2c\sqrt{df}+bf}{f}\left(\right.}\right.\right.$$
$$-\frac{d^2}{2f^2}\ln\left(1\left(2\frac{-b\sqrt{df}+fa+cd}{f}+\frac{1}{f}\left(-2c\sqrt{df}+bf\right)\left(x+\frac{1}{f}\sqrt{df}\right)+2\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\frac{-2c\sqrt{df}}{f}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/f^2*d*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/2*
x*(c*x^2+b*x+a)^(1/2)/c/f+3/4*b*(c*x^2+b*x+a)^(1/2)/c^2/f-3/8/f*b
^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2/f*a/c^
(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2/f^2*d^2/(d*
f)^(1/2)/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f
*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(
1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*
f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)
^(1/2)/f))-1/2/f^2*d^2/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))
^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f
)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d
*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f
*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^4/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^4/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^4}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**4/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x +
 c*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^4/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.95 $\int \frac{x^3}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal. Leaf size=287

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}-\frac{d\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{d\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\sqrt{a+bx+cx^2}}{cf}$$

```
[Out] -(Sqrt[a + b*x + c*x^2]/(c*f)) + (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*f) - (d*ArcTanh[(b*Sqrt[d]
- 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt
[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(3/2)*Sqrt[c*d -
 b*Sqrt[d]*Sqrt[f] + a*f]) + (d*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f]
+ (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] +
a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(3/2)*Sqrt[c*d + b*Sqrt[d]*Sqr
t[f] + a*f])
```

Rubi [A] time = 1.27131, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}-\frac{d\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{d\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\sqrt{a+bx+cx^2}}{cf}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(c*f)) + (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*f) - (d*ArcTanh[(b*Sqrt[d]
- 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt
[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(3/2)*Sqrt[c*d -
 b*Sqrt[d]*Sqrt[f] + a*f]) + (d*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f]
+ (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] +
a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(3/2)*Sqrt[c*d + b*Sqrt[d]*Sqr
t[f] + a*f])
```

Rubi in Sympy [A] time = 124.668, size = 260, normalized size = 0.91

$$\frac{b\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{\frac{3}{2}}f}-\frac{d\operatorname{atanh}\left(\frac{-2a\sqrt{f}-b\sqrt{d}+x\left(-b\sqrt{f}-2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{\frac{3}{2}}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{d\operatorname{atanh}\left(\frac{-2a\sqrt{f}+b\sqrt{d}+x\left(-b\sqrt{f}+2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{\frac{3}{2}}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}-\frac{\sqrt{a+bx+cx^2}}{cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] b*atanh((b + 2*c*x)/(2*sqrt(c)*sqrt(a + b*x + c*x**2)))/(2*c**(3/
2)*f) - d*atanh((-2*a*sqrt(f) - b*sqrt(d) + x*(-b*sqrt(f) - 2*c*s
qrt(d)))/(2*sqrt(a + b*x + c*x**2)*sqrt(a*f + b*sqrt(d)*sqrt(f) +
 c*d)))/(2*f**(3/2)*sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)) - d*atan
h((-2*a*sqrt(f) + b*sqrt(d) + x*(-b*sqrt(f) + 2*c*sqrt(d)))/(2*sq
rt(a + b*x + c*x**2)*sqrt(a*f - b*sqrt(d)*sqrt(f) + c*d)))/(2*f**
(3/2)*sqrt(a*f - b*sqrt(d)*sqrt(f) + c*d)) - sqrt(a + b*x + c*x**
2)/(c*f)
```

Mathematica [A] time = 2.784, size = 404, normalized size = 1.41

$$\frac{\frac{b\sqrt{f}\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)}{c^{3/2}}-\frac{d\log\left(\sqrt{d}\sqrt{f}-fx\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{d\log\left(\sqrt{d}\sqrt{f}+fx\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{d\log\left(2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}+2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}\right.}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}}{2f^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-2*a*Sqrt[f])/(c*Sqrt[a + x*(b + c*x)]) - (2*b*Sqrt[f]*x)/(c*Sq
rt[a + x*(b + c*x)]) - (2*Sqrt[f]*x^2)/Sqrt[a + x*(b + c*x)] - (d
*Log[Sqrt[d]*Sqrt[f] - f*x])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]
- (d*Log[Sqrt[d]*Sqrt[f] + f*x])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a
*f] + (b*Sqrt[f]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]
)/c^(3/2) + (d*Log[-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b
*Sqrt[f]*x + 2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b
+ c*x)]])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] + (d*Log[b*(Sqrt[d]
 + Sqrt[f]*x) + 2*(a*Sqrt[f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[d]
*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d + b*Sqrt[d]*Sqr
t[f] + a*f])/(2*f^(3/2))
```

Maple [A] time = 0.021, size = 410, normalized size = 1.4

$$-\frac{1}{cf}\sqrt{cx^2+bx+a}+\frac{b}{2f}\ln\left(1\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)c^{-\frac{3}{2}}$$
$$+\frac{d}{2f^2}\ln\left(1\left(2\frac{b\sqrt{df}+fa+cd}{f}+\frac{1}{f}\left(2c\sqrt{df}+bf\right)\left(x-\frac{1}{f}\sqrt{df}\right)+2\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{2c\sqrt{df}+bf}{f}\left(\right.}\right.\right.$$
$$+\frac{d}{2f^2}\ln\left(1\left(2\frac{-b\sqrt{df}+fa+cd}{f}+\frac{1}{f}\left(-2c\sqrt{df}+bf\right)\left(x+\frac{1}{f}\sqrt{df}\right)+2\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\frac{-2c\sqrt{df}}{f}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -(c*x^2+b*x+a)^(1/2)/c/f+1/2/f*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))+1/2/f^2*d/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*l
n((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(
1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*
c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*
d)/f)^(1/2))/(x-(d*f)^(1/2)/f))+1/2/f^2*d/(1/f*(-b*(d*f)^(1/2)+f*
a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1
/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)
*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)
/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**3/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x +
 c*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^3/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.96 $\int \frac{x^2}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal. Leaf size=266

$$\frac{\sqrt{d}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

```
[Out] -(ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]
*f)) + (Sqrt[d]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] -
 b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*
x + c*x^2])])/(2*f*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (Sqrt[d
]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)
/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/
(2*f*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rubi [A] time = 0.532928, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$

$$\frac{\sqrt{d}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -(ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]
*f)) + (Sqrt[d]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] -
 b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*
x + c*x^2])])/(2*f*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (Sqrt[d
]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)
/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/
(2*f*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rubi in Sympy [A] time = 78.7894, size = 241, normalized size = 0.91

$$-\frac{\sqrt{d}\,\mathrm{atanh}\left(\frac{-2a\sqrt{f}-b\sqrt{d}+x\left(-b\sqrt{f}-2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\sqrt{d}\,\mathrm{atanh}\left(\frac{2a\sqrt{f}-b\sqrt{d}+x\left(b\sqrt{f}-2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}\right)}{2f\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}-\frac{\mathrm{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -sqrt(d)*atanh((-2*a*sqrt(f) - b*sqrt(d) + x*(-b*sqrt(f) - 2*c*sq
rt(d)))/(2*sqrt(a + b*x + c*x**2)*sqrt(a*f + b*sqrt(d)*sqrt(f) +
c*d)))/(2*f*sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)) - sqrt(d)*atanh(
(2*a*sqrt(f) - b*sqrt(d) + x*(b*sqrt(f) - 2*c*sqrt(d)))/(2*sqrt(a
 + b*x + c*x**2)*sqrt(a*f - b*sqrt(d)*sqrt(f) + c*d)))/(2*f*sqrt(
a*f - b*sqrt(d)*sqrt(f) + c*d)) - atanh((b + 2*c*x)/(2*sqrt(c)*sq
rt(a + b*x + c*x**2)))/(sqrt(c)*f)
```

Mathematica [A] time = 0.788148, size = 353, normalized size = 1.33

$$\frac{-\frac{\sqrt{d}\log\left(\sqrt{d}\sqrt{f}-fx\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}+\frac{\sqrt{d}\log\left(\sqrt{d}\sqrt{f}+fx\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}-\frac{\sqrt{d}\log\left(\sqrt{d}\left(2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}+2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x\right)\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{d}\log\left(\sqrt{d}\left(2\left(\sqrt{a+x(\right.\right.\right.}{}}{2f}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] (-((Sqrt[d]*Log[Sqrt[d]*Sqrt[f] - f*x])/Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f]) + (Sqrt[d]*Log[Sqrt[d]*Sqrt[f] + f*x])/Sqrt[c*d - b*S
qrt[d]*Sqrt[f] + a*f] - (2*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(
b + c*x)]])/Sqrt[c] - (Sqrt[d]*Log[Sqrt[d]*(-(b*Sqrt[d]) + 2*a*Sq
rt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x + 2*Sqrt[c*d - b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f]
+ a*f] + (Sqrt[d]*Log[Sqrt[d]*(b*(Sqrt[d] + Sqrt[f]*x) + 2*(a*Sqr
t[f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
 x*(b + c*x)]))])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])/(2*f)
```

Maple [A] time = 0.02, size = 399, normalized size = 1.5

$$-\frac{1}{f}\ln\left(1\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)\frac{1}{\sqrt{c}}$$

$$+\frac{d}{2f}\ln\left(1\left(2\frac{b\sqrt{df}+fa+cd}{f}+\frac{1}{f}\left(2c\sqrt{df}+bf\right)\left(x-\frac{1}{f}\sqrt{df}\right)+2\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{2c\sqrt{df}+bf}{f}\left(\right.}\right.\right.$$

$$-\frac{d}{2f}\ln\left(1\left(2\frac{-b\sqrt{df}+fa+cd}{f}+\frac{1}{f}\left(-2c\sqrt{df}+bf\right)\left(x+\frac{1}{f}\sqrt{df}\right)+2\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\frac{-2c\sqrt{df}}{f}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/f*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+1/2*d/(d
*f)^(1/2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2
)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f
)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)
+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d
*f)^(1/2)/f))-1/2*d/(d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^
(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)
*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*
f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*
(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**2/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x +
 c*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.97 $\int \frac{x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal. Leaf size=220

$$\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

```
[Out] -ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/
(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2
*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + ArcTanh[(b*Sqrt[d
] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sq
rt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[f]*Sqrt[c*d
+ b*Sqrt[d]*Sqrt[f] + a*f])
```

Rubi [A] time = 0.34055, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/
(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2
*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + ArcTanh[(b*Sqrt[d
] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sq
rt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[f]*Sqrt[c*d
+ b*Sqrt[d]*Sqrt[f] + a*f])
```

Rubi in Sympy [A] time = 57.7182, size = 202, normalized size = 0.92

$$-\frac{\operatorname{atanh}\left(\frac{-2a\sqrt{f}-b\sqrt{d}+x\left(-b\sqrt{f}-2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\operatorname{atanh}\left(\frac{-2a\sqrt{f}+b\sqrt{d}+x\left(-b\sqrt{f}+2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -atanh((-2*a*sqrt(f) - b*sqrt(d) + x*(-b*sqrt(f) - 2*c*sqrt(d)))/
(2*sqrt(a + b*x + c*x**2)*sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)))/(
2*sqrt(f)*sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)) - atanh((-2*a*sqrt
(f) + b*sqrt(d) + x*(-b*sqrt(f) + 2*c*sqrt(d)))/(2*sqrt(a + b*x +
 c*x**2)*sqrt(a*f - b*sqrt(d)*sqrt(f) + c*d)))/(2*sqrt(f)*sqrt(a*
f - b*sqrt(d)*sqrt(f) + c*d))
```

Mathematica [A] time = 0.486266, size = 289, normalized size = 1.31

$$\frac{-\frac{\log\left(\sqrt{d}\sqrt{f}-fx\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\log\left(\sqrt{d}\sqrt{f}+fx\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\log\left(2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}+2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\log\left(2\left(\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}\right.\right.}{\sqrt{af+b}}}{2\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] (-(Log[Sqrt[d]*Sqrt[f] - f*x]/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]
) - Log[Sqrt[d]*Sqrt[f] + f*x]/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f
] + Log[-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x
+ 2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]]/Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] + Log[b*(Sqrt[d] + Sqrt[f]*x) +
 2*(a*Sqrt[f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]
*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])/(2*
Sqrt[f])
```

Maple [B] time = 0.019, size = 354, normalized size = 1.6

$$\frac{1}{2f}\ln\left(1\left(2\,\frac{b\sqrt{df}+fa+cd}{f}+\frac{1}{f}\left(2\,c\sqrt{df}+bf\right)\left(x-\frac{1}{f}\sqrt{df}\right)+2\,\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^{2}c+\frac{2\,c\sqrt{df}+bf}{f}\left(x\right.}\right.\right.$$

$$+\frac{1}{2f}\ln\left(1\left(2\,\frac{-b\sqrt{df}+fa+cd}{f}+\frac{1}{f}\left(-2\,c\sqrt{df}+bf\right)\left(x+\frac{1}{f}\sqrt{df}\right)+2\,\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^{2}c+\frac{-2\,c\sqrt{df}+}{f}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+
c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2
)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/
f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1
/2)/f))+1/2/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d
*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2
*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*
(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+
c*d))^(1/2))/(x+(d*f)^(1/2)/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.641361, size = 3717, normalized size = 16.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((c*d + a*f + (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)
*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2
+ (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d
*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2))*log((2*b*c*d
*x + b^2*d + 2*(b^2*d*f - (c^3*d^3*f + a^3*f^4 - (b^2*c - 3*a*c^2
)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*sqrt(b^2*d/(c^4*d^4*f + a^4*
f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^
2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))*sqrt(c*x^2 + b*x + a)
*sqrt((c*d + a*f + (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sq
rt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (
b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^
4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)) - (2*a*c^2*d^2*
f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (b*c^2*d^2*f + a^2*b*
f^3 - (b^3 - 2*a*b*c)*d*f^2)*x)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 -
 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^
2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/x) - 1/4*sqrt((c*d + a*f +
 (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*
f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c +
6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f +
a^2*f^3 - (b^2 - 2*a*c)*d*f^2))*log((2*b*c*d*x + b^2*d - 2*(b^2*d
*f - (c^3*d^3*f + a^3*f^4 - (b^2*c - 3*a*c^2)*d^2*f^2 - (a*b^2 -
3*a^2*c)*d*f^3)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*
a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b
^2 - 2*a^3*c)*d*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f + (c
^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*f +
 a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a
^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2
*f^3 - (b^2 - 2*a*c)*d*f^2)) - (2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*
b^2 - 2*a^2*c)*d*f^2 + (b*c^2*d^2*f + a^2*b*f^3 - (b^3 - 2*a*b*c)
*d*f^2)*x)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3
)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 -
2*a^3*c)*d*f^4)))/x) + 1/4*sqrt((c*d + a*f - (c^2*d^2*f + a^2*f^3
 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*
c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 -
2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*
c)*d*f^2))*log((2*b*c*d*x + b^2*d + 2*(b^2*d*f + (c^3*d^3*f + a^3
*f^4 - (b^2*c - 3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*sqrt(
b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4
 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4))
)*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f - (c^2*d^2*f + a^2*f^3 -
(b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2
 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(
a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*
d*f^2)) + (2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2
+ (b*c^2*d^2*f + a^2*b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x)*sqrt(b^2*d
/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*
a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/x)
- 1/4*sqrt((c*d + a*f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^
2)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^
2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)
*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2))*log((2*b*c
*d*x + b^2*d - 2*(b^2*d*f + (c^3*d^3*f + a^3*f^4 - (b^2*c - 3*a*c
^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*sqrt(b^2*d/(c^4*d^4*f + a^
4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*
c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))*sqrt(c*x^2 + b*x +
a)*sqrt((c*d + a*f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*
sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 +
 (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*
f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)) + (2*a*c^2*d^
2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (b*c^2*d^2*f + a^2*
b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5
 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*
d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*
x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.98 $\int \frac{1}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal. Leaf size=220

$$\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}$$

```
[Out] ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(
2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*
Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + ArcTanh[(b*Sqrt[d]
 + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[d]*Sqrt[c*d +
 b*Sqrt[d]*Sqrt[f] + a*f])
```

Rubi [A] time = 0.333938, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(
2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*
Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + ArcTanh[(b*Sqrt[d]
 + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[d]*Sqrt[c*d +
 b*Sqrt[d]*Sqrt[f] + a*f])
```

Rubi in Sympy [A] time = 55.7611, size = 202, normalized size = 0.92

$$-\frac{\operatorname{atanh}\left(\frac{-2a\sqrt{f}-b\sqrt{d}+x\left(-b\sqrt{f}-2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\operatorname{atanh}\left(\frac{2a\sqrt{f}-b\sqrt{d}+x\left(b\sqrt{f}-2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -atanh((-2*a*sqrt(f) - b*sqrt(d) + x*(-b*sqrt(f) - 2*c*sqrt(d)))/
(2*sqrt(a + b*x + c*x**2)*sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)))/(
2*sqrt(d)*sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)) - atanh((2*a*sqrt(
f) - b*sqrt(d) + x*(b*sqrt(f) - 2*c*sqrt(d)))/(2*sqrt(a + b*x + c
*x**2)*sqrt(a*f - b*sqrt(d)*sqrt(f) + c*d)))/(2*sqrt(d)*sqrt(a*f
- b*sqrt(d)*sqrt(f) + c*d))
```

Mathematica [A] time = 0.454991, size = 301, normalized size = 1.37

$$\frac{-\frac{\log\left(\sqrt{d}\sqrt{f}-fx\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}+\frac{\log\left(\sqrt{d}\sqrt{f}+fx\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}-\frac{\log\left(\sqrt{d}\left(2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}+2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x\right)\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\log\left(\sqrt{d}\left(2\left(\sqrt{a+x(b+cx)}\sqrt{a\cdots\right.\right.\right.}{\cdots}}{2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] (-(Log[Sqrt[d]*Sqrt[f] - f*x]/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]
) + Log[Sqrt[d]*Sqrt[f] + f*x]/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f
] - Log[Sqrt[d]*(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*S
qrt[f]*x + 2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b +
c*x)])]/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] + Log[Sqrt[d]*(b*(Sqr
t[d] + Sqrt[f]*x) + 2*(a*Sqrt[f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]))]/Sqrt[c*d + b*Sqrt[d]
*Sqrt[f] + a*f])/(2*Sqrt[d])
```

Maple [B] time = 0.02, size = 358, normalized size = 1.6

$$\frac{1}{2}\ln\left(1\left(2\frac{b\sqrt{df}+fa+cd}{f}+\frac{1}{f}\left(2c\sqrt{df}+bf\right)\left(x-\frac{1}{f}\sqrt{df}\right)+2\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{2c\sqrt{df}+bf}{f}\left(x-\cdots\right.}\right.\right.$$

$$-\frac{1}{2}\ln\left(1\left(2\frac{-b\sqrt{df}+fa+cd}{f}+\frac{1}{f}\left(-2c\sqrt{df}+bf\right)\left(x+\frac{1}{f}\sqrt{df}\right)+2\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\frac{-2c\sqrt{df}+\cdots}{f}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/(d*f)^(1/2)/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^
(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*
(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(
1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(
x-(d*f)^(1/2)/f))-1/2/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^
(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)
*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*
f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*
(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.701576, size = 3565, normalized size = 16.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((c*d + a*f + (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)
*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f +
(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2
*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x
 + b^2 + 2*(b*c*d + a*b*f - (b*c^2*d^3 + a^2*b*d*f^2 - (b^3 - 2*a
*b*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c
^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 -
2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f + (c^2*
d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*
d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^
2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^
2 - (b^2 - 2*a*c)*d^2*f)) - (2*a*c^2*d^2 + 2*a^3*f^2 - 2*(a*b^2 -
 2*a^2*c)*d*f + (b*c^2*d^2 + a^2*b*f^2 - (b^3 - 2*a*b*c)*d*f)*x)*
sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (
b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*
f^3)))/x) - 1/4*sqrt((c*d + a*f + (c^2*d^3 + a^2*d*f^2 - (b^2 - 2
*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c
^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 -
2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))*
log((2*b*c*x + b^2 - 2*(b*c*d + a*b*f - (b*c^2*d^3 + a^2*b*d*f^2
- (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2
*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2
*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d +
 a*f + (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^
4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*
c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^
3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)) - (2*a*c^2*d^2 + 2*a^3*f^2
- 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^2*d^2 + a^2*b*f^2 - (b^3 - 2*a*b
*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^
3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2
*a^3*c)*d^2*f^3)))/x) + 1/4*sqrt((c*d + a*f - (c^2*d^3 + a^2*d*f^
2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2
*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2
*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a
*c)*d^2*f))*log((2*b*c*x + b^2 + 2*(b*c*d + a*b*f + (b*c^2*d^3 +
a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*
f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)
*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)
*sqrt((c*d + a*f - (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sq
rt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^
4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^
3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)) + (2*a*c^2*d^2
+ 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^2*d^2 + a^2*b*f^2 -
(b^3 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*
c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*
(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/x) - 1/4*sqrt((c*d + a*f - (c^2*d^
3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*
f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)
*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2
- (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b^2 - 2*(b*c*d + a*b*f + (
b*c^2*d^3 + a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^2*f/(c^4*
d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c
+ 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^
2 + b*x + a)*sqrt((c*d + a*f - (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*
c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)
*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a
^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)) + (
2*a*c^2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^2*d^2 +
a^2*b*f^2 - (b^3 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f
^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*
d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*
x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.99 $\int \frac{1}{x\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal. Leaf size=267

$$-\frac{\sqrt{f}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{f}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d}$$

```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(Sqrt[a]
*d)) - (Sqrt[f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] -
 b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*
x + c*x^2])])/(2*d*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (Sqrt[f
]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)
/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/
(2*d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rubi [A] time = 1.31927, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{\sqrt{f}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{f}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(Sqrt[a]
*d)) - (Sqrt[f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] -
 b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*
x + c*x^2])])/(2*d*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (Sqrt[f
]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)
/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/
(2*d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rubi in Sympy [A] time = 124.81, size = 241, normalized size = 0.9

$$-\frac{\sqrt{f}\operatorname{atanh}\left(\frac{-2a\sqrt{f}-b\sqrt{d}+x\left(-b\sqrt{f}-2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\sqrt{f}\operatorname{atanh}\left(\frac{-2a\sqrt{f}+b\sqrt{d}+x\left(-b\sqrt{f}+2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}-\frac{\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ad}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -sqrt(f)*atanh((-2*a*sqrt(f) - b*sqrt(d) + x*(-b*sqrt(f) - 2*c*sq
rt(d)))/(2*sqrt(a + b*x + c*x**2)*sqrt(a*f + b*sqrt(d)*sqrt(f) +
c*d)))/(2*d*sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)) - sqrt(f)*atanh(
(-2*a*sqrt(f) + b*sqrt(d) + x*(-b*sqrt(f) + 2*c*sqrt(d)))/(2*sqrt
(a + b*x + c*x**2)*sqrt(a*f - b*sqrt(d)*sqrt(f) + c*d)))/(2*d*sqr
t(a*f - b*sqrt(d)*sqrt(f) + c*d)) - atanh((2*a + b*x)/(2*sqrt(a)*
sqrt(a + b*x + c*x**2)))/(sqrt(a)*d)
```

Mathematica [A] time = 1.0315, size = 351, normalized size = 1.31

$$-\frac{\frac{\sqrt{f}\log\left(\sqrt{d}\sqrt{f}-fx\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\sqrt{f}\log\left(\sqrt{d}\sqrt{f}+fx\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{f}\log\left(2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}+2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{f}\log\left(2\left(\sqrt{a+x(b+cx)}\sqrt{af\right.\right.}{}}{2d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((2*Log[x])/Sqrt[a] - (Sqrt[f]*Log[Sqrt[d]*Sqrt[f] - f*x])/Sqrt[c
*d + b*Sqrt[d]*Sqrt[f] + a*f] - (Sqrt[f]*Log[Sqrt[d]*Sqrt[f] + f*
x])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] - (2*Log[2*a + b*x + 2*Sq
rt[a]*Sqrt[a + x*(b + c*x)]])/Sqrt[a] + (Sqrt[f]*Log[-(b*Sqrt[d])
 + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x + 2*Sqrt[c*d - b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]])/Sqrt[c*d - b*Sqrt[d]*
Sqrt[f] + a*f] + (Sqrt[f]*Log[b*(Sqrt[d] + Sqrt[f]*x) + 2*(a*Sqrt
[f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
x*(b + c*x)])])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])/(2*d)
```

Maple [A] time = 0.022, size = 391, normalized size = 1.5

$$-\frac{1}{d}\ln\left(\frac{1}{x}\left(2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)\frac{1}{\sqrt{a}}$$

$$+\frac{1}{2\,d}\ln\left(1\left(2\,\frac{b\sqrt{df}+fa+cd}{f}+\frac{1}{f}\left(2\,c\sqrt{df}+bf\right)\left(x-\frac{1}{f}\sqrt{df}\right)+2\,\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{2\,c\sqrt{df}+bf}{f}\left(\right.}\right.\right.$$

$$+\frac{1}{2\,d}\ln\left(1\left(2\,\frac{-b\sqrt{df}+fa+cd}{f}+\frac{1}{f}\left(-2\,c\sqrt{df}+bf\right)\left(x+\frac{1}{f}\sqrt{df}\right)+2\,\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\frac{-2\,c\sqrt{df}+}{f}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+1/2/d/
((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f
+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+
c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(
d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)
)+1/2/d/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1
/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*
(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*
(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^
(1/2))/(x+(d*f)^(1/2)/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{cx^2 + bx + a}(fx^2 - d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x), x)
```

Fricas [A] time = 71.9786, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*d*sqrt((c*d*f + a*f^2 + (c^2*d^4 + a^2*d^2*f^2 - (b
^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c
^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(
a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a
*c)*d^3*f))*log((2*b*c*f^2*x + b^2*f^2 + 2*(b^2*d*f^2 - (c^3*d^5
+ a^3*d^2*f^3 - (b^2*c - 3*a*c^2)*d^4*f - (a*b^2 - 3*a^2*c)*d^3*f
^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d
^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3
*c)*d^4*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d*f + a*f^2 + (c^2*d
^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a
^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a
^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2
*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)) - (2*a*c^2*d^3*f + 2*a^3*d*f^3 -
 2*(a*b^2 - 2*a^2*c)*d^2*f^2 + (b*c^2*d^3*f + a^2*b*d*f^3 - (b^3
- 2*a*b*c)*d^2*f^2)*x)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b
^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 -
 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/x) - sqrt(a)*d*sqrt((c*d*f + a*
f^2 + (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/
(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a
*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c
^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f))*log((2*b*c*f^2*x + b
^2*f^2 - 2*(b^2*d*f^2 - (c^3*d^5 + a^3*d^2*f^3 - (b^2*c - 3*a*c^2
)*d^4*f - (a*b^2 - 3*a^2*c)*d^3*f^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4*
d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*
c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))*sqrt(c*x^2 + b*x
+ a)*sqrt((c*d*f + a*f^2 + (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)
*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^
3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2
*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f))
 - (2*a*c^2*d^3*f + 2*a^3*d*f^3 - 2*(a*b^2 - 2*a^2*c)*d^2*f^2 + (
b*c^2*d^3*f + a^2*b*d*f^3 - (b^3 - 2*a*b*c)*d^2*f^2)*x)*sqrt(b^2*
f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 -
 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3))
)/x) + sqrt(a)*d*sqrt((c*d*f + a*f^2 - (c^2*d^4 + a^2*d^2*f^2 - (
b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*
c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*
(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*
a*c)*d^3*f))*log((2*b*c*f^2*x + b^2*f^2 + 2*(b^2*d*f^2 + (c^3*d^5
 + a^3*d^2*f^3 - (b^2*c - 3*a*c^2)*d^4*f - (a*b^2 - 3*a^2*c)*d^3*
f^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*
d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^
3*c)*d^4*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d*f + a*f^2 - (c^2*
d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 +
a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*
a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^
2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)) + (2*a*c^2*d^3*f + 2*a^3*d*f^3
```

```
- 2*(a*b^2 - 2*a^2*c)*d^2*f^2 + (b*c^2*d^3*f + a^2*b*d*f^3 - (b^3
 - 2*a*b*c)*d^2*f^2)*x)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(
b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2
- 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/x) - sqrt(a)*d*sqrt((c*d*f + a
*f^2 - (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3
/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*
a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(
c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f))*log((2*b*c*f^2*x +
b^2*f^2 - 2*(b^2*d*f^2 + (c^3*d^5 + a^3*d^2*f^3 - (b^2*c - 3*a*c^
2)*d^4*f - (a*b^2 - 3*a^2*c)*d^3*f^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4
*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2
*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))*sqrt(c*x^2 + b*x
 + a)*sqrt((c*d*f + a*f^2 - (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c
)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c
^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 -
2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)
) + (2*a*c^2*d^3*f + 2*a^3*d*f^3 - 2*(a*b^2 - 2*a^2*c)*d^2*f^2 +
(b*c^2*d^3*f + a^2*b*d*f^3 - (b^3 - 2*a*b*c)*d^2*f^2)*x)*sqrt(b^2
*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4
- 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)
))/x) + 2*log((4*(a*b*x + 2*a^2)*sqrt(c*x^2 + b*x + a) - (8*a*b*x
 + (b^2 + 4*a*c)*x^2 + 8*a^2)*sqrt(a))/x^2))/(sqrt(a)*d), 1/4*(sq
rt(-a)*d*sqrt((c*d*f + a*f^2 + (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*
a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*
a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2
 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3
*f))*log((2*b*c*f^2*x + b^2*f^2 + 2*(b^2*d*f^2 - (c^3*d^5 + a^3*d
^2*f^3 - (b^2*c - 3*a*c^2)*d^4*f - (a*b^2 - 3*a^2*c)*d^3*f^2)*sqr
t(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f +
(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4
*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d*f + a*f^2 + (c^2*d^4 + a^
2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*
f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)
*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^
2 - (b^2 - 2*a*c)*d^3*f)) - (2*a*c^2*d^3*f + 2*a^3*d*f^3 - 2*(a*b
^2 - 2*a^2*c)*d^2*f^2 + (b*c^2*d^3*f + a^2*b*d*f^3 - (b^3 - 2*a*b
*c)*d^2*f^2)*x)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2
- 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2
*b^2 - 2*a^3*c)*d^4*f^3)))/x) - sqrt(-a)*d*sqrt((c*d*f + a*f^2 +
(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d
^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c
 + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4
 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f))*log((2*b*c*f^2*x + b^2*f^2
 - 2*(b^2*d*f^2 - (c^3*d^5 + a^3*d^2*f^3 - (b^2*c - 3*a*c^2)*d^4*
f - (a*b^2 - 3*a^2*c)*d^3*f^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^
4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d
^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))*sqrt(c*x^2 + b*x + a)*s
qrt((c*d*f + a*f^2 + (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f
)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6
*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c
)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)) - (2*
a*c^2*d^3*f + 2*a^3*d*f^3 - 2*(a*b^2 - 2*a^2*c)*d^2*f^2 + (b*c^2*
d^3*f + a^2*b*d*f^3 - (b^3 - 2*a*b*c)*d^2*f^2)*x)*sqrt(b^2*f^3/(c
^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b
^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/x) +
 sqrt(-a)*d*sqrt((c*d*f + a*f^2 - (c^2*d^4 + a^2*d^2*f^2 - (b^2 -
 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 -
 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*
b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*
d^3*f))*log((2*b*c*f^2*x + b^2*f^2 + 2*(b^2*d*f^2 + (c^3*d^5 + a^
3*d^2*f^3 - (b^2*c - 3*a*c^2)*d^4*f - (a*b^2 - 3*a^2*c)*d^3*f^2)*
sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f
 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*
d^4*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d*f + a*f^2 - (c^2*d^4 +
 a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d
^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c
^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2
*f^2 - (b^2 - 2*a*c)*d^3*f)) + (2*a*c^2*d^3*f + 2*a^3*d*f^3 - 2*(
a*b^2 - 2*a^2*c)*d^2*f^2 + (b*c^2*d^3*f + a^2*b*d*f^3 - (b^3 - 2*
a*b*c)*d^2*f^2)*x)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c
^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(
a^2*b^2 - 2*a^3*c)*d^4*f^3)))/x) - sqrt(-a)*d*sqrt((c*d*f + a*f^2
```

```
 - (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^
4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*
d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f))*log((2*b*c*f^2*x + b^2*
f^2 - 2*(b^2*d*f^2 + (c^3*d^5 + a^3*d^2*f^3 - (b^2*c - 3*a*c^2)*d
^4*f - (a*b^2 - 3*a^2*c)*d^3*f^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3
*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2
)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))*sqrt(c*x^2 + b*x + a
)*sqrt((c*d*f + a*f^2 - (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^
3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*
d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^
3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)) +
(2*a*c^2*d^3*f + 2*a^3*d*f^3 - 2*(a*b^2 - 2*a^2*c)*d^2*f^2 + (b*c
^2*d^3*f + a^2*b*d*f^3 - (b^3 - 2*a*b*c)*d^2*f^2)*x)*sqrt(b^2*f^3
/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*
a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/x
) - 4*arctan(1/2*(b*x + 2*a)*sqrt(-a)/(sqrt(c*x^2 + b*x + a)*a)))
/(sqrt(-a)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-dx\sqrt{a+bx+cx^2}+fx^3\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-d*x*sqrt(a + b*x + c*x**2) + f*x**3*sqrt(a + b*x +
c*x**2)), x)
```

GIAC/XCAS [A] time = 0.281477, size = 1, normalized size = 0.

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x),x, algorithm="giac")
```

```
[Out] sage2
```

3.100 $\int \frac{1}{x^2\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal. Leaf size=291

$$\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}+\frac{f\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{f\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\sqrt{a+bx+cx^2}}{adx}$$

```
[Out] -(Sqrt[a + b*x + c*x^2]/(a*d*x)) + (b*ArcTanh[(2*a + b*x)/(2*Sqrt
[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)*d) + (f*ArcTanh[(b*Sqrt[d
] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sq
rt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*Sqrt[c*d
 - b*Sqrt[d]*Sqrt[f] + a*f]) + (f*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f
] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f]
+ a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*Sqrt[c*d + b*Sqrt[d]*S
qrt[f] + a*f])
```

Rubi [A] time = 1.36445, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$

$$\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}+\frac{f\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{f\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\sqrt{a+bx+cx^2}}{adx}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(a*d*x)) + (b*ArcTanh[(2*a + b*x)/(2*Sqrt
[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)*d) + (f*ArcTanh[(b*Sqrt[d
] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sq
rt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*Sqrt[c*d
 - b*Sqrt[d]*Sqrt[f] + a*f]) + (f*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f
] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f]
+ a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*Sqrt[c*d + b*Sqrt[d]*S
qrt[f] + a*f])
```

Rubi in Sympy [A] time = 131.872, size = 262, normalized size = 0.9

$$-\frac{f\,\mathrm{atanh}\left(\frac{-2a\sqrt{f}-b\sqrt{d}+x\left(-b\sqrt{f}-2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{\frac{3}{2}}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{f\,\mathrm{atanh}\left(\frac{2a\sqrt{f}-b\sqrt{d}+x\left(b\sqrt{f}-2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{\frac{3}{2}}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}-\frac{\sqrt{a+bx+cx^2}}{adx}+\frac{b\,\mathrm{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -f*atanh((-2*a*sqrt(f) - b*sqrt(d) + x*(-b*sqrt(f) - 2*c*sqrt(d))
)/(2*sqrt(a + b*x + c*x**2)*sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)))
/(2*d**(3/2)*sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)) - f*atanh((2*a*
sqrt(f) - b*sqrt(d) + x*(b*sqrt(f) - 2*c*sqrt(d)))/(2*sqrt(a + b*
x + c*x**2)*sqrt(a*f - b*sqrt(d)*sqrt(f) + c*d)))/(2*d**(3/2)*sqr
t(a*f - b*sqrt(d)*sqrt(f) + c*d)) - sqrt(a + b*x + c*x**2)/(a*d*x
) + b*atanh((2*a + b*x)/(2*sqrt(a)*sqrt(a + b*x + c*x**2)))/(2*a*
*(3/2)*d)
```

Mathematica [A] time = 2.27305, size = 432, normalized size = 1.48

$$\frac{\frac{b\sqrt{d}\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+bx\right)}{a^{3/2}}-\frac{b\sqrt{d}\log(x)}{a^{3/2}}-\frac{f\log\left(\sqrt{d}\sqrt{f}-fx\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}+\frac{f\log\left(\sqrt{d}\sqrt{f}+fx\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}-\frac{f\log\left(\sqrt{d}\left(2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}+2a\sqrt{f}\cdots\right.\right.}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\cdots}{2d\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-2*b*Sqrt[d])/(a*Sqrt[a + x*(b + c*x)]) - (2*Sqrt[d])/(x*Sqrt[a
 + x*(b + c*x)]) - (2*c*Sqrt[d]*x)/(a*Sqrt[a + x*(b + c*x)]) - (b
*Sqrt[d]*Log[x])/a^(3/2) - (f*Log[Sqrt[d]*Sqrt[f] - f*x])/Sqrt[c*
d + b*Sqrt[d]*Sqrt[f] + a*f] + (f*Log[Sqrt[d]*Sqrt[f] + f*x])/Sqr
t[c*d - b*Sqrt[d]*Sqrt[f] + a*f] + (b*Sqrt[d]*Log[2*a + b*x + 2*S
qrt[a]*Sqrt[a + x*(b + c*x)]])/a^(3/2) - (f*Log[Sqrt[d]*(-(b*Sqrt
[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x + 2*Sqrt[c*d - b
*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d - b*Sqr
t[d]*Sqrt[f] + a*f] + (f*Log[Sqrt[d]*(b*(Sqrt[d] + Sqrt[f]*x) + 2
*(a*Sqrt[f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*S
qrt[a + x*(b + c*x)]))])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])/(2*
d^(3/2))
```

Maple [A] time = 0.024, size = 427, normalized size = 1.5

$$-\frac{1}{adx}\sqrt{cx^2+bx+a}+\frac{b}{2\,d}\ln\left(\frac{1}{x}\left(2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)a^{-\frac{3}{2}}$$

$$+\frac{f}{2\,d}\ln\left(1\left(2\,\frac{b\sqrt{df}+fa+cd}{f}+\frac{1}{f}\left(2\,c\sqrt{df}+bf\right)\left(x-\frac{1}{f}\sqrt{df}\right)+2\,\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{2\,c\sqrt{df}+bf}{f}\left(\cdots\right.}\right.\right.$$

$$-\frac{f}{2\,d}\ln\left(1\left(2\,\frac{-b\sqrt{df}+fa+cd}{f}+\frac{1}{f}\left(-2\,c\sqrt{df}+bf\right)\left(x+\frac{1}{f}\sqrt{df}\right)+2\,\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\frac{-2\,c\sqrt{df}\cdots}{f}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -(c*x^2+b*x+a)^(1/2)/a/d/x+1/2/d*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*
(c*x^2+b*x+a)^(1/2))/x)+1/2*f/d/(d*f)^(1/2)/((b*(d*f)^(1/2)+f*a+c
*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f
)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*
f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f
)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))-1/2*f/d/(d*f)^(1/2)
```

```
/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a
+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*
f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(
1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/
(x+(d*f)^(1/2)/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{cx^2+bx+a}(fx^2-d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-dx^2\sqrt{a+bx+cx^2}+fx^4\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-d*x**2*sqrt(a + b*x + c*x**2) + f*x**4*sqrt(a + b*x
 + c*x**2)), x)
```

GIAC/XCAS [A] time = 0.65815, size = 4, normalized size = 0.01

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.101 $\int \frac{1}{x^3\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal. Leaf size=376

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{f^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{f^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2}-\frac{\sqrt{a+bx+cx^2}}{2adx^2}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(2*a*d*x^2) + (3*b*Sqrt[a + b*x + c*x^2])/
(4*a^2*d*x) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqr
t[a + b*x + c*x^2])])/(8*a^(5/2)*d) - (f*ArcTanh[(2*a + b*x)/(2*S
qrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*d^2) - (f^(3/2)*ArcTanh[
(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c
*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqr
t[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (f^(3/2)*ArcTanh[(b*Sqrt[d] +
 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[c*d + b*Sq
rt[d]*Sqrt[f] + a*f])
```

Rubi [A] time = 1.59301, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{f^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{f^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2}-\frac{\sqrt{a+bx+cx^2}}{2adx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```



```
[Out] -Sqrt[a + b*x + c*x^2]/(2*a*d*x^2) + (3*b*Sqrt[a + b*x + c*x^2])/
(4*a^2*d*x) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqr
t[a + b*x + c*x^2])])/(8*a^(5/2)*d) - (f*ArcTanh[(2*a + b*x)/(2*S
qrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*d^2) - (f^(3/2)*ArcTanh[
(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c
*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqr
t[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (f^(3/2)*ArcTanh[(b*Sqrt[d] +
 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[c*d + b*Sq
rt[d]*Sqrt[f] + a*f])
```

Rubi in Sympy [A] time = 159.291, size = 342, normalized size = 0.91

$$-\frac{f^{\frac{3}{2}}\operatorname{atanh}\left(\frac{-2a\sqrt{f}-b\sqrt{d}+x\left(-b\sqrt{f}-2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{f^{\frac{3}{2}}\operatorname{atanh}\left(\frac{-2a\sqrt{f}+b\sqrt{d}+x\left(-b\sqrt{f}+2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^2\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}-\frac{\sqrt{a+bx+cx^2}}{2adx^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{f\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2}-\frac{\left(-4ac+3b^2\right)\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -f**(3/2)*atanh((-2*a*sqrt(f) - b*sqrt(d) + x*(-b*sqrt(f) - 2*c*s
qrt(d)))/(2*sqrt(a + b*x + c*x**2)*sqrt(a*f + b*sqrt(d)*sqrt(f) +
 c*d)))/(2*d**2*sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)) - f**(3/2)*a
tanh((-2*a*sqrt(f) + b*sqrt(d) + x*(-b*sqrt(f) + 2*c*sqrt(d)))/(2
*sqrt(a + b*x + c*x**2)*sqrt(a*f - b*sqrt(d)*sqrt(f) + c*d)))/(2*
d**2*sqrt(a*f - b*sqrt(d)*sqrt(f) + c*d)) - sqrt(a + b*x + c*x**2
)/(2*a*d*x**2) + 3*b*sqrt(a + b*x + c*x**2)/(4*a**2*d*x) - f*atan
h((2*a + b*x)/(2*sqrt(a)*sqrt(a + b*x + c*x**2)))/(sqrt(a)*d**2)
- (-4*a*c + 3*b**2)*atanh((2*a + b*x)/(2*sqrt(a)*sqrt(a + b*x + c
*x**2)))/(8*a**(5/2)*d)
```

Mathematica [A] time = 2.71356, size = 418, normalized size = 1.11

$$-\frac{2d(2a-3bx)\sqrt{a+x(b+cx)}}{a^2x^2}+\frac{\log(x)\left(8a^2f-4acd+3b^2d\right)}{a^{5/2}}-\frac{\left(8a^2f-4acd+3b^2d\right)\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+bx\right)}{a^{5/2}}-\frac{4f^{3/2}\log\left(\sqrt{d}\sqrt{f}-fx\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{4f^{3/2}\log\left(\sqrt{d}\ldots\right.}{\sqrt{af+b\left(-\sqrt{d}\right)\ldots}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-2*d*(2*a - 3*b*x)*Sqrt[a + x*(b + c*x)])/(a^2*x^2) + ((3*b^2*d
 - 4*a*c*d + 8*a^2*f)*Log[x])/a^(5/2) - (4*f^(3/2)*Log[Sqrt[d]*Sq
rt[f] - f*x])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f] - (4*f^(3/2)*Lo
g[Sqrt[d]*Sqrt[f] + f*x])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] - (
(3*b^2*d - 4*a*c*d + 8*a^2*f)*Log[2*a + b*x + 2*Sqrt[a]*Sqrt[a +
x*(b + c*x)]])/a^(5/2) + (4*f^(3/2)*Log[-(b*Sqrt[d]) + 2*a*Sqrt[f
] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x + 2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f]
+ a*f]*Sqrt[a + x*(b + c*x)]])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f
] + (4*f^(3/2)*Log[b*(Sqrt[d] + Sqrt[f]*x) + 2*(a*Sqrt[f] + c*Sqr
t[d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)
])])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])/(8*d^2)
```

Maple [A] time = 0.026, size = 519, normalized size = 1.4

$$-\frac{1}{2\,adx^{2}}\sqrt{cx^{2}+bx+a}+\frac{3\,b}{4\,a^{2}dx}\sqrt{cx^{2}+bx+a}-\frac{3\,b^{2}}{8\,d}\ln\left(\frac{1}{x}\left(2\,a+bx+2\,\sqrt{a}\sqrt{cx^{2}+bx+a}\right)\right)a^{-\frac{5}{2}}+\frac{c}{2\,d}\ln\left(\frac{1}{x}\left(2\,a+bx+2\,\sqrt{a}\sqrt{cx^{2}+bx+a}\right)\right)a^{-\frac{3}{2}}-\frac{f}{d^{2}}\ln\left(\frac{1}{x}\left(2\,a+bx+2\,\sqrt{a}\sqrt{cx^{2}+bx+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{f}{2\,d^{2}}\ln\left(1\left(2\,\frac{b\sqrt{df}+fa+cd}{f}+\frac{1}{f}\left(2\,c\sqrt{df}+bf\right)\left(x-\frac{1}{f}\sqrt{df}\right)+2\,\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^{2}c+\frac{2\,c\sqrt{df}+bf}{f}\left(\right.}\right.\right.+\frac{f}{2\,d^{2}}\ln\left(1\left(2\,\frac{-b\sqrt{df}+fa+cd}{f}+\frac{1}{f}\left(-2\,c\sqrt{df}+bf\right)\left(x+\frac{1}{f}\sqrt{df}\right)+2\,\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^{2}c+\frac{-2\,c\sqrt{df}}{f}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2*(c*x^2+b*x+a)^(1/2)/a/d/x^2+3/4*b*(c*x^2+b*x+a)^(1/2)/a^2/d/
x-3/8/d*b^2/a^(5/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
+1/2/d*c/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-f/
d^2/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+1/2*f/d
^2/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d
)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f
*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(
x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)
/f))+1/2*f/d^2/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(
d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+
2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f
*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a
+c*d))^(1/2))/(x+(d*f)^(1/2)/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{\sqrt{cx^{2}+bx+a}(fx^{2}-d)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^3),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{-dx^{3}\sqrt{a+bx+cx^{2}}+fx^{5}\sqrt{a+bx+cx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-d*x**3*sqrt(a + b*x + c*x**2) + f*x**5*sqrt(a + b*x
 + c*x**2)), x)
```

GIAC/XCAS [A] time = 0.677075, size = 4, normalized size = 0.01

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^3),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.102 $\int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

Optimal. Leaf size=466

$$-\frac{2d^2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{2d(b+2cx)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{2b\sqrt{a+bx+cx^2}}{cf\left(b^2-4ac\right)}-\frac{2x(2a+bx)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}f}+\frac{d^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{d^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

```
[Out] (-2*x*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) + (2*d
*(b + 2*c*x))/((b^2 - 4*a*c)*f^2*Sqrt[a + b*x + c*x^2]) - (2*d^2*
(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/
((b^2 - 4*a*c)*f^2*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2
]) + (2*b*Sqrt[a + b*x + c*x^2])/(c*(b^2 - 4*a*c)*f) - ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(c^(3/2)*f) + (d^(3/
2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x
)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])
/(2*f*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d^(3/2)*ArcTanh[(
b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*
d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f*(c*d +
 b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rubi [A] time = 2.90387, antiderivative size = 466, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$

$$-\frac{2d^2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{2d(b+2cx)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{2b\sqrt{a+bx+cx^2}}{cf\left(b^2-4ac\right)}-\frac{2x(2a+bx)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}f}+\frac{d^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{d^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```


```
[Out] (-2*x*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) + (2*d
*(b + 2*c*x))/((b^2 - 4*a*c)*f^2*Sqrt[a + b*x + c*x^2]) - (2*d^2*
(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/
((b^2 - 4*a*c)*f^2*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2
]) + (2*b*Sqrt[a + b*x + c*x^2])/(c*(b^2 - 4*a*c)*f) - ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(c^(3/2)*f) + (d^(3/
2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x
)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])
/(2*f*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d^(3/2)*ArcTanh[(
b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*
d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f*(c*d +
 b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 5.48617, size = 480, normalized size = 1.03

$$\frac{1}{2}\left(\frac{4\left(-a^3f(b-2cx)-a^2\left(b^2fx+3bcd-2c^2dx\right)+ab^2d(b-4cx)+b^4dx\right)}{c\left(4ac-b^2\right)\sqrt{a+x(b+cx)}\left(f\left(a^2f-b^2d\right)+2acdf+c^2d^2\right)}-\frac{2\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)}{c^{3/2}f}-\frac{d^{3/2}\log\left(\sqrt{d}\sqrt{f}-fx\right)}{f\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}+\frac{d^{3/2}\log\left(\sqrt{d}\sqrt{f}+fx\right)}{f\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}-\frac{d^{3/2}\log\left(\sqrt{d}\left(2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}+2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x\right)\right)}{f\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{d^{3/2}\log\left(\sqrt{d}\left(2\left(\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}+a\sqrt{f}+c\sqrt{d}x\right)+b\left(\sqrt{d}+\sqrt{f}x\right)\right)\right)}{f\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] ((4*(b^4*d*x + a*b^2*d*(b - 4*c*x) - a^3*f*(b - 2*c*x) - a^2*(3*b
*c*d - 2*c^2*d*x + b^2*f*x)))/(c*(-b^2 + 4*a*c)*(c^2*d^2 + 2*a*c*
d*f + f*(-(b^2*d) + a^2*f))*Sqrt[a + x*(b + c*x)]) - (d^(3/2)*Log
[Sqrt[d]*Sqrt[f] - f*x])/(f*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)
) + (d^(3/2)*Log[Sqrt[d]*Sqrt[f] + f*x])/(f*(c*d - b*Sqrt[d]*Sqrt
[f] + a*f)^(3/2)) - (2*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b +
c*x)]])/(c^(3/2)*f) - (d^(3/2)*Log[Sqrt[d]*(-(b*Sqrt[d]) + 2*a*Sq
rt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x + 2*Sqrt[c*d - b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(f*(c*d - b*Sqrt[d]*Sqrt[f] +
 a*f)^(3/2)) + (d^(3/2)*Log[Sqrt[d]*(b*(Sqrt[d] + Sqrt[f]*x) + 2*
(a*Sqrt[f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sq
rt[a + x*(b + c*x)]))])/(f*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
)/2
```

Maple [B] time = 0.025, size = 1648, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -4/f^2*d/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*c*x-2/f^2*d/(4*a*c-b^2)/
(c*x^2+b*x+a)^(1/2)*b+1/f*x/c/(c*x^2+b*x+a)^(1/2)-1/2/f*b/c^2/(c*
x^2+b*x+a)^(1/2)-1/f*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-1/2/
f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/f/c^(3/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2/f*d^2/(d*f)^(1/2)/(b*(d*f)^(
1/2)+f*a+c*d)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(
d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)+2/f^2*d^2/(b*(d*f)
^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*x*c^
2+1/f*d^2/(d*f)^(1/2)/(b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-(d*
f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f
)^(1/2)+f*a+c*d)/f)^(1/2)*x*b*c+1/f^2*d^2/(b*(d*f)^(1/2)+f*a+c*d)
/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d
*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*b*c+1/2/f*d^2/(d*f)
^(1/2)/(b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c
+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d
)/f)^(1/2)*b^2+1/2/f*d^2/(d*f)^(1/2)/(b*(d*f)^(1/2)+f*a+c*d)/((b*
(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*
c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)
/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)
^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))+1/
2/f*d^2/(d*f)^(1/2)/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*f)^(1/2)/f)^2
*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/
2)+f*a+c*d))^(1/2)+2/f^2*d^2/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)
/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)
/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*x*c^2-1/f*d^2/(d*f)^(1/2)
/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*
(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+
c*d))^(1/2)*x*b*c+1/f^2*d^2/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/
((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/
f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*b*c-1/2/f*d^2/(d*f)^(1/2)/
(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(
-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c
*d))^(1/2)*b^2-1/2/f*d^2/(d*f)^(1/2)/(-b*(d*f)^(1/2)+f*a+c*d)/(1/
f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d
)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(
1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)
+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(
d*f)^(1/2)/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^4/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^4/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^4/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.103 $\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

Optimal. Leaf size=341

$$-\frac{2d\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}-\frac{2(2a+bx)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{d\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{d\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

```
[Out] (-2*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) - (2*d*(
a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c
)*f*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - (d*ArcTanh
[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[f
]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d*ArcTanh[(b*Sqrt[d]
+ 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt
[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[f]*(c*d + b*S
qrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rubi [A] time = 2.18094, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$-\frac{2d\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}-\frac{2(2a+bx)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{d\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{d\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) - (2*d*(
a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c
)*f*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - (d*ArcTanh
[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[f
]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d*ArcTanh[(b*Sqrt[d]
+ 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt
[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[f]*(c*d + b*S
qrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 3.65805, size = 414, normalized size = 1.21

$$\frac{1}{2}\left(\frac{8a^3f+4a^2(bfx+2cd)-4abd(b-3cx)-4b^3dx}{(b^2-4ac)\sqrt{a+x(b+cx)}\,(f(b^2d-a^2f)-2acdf-c^2d^2)}-\frac{d\log\left(\sqrt{d}\sqrt{f}-fx\right)}{\sqrt{f}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}-\frac{d\log\left(\sqrt{d}\sqrt{f}+fx\right)}{\sqrt{f}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{d\log\left(\sqrt{d}\left(2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}+2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x\right)\right)}{\sqrt{f}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{d\log\left(\sqrt{d}\left(2\left(\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}+a\sqrt{f}+c\sqrt{d}x\right)+b\left(\sqrt{d}+\sqrt{f}x\right)\right)\right)}{\sqrt{f}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] ((8*a^3*f - 4*b^3*d*x - 4*a*b*d*(b - 3*c*x) + 4*a^2*(2*c*d + b*f*
x))/((b^2 - 4*a*c)*(-(c^2*d^2) - 2*a*c*d*f + f*(b^2*d - a^2*f))*S
qrt[a + x*(b + c*x)]) - (d*Log[Sqrt[d]*Sqrt[f] - f*x])/(Sqrt[f]*(
c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) - (d*Log[Sqrt[d]*Sqrt[f] +
f*x])/(Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d*Log[Sq
rt[d]*(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x +
 2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(
Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d*Log[Sqrt[d]*(
b*(Sqrt[d] + Sqrt[f]*x) + 2*(a*Sqrt[f] + c*Sqrt[d]*x + Sqrt[c*d +
 b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]))])/(Sqrt[f]*(c*d
 + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)))/2
```

Maple [B] time = 0.023, size = 1480, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/f/c/(c*x^2+b*x+a)^(1/2)+2/f*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x
+1/f*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/2/f*d/(b*(d*f)^(1/2)
+f*a+c*d)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)
^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)+2/f^2*d/(b*(d*f)^(1/2)
+f*a+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f
)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*(d*f)^(1/2
)*x*c^2+1/f*d/(b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)
/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+
f*a+c*d)/f)^(1/2)*x*b*c+1/f^2*d/(b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^
2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/
f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*(d*f)^(1/2)*b*c+1/2/f*d/(b*(d
*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^
(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*b
^2+1/2/f*d/(b*(d*f)^(1/2)+f*a+c*d)/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1
/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d
*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/
f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f
*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))-1/2/f*d/(-b*(d*f)^(1/2)+f*a+
```

```
c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(
1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-2/f^2*d/(-b*(d*f)^(1/
2)+f*a+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1
/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*(d
*f)^(1/2)*x*c^2+1/f*d/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x+(d
*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f
*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*x*b*c-1/f^2*d/(-b*(d*f)^(1/2)+f*
a+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b
*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*(d*f)^(
1/2)*b*c+1/2/f*d/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x+(d*f)^(
1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*
(d*f)^(1/2)+f*a+c*d))^(1/2)*b^2+1/2/f*d/(-b*(d*f)^(1/2)+f*a+c*d)/
(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+
c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f
)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1
/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(
x+(d*f)^(1/2)/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^3/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.104 $\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

Optimal. Leaf size=297

$$\frac{2\left(cx\left(b^2d-2a(af+cd)\right)+ab(cd-af)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

```
[Out] (2*(a*b*(c*d - a*f) + c*(b^2*d - 2*a*(c*d + a*f))*x))/((b^2 - 4*a
*c)*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (Sqrt[d]*A
rcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2
*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*
(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (Sqrt[d]*ArcTanh[(b*Sqrt
[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*
Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*(c*d + b*Sqrt[
d]*Sqrt[f] + a*f)^(3/2))
```

Rubi [A] time = 0.935274, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{2\left(cx\left(b^2d-2a(af+cd)\right)+ab(cd-af)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*(a*b*(c*d - a*f) + c*(b^2*d - 2*a*(c*d + a*f))*x))/((b^2 - 4*a
*c)*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (Sqrt[d]*A
rcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2
*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*
(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (Sqrt[d]*ArcTanh[(b*Sqrt
[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*
Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*(c*d + b*Sqrt[
d]*Sqrt[f] + a*f)^(3/2))
```

Rubi in Sympy [A] time = 167.761, size = 338, normalized size = 1.14

$$\frac{\sqrt{d}\left(af-b\sqrt{d}\sqrt{f}+cd\right)\operatorname{atanh}\left(\frac{-2a\sqrt{f}-b\sqrt{d}+x\left(-b\sqrt{f}-2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\left(b^2df-(af+cd)^2\right)\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\sqrt{d}\left(af+b\sqrt{d}\sqrt{f}+cd\right)\operatorname{atanh}\left(\frac{-2a\sqrt{f}+b\sqrt{d}+x\left(-b\sqrt{f}+2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}\right)}{2\left(b^2df-(af+cd)^2\right)\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}-\frac{2\left(ab(af-cd)+cx\left(2a(af+cd)-b^2d\right)\right)}{(-4ac+b^2)\left(b^2df-(af+cd)^2\right)\sqrt{a+bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] sqrt(d)*(a*f - b*sqrt(d)*sqrt(f) + c*d)*atanh((-2*a*sqrt(f) - b*s
qrt(d) + x*(-b*sqrt(f) - 2*c*sqrt(d)))/(2*sqrt(a + b*x + c*x**2)*
sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)))/(2*(b**2*d*f - (a*f + c*d)*
*2)*sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)) - sqrt(d)*(a*f + b*sqrt(
d)*sqrt(f) + c*d)*atanh((-2*a*sqrt(f) + b*sqrt(d) + x*(-b*sqrt(f)
 + 2*c*sqrt(d)))/(2*sqrt(a + b*x + c*x**2)*sqrt(a*f - b*sqrt(d)*s
qrt(f) + c*d)))/(2*(b**2*d*f - (a*f + c*d)**2)*sqrt(a*f - b*sqrt(
d)*sqrt(f) + c*d)) - 2*(a*b*(a*f - c*d) + c*x*(2*a*(a*f + c*d) -
b**2*d))/((-4*a*c + b**2)*(b**2*d*f - (a*f + c*d)**2)*sqrt(a + b*
x + c*x**2))
```

Mathematica [A] time = 1.37342, size = 583, normalized size = 1.96

$$\frac{4\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\left(a^2f(b+2cx)+acd(2cx-b)-b^2cdx\right)+\sqrt{d}\left(4ac-b^2\right)\sqrt{a+x(b+cx)}\log}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (4*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[
f] + a*f]*(-(b^2*c*d*x) + a*c*d*(-b + 2*c*x) + a^2*f*(b + 2*c*x))
 + (-b^2 + 4*a*c)*Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*S
qrt[a + x*(b + c*x)]*Log[Sqrt[d]*Sqrt[f] - f*x] + (b^2 - 4*a*c)*S
qrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Sqrt[a + x*(b + c*x)
]*Log[Sqrt[d]*Sqrt[f] + f*x] + (-b^2 + 4*a*c)*Sqrt[d]*(c*d + b*Sq
rt[d]*Sqrt[f] + a*f)^(3/2)*Sqrt[a + x*(b + c*x)]*Log[Sqrt[d]*(-(b
*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x + 2*Sqrt[c*
d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + (b^2 - 4*a
*c)*Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Sqrt[a + x*(b +
 c*x)]*Log[Sqrt[d]*(b*(Sqrt[d] + Sqrt[f]*x) + 2*(a*Sqrt[f] + c*Sq
rt[d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x
)]))])/(2*(b^2 - 4*a*c)*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[
c*d + b*Sqrt[d]*Sqrt[f] + a*f]*(c^2*d^2 + 2*a*c*d*f + f*(-(b^2*d)
 + a^2*f))*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.023, size = 1427, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -2/f*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/2*d/(d*f)^(1/2)/
(b*(d*f)^(1/2)+f*a+c*d)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b
*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)+2*d/f/(b
*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*
f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2
)*x*c^2+d/(d*f)^(1/2)/(b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-(d*
f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f
)^(1/2)+f*a+c*d)/f)^(1/2)*x*b*c+d/f/(b*(d*f)^(1/2)+f*a+c*d)/(4*a*
c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1
/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*b*c+1/2*d/(d*f)^(1/2)/(b*
(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)
*b^2+1/2*d/(d*f)^(1/2)/(b*(d*f)^(1/2)+f*a+c*d)/((b*(d*f)^(1/2)+f*
a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+
b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-
(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(
d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))+1/2*d/(d*f)^(1/2
)/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)
^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)
+2*d/f/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*
c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2
)+f*a+c*d))^(1/2)*x*c^2-d/(d*f)^(1/2)/(-b*(d*f)^(1/2)+f*a+c*d)/(4
*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d
*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*x*b*c+d/f/(-b*(d
*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(
d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(
1/2)*b*c-1/2*d/(d*f)^(1/2)/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/(
(x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f
)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*b^2-1/2*d/(d*f)^(1/2)/(-b*(
d*f)^(1/2)+f*a+c*d)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*
(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2
)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*
c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2
)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.105 $\int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

Optimal. Leaf size=299

$$-\frac{2\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}-\frac{\sqrt{f}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\sqrt{f}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

```
[Out] (-2*(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 -
4*a*c)*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - (Sqrt[f
]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)
/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/
(2*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (Sqrt[f]*ArcTanh[(b*S
qrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d +
 b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*(c*d + b*Sq
rt[d]*Sqrt[f] + a*f)^(3/2))
```

Rubi [A] time = 0.862, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{2\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}-\frac{\sqrt{f}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\sqrt{f}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```


```
[Out] (-2*(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 -
4*a*c)*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - (Sqrt[f
]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)
/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/
(2*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (Sqrt[f]*ArcTanh[(b*S
qrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d +
 b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*(c*d + b*Sq
rt[d]*Sqrt[f] + a*f)^(3/2))
```

Rubi in Sympy [A] time = 156.897, size = 342, normalized size = 1.14

$$\frac{\sqrt{f}\left(af-b\sqrt{d}\sqrt{f}+cd\right)\operatorname{atanh}\left(\frac{-2a\sqrt{f}-b\sqrt{d}+x\left(-b\sqrt{f}-2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\left(b^2df-(af+cd)^2\right)\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}+\frac{\sqrt{f}\left(af+b\sqrt{d}\sqrt{f}+cd\right)\operatorname{atanh}\left(\frac{-2a\sqrt{f}+b\sqrt{d}+x\left(-b\sqrt{f}+2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}\right)}{2\left(b^2df-(af+cd)^2\right)\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}+\frac{2\left(a\left(-2acf+b^2f-2c^2d\right)+bcx\left(af-cd\right)\right)}{\left(-4ac+b^2\right)\left(b^2df-(af+cd)^2\right)\sqrt{a+bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] sqrt(f)*(a*f - b*sqrt(d)*sqrt(f) + c*d)*atanh((-2*a*sqrt(f) - b*s
qrt(d) + x*(-b*sqrt(f) - 2*c*sqrt(d)))/(2*sqrt(a + b*x + c*x**2)*
sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)))/(2*(b**2*d*f - (a*f + c*d)*
*2)*sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)) + sqrt(f)*(a*f + b*sqrt(
d)*sqrt(f) + c*d)*atanh((-2*a*sqrt(f) + b*sqrt(d) + x*(-b*sqrt(f)
 + 2*c*sqrt(d)))/(2*sqrt(a + b*x + c*x**2)*sqrt(a*f - b*sqrt(d)*s
qrt(f) + c*d)))/(2*(b**2*d*f - (a*f + c*d)**2)*sqrt(a*f - b*sqrt(
d)*sqrt(f) + c*d)) + 2*(a*(-2*a*c*f + b**2*f - 2*c**2*d) + b*c*x*
(a*f - c*d))/((-4*a*c + b**2)*(b**2*d*f - (a*f + c*d)**2)*sqrt(a
+ b*x + c*x**2))
```

Mathematica [A] time = 2.44677, size = 407, normalized size = 1.36

$$\frac{1}{2}\left(\frac{-8a^2cf+4a\left(b^2f+bcfx-2c^2d\right)-4bc^2dx}{(b^2-4ac)\sqrt{a+x(b+cx)}\left(f\left(b^2d-a^2f\right)-2acdf-c^2d^2\right)}-\frac{\sqrt{f}\log\left(\sqrt{d}\sqrt{f}-fx\right)}{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}-\frac{\sqrt{f}\log\left(\sqrt{d}\sqrt{f}+fx\right)}{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\sqrt{f}\log\left(\sqrt{d}\left(2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}+2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x\right)\right)}{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\sqrt{f}\log\left(\sqrt{d}\left(2\left(\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}+a\sqrt{f}+c\sqrt{d}x\right)+b\left(\sqrt{d}+\sqrt{f}x\right)\right)\right)}{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] ((-8*a^2*c*f - 4*b*c^2*d*x + 4*a*(-2*c^2*d + b^2*f + b*c*f*x))/((
b^2 - 4*a*c)*(-(c^2*d^2) - 2*a*c*d*f + f*(b^2*d - a^2*f))*Sqrt[a
+ x*(b + c*x)]) - (Sqrt[f]*Log[Sqrt[d]*Sqrt[f] - f*x])/(c*d + b*S
qrt[d]*Sqrt[f] + a*f)^(3/2) - (Sqrt[f]*Log[Sqrt[d]*Sqrt[f] + f*x]
)/(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2) + (Sqrt[f]*Log[Sqrt[d]*(-
(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x + 2*Sqrt[
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(c*d - b*
Sqrt[d]*Sqrt[f] + a*f)^(3/2) + (Sqrt[f]*Log[Sqrt[d]*(b*(Sqrt[d] +
 Sqrt[f]*x) + 2*(a*Sqrt[f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[d]*S
qrt[f] + a*f]*Sqrt[a + x*(b + c*x)]))])/(c*d + b*Sqrt[d]*Sqrt[f]
+ a*f)^(3/2))/2
```

Maple [B] time = 0.021, size = 1360, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/(b*(d*f)^(1/2)+f*a+c*d)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1
/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)+2/f
```

```
/(b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*
(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(
1/2)*(d*f)^(1/2)*x*c^2+1/(b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-
(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(
d*f)^(1/2)+f*a+c*d)/f)^(1/2)*x*b*c+1/f/(b*(d*f)^(1/2)+f*a+c*d)/(4
*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)
^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*(d*f)^(1/2)*b*c+1/2/(b
*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*
f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2
)*b^2+1/2/(b*(d*f)^(1/2)+f*a+c*d)/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/
2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*
f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f
)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*
a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))-1/2/(-b*(d*f)^(1/2)+f*a+c*d)/
((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/
f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-2/f/(-b*(d*f)^(1/2)+f*a+c*
d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*
(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*(d*f)^(1/2)
*x*c^2+1/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^
2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1
/2)+f*a+c*d))^(1/2)*x*b*c-1/f/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2
)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2
)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*(d*f)^(1/2)*b*c+1/2/(-b*
(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c
*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))
^(1/2)*b^2+1/2/(-b*(d*f)^(1/2)+f*a+c*d)/(1/f*(-b*(d*f)^(1/2)+f*a+
c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2
)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*(
(x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f
)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.106 $\int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

Optimal. Leaf size=310

$$-\frac{2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}+\frac{f\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{f\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

```
[Out] (-2*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*
x))/((b^2 - 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2
]) + (f*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[
f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^
2])])/(2*Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (f*ArcT
anh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sq
rt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqr
t[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rubi [A] time = 0.926525, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$-\frac{2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}+\frac{f\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{f\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```


```
[Out] (-2*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*
x))/((b^2 - 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2
]) + (f*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[
f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^
2])])/(2*Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (f*ArcT
anh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sq
rt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqr
t[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rubi in Sympy [A] time = 151.232, size = 352, normalized size = 1.14

$$-\frac{2\left(b\left(b^2f-c\left(3af+cd\right)\right)+cx\left(-2acf+b^2f-2c^2d\right)\right)}{\left(-4ac+b^2\right)\left(b^2df-\left(af+cd\right)^2\right)\sqrt{a+bx+cx^2}}+\frac{f\left(af-b\sqrt{d}\sqrt{f}+cd\right)\operatorname{atanh}\left(\frac{-2a\sqrt{f}-b\sqrt{d}+x\left(-b\sqrt{f}-2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(b^2df-\left(af+cd\right)^2\right)\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{f\left(af+b\sqrt{d}\sqrt{f}+cd\right)\operatorname{atanh}\left(\frac{-2a\sqrt{f}+b\sqrt{d}+x\left(-b\sqrt{f}+2c\sqrt{d}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(b^2df-\left(af+cd\right)^2\right)\sqrt{af-b\sqrt{d}\sqrt{f}+cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -2*(b*(b**2*f - c*(3*a*f + c*d)) + c*x*(-2*a*c*f + b**2*f - 2*c**
2*d))/((-4*a*c + b**2)*(b**2*d*f - (a*f + c*d)**2)*sqrt(a + b*x +
 c*x**2)) + f*(a*f - b*sqrt(d)*sqrt(f) + c*d)*atanh((-2*a*sqrt(f)
 - b*sqrt(d) + x*(-b*sqrt(f) - 2*c*sqrt(d)))/(2*sqrt(a + b*x + c*
x**2)*sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)))/(2*sqrt(d)*(b**2*d*f
- (a*f + c*d)**2)*sqrt(a*f + b*sqrt(d)*sqrt(f) + c*d)) - f*(a*f +
 b*sqrt(d)*sqrt(f) + c*d)*atanh((-2*a*sqrt(f) + b*sqrt(d) + x*(-b
*sqrt(f) + 2*c*sqrt(d)))/(2*sqrt(a + b*x + c*x**2)*sqrt(a*f - b*s
qrt(d)*sqrt(f) + c*d)))/(2*sqrt(d)*(b**2*d*f - (a*f + c*d)**2)*sq
rt(a*f - b*sqrt(d)*sqrt(f) + c*d))
```

Mathematica [A] time = 1.40269, size = 584, normalized size = 1.88

$$\frac{f\left(4ac-b^2\right)\sqrt{a+x(b+cx)}\log\left(\sqrt{d}\sqrt{f}-fx\right)\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}+f\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(af+b\left(-\sqrt{d}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-4*Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[c*d + b*Sqrt
[d]*Sqrt[f] + a*f]*(-(b^3*f) + b*c*(c*d + 3*a*f) - b^2*c*f*x + 2*
c^2*(c*d + a*f)*x) + (-b^2 + 4*a*c)*f*(c*d - b*Sqrt[d]*Sqrt[f] +
a*f)^(3/2)*Sqrt[a + x*(b + c*x)]*Log[Sqrt[d]*Sqrt[f] - f*x] + (b^
2 - 4*a*c)*f*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Sqrt[a + x*(b
+ c*x)]*Log[Sqrt[d]*Sqrt[f] + f*x] + (-b^2 + 4*a*c)*f*(c*d + b*Sq
rt[d]*Sqrt[f] + a*f)^(3/2)*Sqrt[a + x*(b + c*x)]*Log[Sqrt[d]*(-(b
*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x + 2*Sqrt[c*
d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + (b^2 - 4*a
*c)*f*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Sqrt[a + x*(b + c*x)]
*Log[Sqrt[d]*(b*(Sqrt[d] + Sqrt[f]*x) + 2*(a*Sqrt[f] + c*Sqrt[d]*
x + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]))])
/(2*(b^2 - 4*a*c)*Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqr
t[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*(c^2*d^2 + 2*a*c*d*f + f*(-(b^2*
d) + a^2*f))*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.02, size = 1376, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/(d*f)^(1/2)/(b*(d*f)^(1/2)+f*a+c*d)*f/((x-(d*f)^(1/2)/f)^2*c
+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d
)/f)^(1/2)+2/(b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/
f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f
*a+c*d)/f)^(1/2)*x*c^2+1/(d*f)^(1/2)/(b*(d*f)^(1/2)+f*a+c*d)/(4*a
*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(
1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*x*b*c*f+1/(b*(d*f)^(1/2)
+f*a+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f
)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*b*c+1/2/(d
*f)^(1/2)/(b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^
2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+
```

```
c*d)/f)^(1/2)*b^2*f+1/2/(d*f)^(1/2)/(b*(d*f)^(1/2)+f*a+c*d)*f/((b
*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2
*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d
)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f
)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))+1
/2/(d*f)^(1/2)*f/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*f)^(1/2)/f)^2*c+
1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+
f*a+c*d))^(1/2)+2/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x+(d*f)^
(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b
*(d*f)^(1/2)+f*a+c*d))^(1/2)*x*c^2-1/(d*f)^(1/2)/(-b*(d*f)^(1/2)+
f*a+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)
+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*x*b*c
*f+1/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+
1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+
f*a+c*d))^(1/2)*b*c-1/2/(d*f)^(1/2)/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a
*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f
)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*b^2*f-1/2/(d*f)^(1
/2)*f/(-b*(d*f)^(1/2)+f*a+c*d)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/
2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x
+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^
(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b
*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.107 $\int \frac{1}{x(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

Optimal. Leaf size=394

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d}-\frac{2f\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ad\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{f^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{f^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2
]) - (2*f*(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((
b^2 - 4*a*c)*d*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) -
 ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(a^(3/2)*
d) - (f^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b
*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
+ c*x^2])])/(2*d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (f^(3/2
)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)
/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/
(2*d*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rubi [A] time = 2.33411, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d}-\frac{2f\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ad\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{f^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{f^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```


```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2
]) - (2*f*(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((
b^2 - 4*a*c)*d*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) -
 ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(a^(3/2)*
d) - (f^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b
*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
+ c*x^2])])/(2*d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (f^(3/2
)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)
/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/
(2*d*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 6.4807, size = 742, normalized size = 1.88

$$-\frac{\log\left(2\sqrt{a}\sqrt{a+bx+cx^2}+2a+bx\right)}{a^{3/2}d}+\frac{\log(x)}{a^{3/2}d}$$
$$+\frac{f\left(a\sqrt{d}f^{3/2}+bdf+cd^{3/2}\sqrt{f}\right)\log\left(2\sqrt{d}\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}+2a\sqrt{d}\sqrt{f}+b\sqrt{d}\sqrt{f}x-bd-2cdx\right)}{2d^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,(a^2f^2+2acdf+b^2(-d)f+c^2d^2)}$$
$$+\frac{f\left(a\sqrt{d}f^{3/2}-bdf+cd^{3/2}\sqrt{f}\right)\log\left(2\sqrt{d}\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}+2a\sqrt{d}\sqrt{f}+b\sqrt{d}\sqrt{f}x+bd+2cdx\right)}{2d^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,(a^2f^2+2acdf+b^2(-d)f+c^2d^2)}$$
$$-\frac{f\log\left(\sqrt{d}\sqrt{f}-fx\right)\left(a\sqrt{d}f^{3/2}-bdf+cd^{3/2}\sqrt{f}\right)}{2d^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,(a^2f^2+2acdf+b^2(-d)f+c^2d^2)}$$
$$-\frac{f\log\left(\sqrt{d}\sqrt{f}+fx\right)\left(a\sqrt{d}f^{3/2}+bdf+cd^{3/2}\sqrt{f}\right)}{2d^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,(a^2f^2+2acdf+b^2(-d)f+c^2d^2)}$$
$$+\frac{2\left(2a^2c^2f-4ab^2cf-3abc^2fx+2ac^3d+b^4f+b^3cfx-b^2c^2d-bc^3dx\right)}{a(4ac-b^2)\sqrt{a+bx+cx^2}\,(a^2f^2+2acdf+b^2(-d)f+c^2d^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*(-(b^2*c^2*d) + 2*a*c^3*d + b^4*f - 4*a*b^2*c*f + 2*a^2*c^2*f
- b*c^3*d*x + b^3*c*f*x - 3*a*b*c^2*f*x))/(a*(-b^2 + 4*a*c)*(c^2*
d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2)*Sqrt[a + b*x + c*x^2]) + Log
[x]/(a^(3/2)*d) - (f*(c*d^(3/2)*Sqrt[f] - b*d*f + a*Sqrt[d]*f^(3/
2))*Log[Sqrt[d]*Sqrt[f] - f*x])/(2*d^(3/2)*Sqrt[c*d + b*Sqrt[d]*S
qrt[f] + a*f]*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2)) - (f*(c*
d^(3/2)*Sqrt[f] + b*d*f + a*Sqrt[d]*f^(3/2))*Log[Sqrt[d]*Sqrt[f]
+ f*x])/(2*d^(3/2)*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*(c^2*d^2 -
 b^2*d*f + 2*a*c*d*f + a^2*f^2)) - Log[2*a + b*x + 2*Sqrt[a]*Sqrt
[a + b*x + c*x^2]]/(a^(3/2)*d) + (f*(c*d^(3/2)*Sqrt[f] + b*d*f +
a*Sqrt[d]*f^(3/2))*Log[-(b*d) + 2*a*Sqrt[d]*Sqrt[f] - 2*c*d*x + b
*Sqrt[d]*Sqrt[f]*x + 2*Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f
]*Sqrt[a + b*x + c*x^2]])/(2*d^(3/2)*Sqrt[c*d - b*Sqrt[d]*Sqrt[f]
 + a*f]*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2)) + (f*(c*d^(3/2
)*Sqrt[f] - b*d*f + a*Sqrt[d]*f^(3/2))*Log[b*d + 2*a*Sqrt[d]*Sqrt
[f] + 2*c*d*x + b*Sqrt[d]*Sqrt[f]*x + 2*Sqrt[d]*Sqrt[c*d + b*Sqrt
[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2]])/(2*d^(3/2)*Sqrt[c*d +
b*Sqrt[d]*Sqrt[f] + a*f]*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2
))
```

Maple [B] time = 0.024, size = 1518, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/d/a/(c*x^2+b*x+a)^(1/2)-2/d*b/a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
*c*x-1/d*b^2/a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/d/a^(3/2)*ln((2*
a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-1/2/d/(b*(d*f)^(1/2)+f*a+
c*d)*f/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1
/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)+2/d/(b*(d*f)^(1/2)+f*a+c*
d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-
(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*(d*f)^(1/2)*x*c^2
+1/d/(b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(
2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/
f)^(1/2)*x*b*c*f+1/d/(b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-(d*f
)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)
^(1/2)+f*a+c*d)/f)^(1/2)*(d*f)^(1/2)*b*c+1/2/d/(b*(d*f)^(1/2)+f*a
+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*
(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*b^2*f+1/2/d/(b
*(d*f)^(1/2)+f*a+c*d)*f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(
b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f
)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c
*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^
(1/2))/(x-(d*f)^(1/2)/f))-1/2/d*f/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d
*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f
*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-2/d/(-b*(d*f)^(1/2)+f*a+c*d)/(4*
a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*
f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*(d*f)^(1/2)*x*c^2
+1/d/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+
1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+
f*a+c*d))^(1/2)*x*b*c*f-1/d/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/
((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/
f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*(d*f)^(1/2)*b*c+1/2/d/(-b*
(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c
*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))
^(1/2)*b^2*f+1/2/d*f/(-b*(d*f)^(1/2)+f*a+c*d)/(1/f*(-b*(d*f)^(1/2
)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f
)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(
1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(
1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(cx^2+bx+a)^{\frac{3}{2}}(fx^2-d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.294934, size = 1, normalized size = 0.

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x),x, algorithm="giac")
```

```
[Out] sage2
```

3.108 $\int \frac{1}{x^2(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

Optimal. Leaf size=454

$$\frac{3b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{5/2}d}-\frac{(3b^2-8ac)\sqrt{a+bx+cx^2}}{a^2dx(b^2-4ac)}-\frac{2f\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{d(b^2-4ac)\sqrt{a+bx+cx^2}(b^2df-(af+cd)^2)}+\frac{2\left(-2ac+b^2+bcx\right)}{adx(b^2-4ac)\sqrt{a+bx+cx^2}}+\frac{f^2\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{f^2\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*x*Sqrt[a + b*x + c*x
^2]) - (2*f*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2
*a*c*f)*x))/((b^2 - 4*a*c)*d*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b
*x + c*x^2]) - ((3*b^2 - 8*a*c)*Sqrt[a + b*x + c*x^2])/(a^2*(b^2
- 4*a*c)*d*x) + (3*b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x
+ c*x^2])])/(2*a^(5/2)*d) + (f^2*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f]
 + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] +
 a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*(c*d - b*Sqrt[d]*Sqrt[f
] + a*f)^(3/2)) + (f^2*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sq
rt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt
[a + b*x + c*x^2])])/(2*d^(3/2)*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(
3/2))
```

Rubi [A] time = 2.75034, antiderivative size = 454, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{3b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{5/2}d}-\frac{(3b^2-8ac)\sqrt{a+bx+cx^2}}{a^2dx(b^2-4ac)}-\frac{2f\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{d(b^2-4ac)\sqrt{a+bx+cx^2}(b^2df-(af+cd)^2)}+\frac{2\left(-2ac+b^2+bcx\right)}{adx(b^2-4ac)\sqrt{a+bx+cx^2}}+\frac{f^2\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{f^2\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*x*Sqrt[a + b*x + c*x
^2]) - (2*f*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2
*a*c*f)*x))/((b^2 - 4*a*c)*d*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b
*x + c*x^2]) - ((3*b^2 - 8*a*c)*Sqrt[a + b*x + c*x^2])/(a^2*(b^2
- 4*a*c)*d*x) + (3*b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x
+ c*x^2])])/(2*a^(5/2)*d) + (f^2*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f]
 + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] +
 a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*(c*d - b*Sqrt[d]*Sqrt[f
] + a*f)^(3/2)) + (f^2*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sq
rt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt
[a + b*x + c*x^2])])/(2*d^(3/2)*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(
3/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 3.39167, size = 537, normalized size = 1.18

$$\frac{1}{2}\left(\frac{3b\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+bx\right)}{a^{5/2}d}-\frac{3b\log(x)}{a^{5/2}d}-\frac{4\left(-b^3c(5af+cd)-b^2c^2x(4af+cd)+abc^2(5af+3cd)+2ac^3x(af+cd)+b^5f+b^4cfx\right)}{a^2\left(4ac-b^2\right)\sqrt{a+x(b+cx)}\left(f\left(a^2f-b^2d\right)+2acdf+c^2d^2\right)}-\frac{2\sqrt{a+x(b+cx)}}{a^2dx}-\frac{f^2\log\left(\sqrt{d}\sqrt{f}-fx\right)}{d^{3/2}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}+\frac{f^2\log\left(\sqrt{d}\sqrt{f}+fx\right)}{d^{3/2}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}-\frac{f^2\log\left(\sqrt{d}\left(2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}+2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x\right)\right)}{d^{3/2}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{f^2\log\left(\sqrt{d}\left(2\left(\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}+a\sqrt{f}+c\sqrt{d}x\right)+b\left(\sqrt{d}+\sqrt{f}x\right)\right)\right)}{d^{3/2}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] ((-4*(b^5*f - b^3*c*(c*d + 5*a*f) + a*b*c^2*(3*c*d + 5*a*f) + b^4
*c*f*x + 2*a*c^3*(c*d + a*f)*x - b^2*c^2*(c*d + 4*a*f)*x))/(a^2*(
-b^2 + 4*a*c)*(c^2*d^2 + 2*a*c*d*f + f*(-(b^2*d) + a^2*f))*Sqrt[a
 + x*(b + c*x)]) - (2*Sqrt[a + x*(b + c*x)])/(a^2*d*x) - (3*b*Log
[x])/(a^(5/2)*d) - (f^2*Log[Sqrt[d]*Sqrt[f] - f*x])/(d^(3/2)*(c*d
 + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (f^2*Log[Sqrt[d]*Sqrt[f] + f
*x])/(d^(3/2)*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (3*b*Log[2
*a + b*x + 2*Sqrt[a]*Sqrt[a + x*(b + c*x)]])/(a^(5/2)*d) - (f^2*L
og[Sqrt[d]*(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f
]*x + 2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]
)])/(d^(3/2)*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (f^2*Log[Sq
rt[d]*(b*(Sqrt[d] + Sqrt[f]*x) + 2*(a*Sqrt[f] + c*Sqrt[d]*x + Sqr
t[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]))])/(d^(3/
2)*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)))/2
```

Maple [B] time = 0.025, size = 1656, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -1/d/a/x/(c*x^2+b*x+a)^(1/2)-3/2/d*b/a^2/(c*x^2+b*x+a)^(1/2)+3/d*
b^2/a^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*c*x+3/2/d*b^3/a^2/(4*a*c-
b^2)/(c*x^2+b*x+a)^(1/2)+3/2/d*b/a^(5/2)*ln((2*a+b*x+2*a^(1/2)*(c
*x^2+b*x+a)^(1/2))/x)-8/d*c^2/a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x
-4/d*c/a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b-1/2*f^2/d/(d*f)^(1/2)/
(b*(d*f)^(1/2)+f*a+c*d)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b
*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)+2*f/d/(b
*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*
f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2
)*x*c^2+f^2/d/(d*f)^(1/2)/(b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x
-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*
(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*x*b*c+f/d/(b*(d*f)^(1/2)+f*a+c*d)/(
4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f
)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*b*c+1/2*f^2/d/(d*f)^(
1/2)/(b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(
2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/
f)^(1/2)*b^2+1/2*f^2/d/(d*f)^(1/2)/(b*(d*f)^(1/2)+f*a+c*d)/((b*(d
*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*
(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f
)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(
1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))+1/2*
f^2/d/(d*f)^(1/2)/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*f)^(1/2)/f)^2*c
+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)
+f*a+c*d))^(1/2)+2*f/d/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x+(
d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/
f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*x*c^2-f^2/d/(d*f)^(1/2)/(-b*(d*
f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d
*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1
/2)*x*b*c+f/d/(-b*(d*f)^(1/2)+f*a+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2
)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*
f)^(1/2)+f*a+c*d))^(1/2)*b*c-1/2*f^2/d/(d*f)^(1/2)/(-b*(d*f)^(1/2
)+f*a+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/
2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*b^2
-1/2*f^2/d/(d*f)^(1/2)/(-b*(d*f)^(1/2)+f*a+c*d)/(1/f*(-b*(d*f)^(1
/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d
*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))
^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(cx^2+bx+a)^{\frac{3}{2}}(fx^2-d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.109 $\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=761

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)}{8c^{3/2}f^3}$$
$$-\frac{\left(f\left(af\left(-e\sqrt{e^2-4df}-2df+e^2\right)-b\left(-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}-3def+e^3\right)\right)+c\left(2d^2f^2-4de^2f+2def\sqrt{e^2-\right.\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{\right.}}}$$
$$+\frac{\left(f\left(af\left(e\sqrt{e^2-4df}-2df+e^2\right)-b\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)\right)+c\left(2d^2f^2-4de^2f-2def\sqrt{e^2-4d\right.\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2\right.}}}$$
$$-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

```
[Out] -((4*c*e - b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(4*c*f^2) - ((b^
2*f^2 + 4*c*f*(b*e - a*f) - 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x
)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2)*f^3) - ((c*(e^4
- 4*d*e^2*f + 2*d^2*f^2 - e^3*Sqrt[e^2 - 4*d*f] + 2*d*e*f*Sqrt[e^
2 - 4*d*f]) + f*(a*f*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) - b*(e^3
 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f])))*Arc
Tanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^
2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[
2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f
]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]) + ((c*(e^4 - 4*d*e^2
*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f
]) + f*(a*f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) - b*(e^3 - 3*d*e*
f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a
*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f
]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e -
 b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^3*Sq
rt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2
*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])
```

Rubi [A] time = 8.37725, antiderivative size = 761, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)}{8c^{3/2}f^3}$$
$$-\frac{\left(f\left(af\left(-e\sqrt{e^2-4df}-2df+e^2\right)-b\left(-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}-3def+e^3\right)\right)+c\left(2d^2f^2-4de^2f+2def\sqrt{e^2-\right.\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{\right.}}}$$
$$+\frac{\left(f\left(af\left(e\sqrt{e^2-4df}-2df+e^2\right)-b\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)\right)+c\left(2d^2f^2-4de^2f-2def\sqrt{e^2-4d\right.\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2\right.}}}$$
$$-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(x^2*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] -((4*c*e - b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(4*c*f^2) - ((b^
2*f^2 + 4*c*f*(b*e - a*f) - 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x
)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2)*f^3) - ((c*(e^4
- 4*d*e^2*f + 2*d^2*f^2 - e^3*Sqrt[e^2 - 4*d*f] + 2*d*e*f*Sqrt[e^
2 - 4*d*f]) + f*(a*f*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) - b*(e^3
 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f])))*Arc
Tanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^
2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[
2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f
]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]) + ((c*(e^4 - 4*d*e^2
*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f
]) + f*(a*f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) - b*(e^3 - 3*d*e*
f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a
*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f
]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e -
 b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^3*Sq
rt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2
*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [B] time = 6.3054, size = 1546, normalized size = 2.03

$$\sqrt{a+x(b+cx)}\left(\frac{x}{2f}-\frac{4ce-bf}{4cf^2}\right)$$

$$-\frac{\left(-ce^4+bfe^3+c\sqrt{e^2-4df}e^3-af^2e^2+4cdfe^2-bf\sqrt{e^2-4df}e^2-3bdf^2e+af^2\sqrt{e^2-4df}e-2cdf\sqrt{e^2-4df}e+2ac\cdots\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{ce^2-bfe-c\sqrt{e^2-4df}e+2af^2-2cdf+bf\sqrt{\cdots}}}$$

$$-\frac{\left(ce^4-bfe^3+c\sqrt{e^2-4df}e^3+af^2e^2-4cdfe^2-bf\sqrt{e^2-4df}e^2+3bdf^2e+af^2\sqrt{e^2-4df}e-2cdf\sqrt{e^2-4df}e-2adf\cdots\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{ce^2-bfe+c\sqrt{e^2-4df}e+2af^2-2cdf-bf\sqrt{e^2\cdots}}}$$

$$+\frac{\left(8e^2c^2-8dfc^2+4af^2c-4befc-b^2f^2\right)\sqrt{a+x(b+cx)}\log\left(b+2cx+2\sqrt{c}\sqrt{cx^2+bx+a}\right)}{8c^{3/2}f^3\sqrt{cx^2+bx+a}}$$

$$+\frac{\left(ce^4-bfe^3+c\sqrt{e^2-4df}e^3+af^2e^2-4cdfe^2-bf\sqrt{e^2-4df}e^2+3bdf^2e+af^2\sqrt{e^2-4df}e-2cdf\sqrt{e^2-4df}e-2adf\cdots\right.}{\cdots}$$

$$+\frac{\left(-ce^4+bfe^3+c\sqrt{e^2-4df}e^3-af^2e^2+4cdfe^2-bf\sqrt{e^2-4df}e^2-3bdf^2e+af^2\sqrt{e^2-4df}e-2cdf\sqrt{e^2-4df}e+2ad\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] (-(4*c*e - b*f)/(4*c*f^2) + x/(2*f))*Sqrt[a + x*(b + c*x)] - ((-(
c*e^4) + 4*c*d*e^2*f + b*e^3*f - 2*c*d^2*f^2 - 3*b*d*e*f^2 - a*e^
2*f^2 + 2*a*d*f^3 + c*e^3*Sqrt[e^2 - 4*d*f] - 2*c*d*e*f*Sqrt[e^2
```

```
- 4*d*f] - b*e^2*f*Sqrt[e^2 - 4*d*f] + b*d*f^2*Sqrt[e^2 - 4*d*f]
+ a*e*f^2*Sqrt[e^2 - 4*d*f])*Sqrt[a + x*(b + c*x)]*Log[-e + Sqrt[
e^2 - 4*d*f] - 2*f*x])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^
2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]) - ((c*e^4 - 4*c*d*e^2*f - b*e^
3*f + 2*c*d^2*f^2 + 3*b*d*e*f^2 + a*e^2*f^2 - 2*a*d*f^3 + c*e^3*S
qrt[e^2 - 4*d*f] - 2*c*d*e*f*Sqrt[e^2 - 4*d*f] - b*e^2*f*Sqrt[e^2
 - 4*d*f] + b*d*f^2*Sqrt[e^2 - 4*d*f] + a*e*f^2*Sqrt[e^2 - 4*d*f]
)*Sqrt[a + x*(b + c*x)]*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt
[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
+ c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c
*x^2]) + ((8*c^2*e^2 - 8*c^2*d*f - 4*b*c*e*f - b^2*f^2 + 4*a*c*f^
2)*Sqrt[a + x*(b + c*x)]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2]])/(8*c^(3/2)*f^3*Sqrt[a + b*x + c*x^2]) + ((c*e^4 - 4*c*d
*e^2*f - b*e^3*f + 2*c*d^2*f^2 + 3*b*d*e*f^2 + a*e^2*f^2 - 2*a*d*
f^3 + c*e^3*Sqrt[e^2 - 4*d*f] - 2*c*d*e*f*Sqrt[e^2 - 4*d*f] - b*e
^2*f*Sqrt[e^2 - 4*d*f] + b*d*f^2*Sqrt[e^2 - 4*d*f] + a*e*f^2*Sqrt
[e^2 - 4*d*f])*Sqrt[a + x*(b + c*x)]*Log[-(b*e^2) + 4*b*d*f - b*e
*Sqrt[e^2 - 4*d*f] + 4*a*f*Sqrt[e^2 - 4*d*f] - 2*c*e^2*x + 8*c*d*
f*x - 2*c*e*Sqrt[e^2 - 4*d*f]*x + 2*b*f*Sqrt[e^2 - 4*d*f]*x + 2*S
qrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 +
 c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*
x^2]])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*
f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt
[a + b*x + c*x^2]) + ((-(c*e^4) + 4*c*d*e^2*f + b*e^3*f - 2*c*d^2
*f^2 - 3*b*d*e*f^2 - a*e^2*f^2 + 2*a*d*f^3 + c*e^3*Sqrt[e^2 - 4*d
*f] - 2*c*d*e*f*Sqrt[e^2 - 4*d*f] - b*e^2*f*Sqrt[e^2 - 4*d*f] + b
*d*f^2*Sqrt[e^2 - 4*d*f] + a*e*f^2*Sqrt[e^2 - 4*d*f])*Sqrt[a + x*
(b + c*x)]*Log[b*e^2 - 4*b*d*f - b*e*Sqrt[e^2 - 4*d*f] + 4*a*f*Sq
rt[e^2 - 4*d*f] + 2*c*e^2*x - 8*c*d*f*x - 2*c*e*Sqrt[e^2 - 4*d*f]
*x + 2*b*f*Sqrt[e^2 - 4*d*f]*x + 2*Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt
[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*
Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]])/(Sqrt[2]*f^3*Sqrt[e^2
- 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 -
4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])
```

Maple [B] time = 0.028, size = 14815, normalized size = 19.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)*x^2/(f*x^2 + e*x + d),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^2 + b*x + a)*x^2/(f*x^2 + e*x + d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{a + bx + cx^2}}{d + ex + fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**2*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^2 + b*x + a)*x^2/(f*x^2 + e*x + d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.110 $\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=549

$$-\frac{\left(\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c\left(e^2-df\right)\right)+2df(ce-bf)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{\left(\left(\sqrt{e^2-4df}+e\right)\left(f(be-af)-c\left(e^2-df\right)\right)+2df(ce-bf)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{(2ce-bf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2}+\frac{\sqrt{a+bx+cx^2}}{f}$$

```
[Out] Sqrt[a + b*x + c*x^2]/f - ((2*c*e - b*f)*ArcTanh[(b + 2*c*x)/(2*S
qrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2) - ((2*d*f*(c*e -
b*f) + (e - Sqrt[e^2 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*A
rcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[
e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqr
t[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((2*d*f*(c*e - b*f) + (e + S
qrt[e^2 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f
 - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f])
)*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b
*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt
[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f
)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 14.0458, antiderivative size = 549, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$-\frac{\left(\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c\left(e^2-df\right)\right)+2df(ce-bf)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{\left(\left(\sqrt{e^2-4df}+e\right)\left(f(be-af)-c\left(e^2-df\right)\right)+2df(ce-bf)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{(2ce-bf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2}+\frac{\sqrt{a+bx+cx^2}}{f}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(x*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```


```
[Out] Sqrt[a + b*x + c*x^2]/f - ((2*c*e - b*f)*ArcTanh[(b + 2*c*x)/(2*S
qrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2) - ((2*d*f*(c*e -
b*f) + (e - Sqrt[e^2 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*A
rcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[
e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqr
t[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((2*d*f*(c*e - b*f) + (e + S
qrt[e^2 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f
 - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f])
```

```
)*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b
*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt
[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f
)*Sqrt[e^2 - 4*d*f]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [B] time = 2.86325, size = 1112, normalized size = 2.03

$$2\sqrt{a+x(b+cx)}f+\frac{\sqrt{2}\left(c\left(-e^3+\sqrt{e^2-4df}e^2+3dfe-df\sqrt{e^2-4df}\right)+f\left(af\left(\sqrt{e^2-4df}-e\right)+b\left(e^2-\sqrt{e^2-4df}e-2df\right)\right)\right)\log\left(-e-2fx+\sqrt{e^2-4df}\right)}{\sqrt{e^2-4df}\sqrt{c\left(e^2-\sqrt{e^2-4df}e-2df\right)+f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)}}+\frac{\sqrt{2}\left(c\left(e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] (2*f*Sqrt[a + x*(b + c*x)] + (Sqrt[2]*(c*(-e^3 + 3*d*e*f + e^2*Sq
rt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]) + f*(a*f*(-e + Sqrt[e^2
- 4*d*f]) + b*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])))*Log[-e + Sqrt
[e^2 - 4*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f -
e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]) +
 (Sqrt[2]*(c*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^
2 - 4*d*f]) + f*(a*f*(e + Sqrt[e^2 - 4*d*f]) - b*(e^2 - 2*d*f + e
*Sqrt[e^2 - 4*d*f])))*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[e
^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f
 - b*(e + Sqrt[e^2 - 4*d*f]))]) - ((2*c*e - b*f)*Log[b + 2*c*x +
2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/Sqrt[c] - (Sqrt[2]*(c*(-e^3 + 3
*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]) + f*(a*f*
(-e + Sqrt[e^2 - 4*d*f]) + b*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))
)*Log[4*a*f*Sqrt[e^2 - 4*d*f] + 2*c*e^2*x - 8*c*d*f*x - 2*c*e*Sqr
t[e^2 - 4*d*f]*x + b*(e^2 - 4*d*f - e*Sqrt[e^2 - 4*d*f] + 2*f*Sqr
t[e^2 - 4*d*f]*x) + 2*Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[f*(-(b*e) +
2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*
f])]*Sqrt[a + x*(b + c*x)]])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d
*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f])
)]) - (Sqrt[2]*(c*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sq
rt[e^2 - 4*d*f]) + f*(a*f*(e + Sqrt[e^2 - 4*d*f]) - b*(e^2 - 2*d*
f + e*Sqrt[e^2 - 4*d*f])))*Log[4*a*f*Sqrt[e^2 - 4*d*f] - 2*c*e^2*
x + 8*c*d*f*x - 2*c*e*Sqrt[e^2 - 4*d*f]*x + 2*Sqrt[2]*Sqrt[e^2 -
4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*
(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)] - b*(e^2 + e*Sqrt
[e^2 - 4*d*f] - 2*f*(2*d + Sqrt[e^2 - 4*d*f]*x))])/(Sqrt[e^2 - 4*
d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e
 + Sqrt[e^2 - 4*d*f]))]))/(2*f^2)
```

Maple [B] time = 0.022, size = 10138, normalized size = 18.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)*x/(f*x^2 + e*x + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)*x/(f*x^2 + e*x + d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{a + bx + cx^2}}{d + ex + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)*x/(f*x^2 + e*x + d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.111 $\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=431

$$-\frac{\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\frac{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}$$

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/
f - (Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e
 - Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]
) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt
[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]) + (Sqrt[c*(e^2
 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*
d*f]))]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(
e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*
f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2
])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])
```

Rubi [A] time = 1.63642, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185

$$-\frac{\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\frac{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```


```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/
f - (Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e
 - Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]
) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt
[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]) + (Sqrt[c*(e^2
 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*
d*f]))]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(
e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*
f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2
])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])
```

Rubi in Sympy [A] time = 157.719, size = 420, normalized size = 0.97

$$\frac{\sqrt{c}\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\sqrt{2}\sqrt{2af^2-bef-bf\sqrt{-4df+e^2}-2cdf+ce^2+ce\sqrt{-4df+e^2}}\operatorname{atanh}\left(\frac{\sqrt{2}\left(4af-b\left(e+\sqrt{-4df+e^2}\right)+x\left(2bf-2c\left(e+\sqrt{-4df+e^2}\right)\right)\right)}{4\sqrt{a+bx+cx^2}\sqrt{2af^2-bef-2cdf+ce^2-(bf-ce)\sqrt{-4df+e^2}}}\right)}{2f\sqrt{-4df+e^2}} - \frac{\sqrt{2}\sqrt{2af^2-bef+bf\sqrt{-4df+e^2}-2cdf+ce^2-ce\sqrt{-4df+e^2}}\operatorname{atanh}\left(\frac{\sqrt{2}\left(4af-be+b\sqrt{-4df+e^2}+x\left(2bf-2ce+2c\sqrt{-4df+e^2}\right)\right)}{4\sqrt{a+bx+cx^2}\sqrt{2af^2-bef-2cdf+ce^2+(bf-ce)\sqrt{-4df+e^2}}}\right)}{2f\sqrt{-4df+e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] sqrt(c)*atanh((b + 2*c*x)/(2*sqrt(c)*sqrt(a + b*x + c*x**2)))/f +
 sqrt(2)*sqrt(2*a*f**2 - b*e*f - b*f*sqrt(-4*d*f + e**2) - 2*c*d*
f + c*e**2 + c*e*sqrt(-4*d*f + e**2))*atanh(sqrt(2)*(4*a*f - b*(e
 + sqrt(-4*d*f + e**2)) + x*(2*b*f - 2*c*(e + sqrt(-4*d*f + e**2)
)))/(4*sqrt(a + b*x + c*x**2)*sqrt(2*a*f**2 - b*e*f - 2*c*d*f + c
*e**2 - (b*f - c*e)*sqrt(-4*d*f + e**2))))/(2*f*sqrt(-4*d*f + e**
2)) - sqrt(2)*sqrt(2*a*f**2 - b*e*f + b*f*sqrt(-4*d*f + e**2) - 2
*c*d*f + c*e**2 - c*e*sqrt(-4*d*f + e**2))*atanh(sqrt(2)*(4*a*f -
 b*e + b*sqrt(-4*d*f + e**2) + x*(2*b*f - 2*c*e + 2*c*sqrt(-4*d*f
 + e**2)))/(4*sqrt(a + b*x + c*x**2)*sqrt(2*a*f**2 - b*e*f - 2*c*
d*f + c*e**2 + (b*f - c*e)*sqrt(-4*d*f + e**2))))/(2*f*sqrt(-4*d*
f + e**2))
```

Mathematica [A] time = 4.70277, size = 699, normalized size = 1.62

$$\frac{\sqrt{2}\log\left(\sqrt{e^2-4df}-e-2fx\right)\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}-\sqrt{2}\log\left(\sqrt{e^2-4df}+e\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 -
 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x
] - Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f
 - b*(e + Sqrt[e^2 - 4*d*f]))]*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x]
 + 2*Sqrt[c]*Sqrt[e^2 - 4*d*f]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a +
 x*(b + c*x)]] - Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*
d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Log[4*a*f*Sqrt[e^2
 - 4*d*f] + 2*c*e^2*x - 8*c*d*f*x - 2*c*e*Sqrt[e^2 - 4*d*f]*x + b
*(e^2 - 4*d*f - e*Sqrt[e^2 - 4*d*f] + 2*f*Sqrt[e^2 - 4*d*f]*x) +
2*Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 -
 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b +
 c*x)]] + Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*
(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Log[4*a*f*Sqrt[e^2 - 4*d*f]
- 2*c*e^2*x + 8*c*d*f*x - 2*c*e*Sqrt[e^2 - 4*d*f]*x + 2*Sqrt[2]*S
qrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(
2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)] - b*(e^
2 + e*Sqrt[e^2 - 4*d*f] - 2*f*(2*d + Sqrt[e^2 - 4*d*f]*x))])/(2*f
*Sqrt[e^2 - 4*d*f])
```

Maple [B] time = 0., size = 6019, normalized size = 14.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)/(f*x^2 + e*x + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)/(f*x^2 + e*x + d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{d + ex + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)/(f*x^2 + e*x + d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.112 $\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex+fx^2)}\,dx$

Optimal. Leaf size=523

$$\frac{\left(cd\left(e-\sqrt{e^2-4df}\right)-f\left(2bd-a\left(\sqrt{e^2-4df}+e\right)\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(cd\left(\sqrt{e^2-4df}+e\right)-f\left(2bd-a\left(e-\sqrt{e^2-4df}\right)\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$

```
[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]
)/d) + ((c*d*(e - Sqrt[e^2 - 4*d*f]) - f*(2*b*d - a*(e + Sqrt[e^2
 - 4*d*f])))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f
- c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f -
 b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x +
c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sq
rt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]) - ((c*
d*(e + Sqrt[e^2 - 4*d*f]) - f*(2*b*d - a*(e - Sqrt[e^2 - 4*d*f]))
)*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sq
rt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*
a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(
Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*
d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])
```

Rubi [A] time = 7.14029, antiderivative size = 521, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{\left(-af\left(\sqrt{e^2-4df}+e\right)+2bdf-cd\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(-af\left(e-\sqrt{e^2-4df}\right)+2bdf-cd\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[Sqrt[a + b*x + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]
)/d) - ((2*b*d*f - c*d*(e - Sqrt[e^2 - 4*d*f]) - a*f*(e + Sqrt[e^
2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f
- c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f -
 b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x +
c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sq
rt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]) + ((2*
b*d*f - a*f*(e - Sqrt[e^2 - 4*d*f]) - c*d*(e + Sqrt[e^2 - 4*d*f])
)*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sq
rt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*
```

```
a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(
Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*
d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)**(1/2)/x/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.86771, size = 903, normalized size = 1.73

$$2\sqrt{a}\log(x)+\frac{\sqrt{2}\left(2bdf-a\left(e+\sqrt{e^2-4df}\right)f+cd\left(\sqrt{e^2-4df}-e\right)\right)\log\left(-e-2fx+\sqrt{e^2-4df}\right)}{\sqrt{e^2-4df}\sqrt{c\left(e^2-\sqrt{e^2-4df}\,e-2df\right)+f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)}}+\frac{\sqrt{2}\left(-2bdf+a\left(e-\sqrt{e^2-4df}\right)f+cd\left(e+\sqrt{e^2-4df}\right)\right)\log\left(e+2fx+\right.}{\sqrt{e^2-4df}\sqrt{c\left(e^2+\sqrt{e^2-4df}\,e-2df\right)+f\left(2af-b\left(e+\sqrt{e^2-4d\right.\right.}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] (2*Sqrt[a]*Log[x] + (Sqrt[2]*(2*b*d*f + c*d*(-e + Sqrt[e^2 - 4*d*
f]) - a*f*(e + Sqrt[e^2 - 4*d*f]))*Log[-e + Sqrt[e^2 - 4*d*f] - 2
*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*
f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]) + (Sqrt[2]*(-2*b*d
*f + a*f*(e - Sqrt[e^2 - 4*d*f]) + c*d*(e + Sqrt[e^2 - 4*d*f]))*L
og[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2
 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*
d*f]))]) - 2*Sqrt[a]*Log[2*a + b*x + 2*Sqrt[a]*Sqrt[a + x*(b + c*
x)]] - (Sqrt[2]*(2*b*d*f + c*d*(-e + Sqrt[e^2 - 4*d*f]) - a*f*(e
+ Sqrt[e^2 - 4*d*f]))*Log[4*a*f*Sqrt[e^2 - 4*d*f] + 2*c*e^2*x - 8
*c*d*f*x - 2*c*e*Sqrt[e^2 - 4*d*f]*x + b*(e^2 - 4*d*f - e*Sqrt[e^
2 - 4*d*f] + 2*f*Sqrt[e^2 - 4*d*f]*x) + 2*Sqrt[2]*Sqrt[e^2 - 4*d*
f]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f
 - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)]])/(Sqrt[e^2 - 4*d*
f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e
+ Sqrt[e^2 - 4*d*f]))]) - (Sqrt[2]*(-2*b*d*f + a*f*(e - Sqrt[e^2
- 4*d*f]) + c*d*(e + Sqrt[e^2 - 4*d*f]))*Log[4*a*f*Sqrt[e^2 - 4*d
*f] - 2*c*e^2*x + 8*c*d*f*x - 2*c*e*Sqrt[e^2 - 4*d*f]*x + 2*Sqrt[
2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) +
 f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)] - b
*(e^2 + e*Sqrt[e^2 - 4*d*f] - 2*f*(2*d + Sqrt[e^2 - 4*d*f]*x))])/
(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f
*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]))/(2*d)
```

Maple [B] time = 0.025, size = 6460, normalized size = 12.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 + ex + d)x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{x\,(d + ex + fx^2)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)**(1/2)/x/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(x*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.113 $\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex+fx^2)}\,dx$

Optimal. Leaf size=736

$$-\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)-bd\left(\sqrt{e^2-4df}+e\right)+2cd^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$+\frac{f\left(a\left(-e\sqrt{e^2-4df}-2df+e^2\right)-bd\left(e-\sqrt{e^2-4df}\right)+2cd^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$+\frac{\sqrt{a}e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{be\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^2}$$
$$-\frac{\sqrt{a+bx+cx^2}}{dx}-\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{d}$$

```
[Out] -(Sqrt[a + b*x + c*x^2]/(d*x)) - (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a
]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a]*d) + (Sqrt[a]*e*ArcTanh[(2*
a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d^2 + (Sqrt[c]*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/d - (b*e*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*d^2
) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
c*x^2])])/(2*Sqrt[c]*d^2) - (f*(2*c*d^2 - b*d*(e + Sqrt[e^2 - 4*d
*f]) + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*
(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/
(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*S
qrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2
- 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f -
b*(e - Sqrt[e^2 - 4*d*f]))]) + (f*(2*c*d^2 - b*d*(e - Sqrt[e^2 -
4*d*f]) + a*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f -
 b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*
x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e
^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f
 - b*(e + Sqrt[e^2 - 4*d*f]))])
```

Rubi [A] time = 8.04169, antiderivative size = 736, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)-bd\left(\sqrt{e^2-4df}+e\right)+2cd^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$+\frac{f\left(a\left(-e\sqrt{e^2-4df}-2df+e^2\right)-bd\left(e-\sqrt{e^2-4df}\right)+2cd^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$+\frac{\sqrt{a}e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{be\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^2}$$
$$-\frac{\sqrt{a+bx+cx^2}}{dx}-\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(d*x)) - (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a
]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a]*d) + (Sqrt[a]*e*ArcTanh[(2*
a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d^2 + (Sqrt[c]*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/d - (b*e*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*d^2
) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
c*x^2])])/(2*Sqrt[c]*d^2) - (f*(2*c*d^2 - b*d*(e + Sqrt[e^2 - 4*d
*f]) + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*
(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/
(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*S
qrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2
- 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f -
b*(e - Sqrt[e^2 - 4*d*f]))]) + (f*(2*c*d^2 - b*d*(e - Sqrt[e^2 -
4*d*f]) + a*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f -
 b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*
x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e
^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f
 - b*(e + Sqrt[e^2 - 4*d*f]))])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)**(1/2)/x**2/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 3.7545, size = 972, normalized size = 1.32

$$\frac{-\frac{2\sqrt{a+x(b+cx)}d}{x}+\frac{(bd-2ae)\log(x)}{\sqrt{a}}+\frac{\sqrt{2}f\left(2cd^2-b\left(e+\sqrt{e^2-4df}\right)d+a\left(e^2+\sqrt{e^2-4df}e-2df\right)\right)\log\left(-e-2fx+\sqrt{e^2-4df}\right)}{\sqrt{e^2-4df}\sqrt{c\left(e^2-\sqrt{e^2-4df}e-2df\right)+f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)}}+\frac{\sqrt{2}f\left(-2cd^2+b\left(e-\sqrt{e^2-4df}\right)d+\cdots}{\sqrt{e^2-4df}\sqrt{c\left(e^2+\sqrt{e^2\cdots}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] ((-2*d*Sqrt[a + x*(b + c*x)])/x + ((b*d - 2*a*e)*Log[x])/Sqrt[a]
+ (Sqrt[2]*f*(2*c*d^2 - b*d*(e + Sqrt[e^2 - 4*d*f]) + a*(e^2 - 2*
d*f + e*Sqrt[e^2 - 4*d*f]))*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x])/
(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f
*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]) + (Sqrt[2]*f*(-2*c*d^2 +
b*d*(e - Sqrt[e^2 - 4*d*f]) + a*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*
f]))*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[
c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^
2 - 4*d*f]))]) + ((-(b*d) + 2*a*e)*Log[2*a + b*x + 2*Sqrt[a]*Sqrt
[a + x*(b + c*x)]])/Sqrt[a] + (Sqrt[2]*f*(-2*c*d^2 + b*d*(e + Sqr
t[e^2 - 4*d*f]) - a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Log[4*a*
f*Sqrt[e^2 - 4*d*f] + 2*c*e^2*x - 8*c*d*f*x - 2*c*e*Sqrt[e^2 - 4*
d*f]*x + b*(e^2 - 4*d*f - e*Sqrt[e^2 - 4*d*f] + 2*f*Sqrt[e^2 - 4*
d*f]*x) + 2*Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[f*(-(b*e) + 2*a*f + b*
Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[
a + x*(b + c*x)]])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqr
t[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]) - (Sqr
t[2]*f*(-2*c*d^2 + b*d*(e - Sqrt[e^2 - 4*d*f]) + a*(-e^2 + 2*d*f
```

```
+ e*Sqrt[e^2 - 4*d*f]))*Log[4*a*f*Sqrt[e^2 - 4*d*f] - 2*c*e^2*x +
 8*c*d*f*x - 2*c*e*Sqrt[e^2 - 4*d*f]*x + 2*Sqrt[2]*Sqrt[e^2 - 4*d
*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e
+ Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)] - b*(e^2 + e*Sqrt[e^
2 - 4*d*f] - 2*f*(2*d + Sqrt[e^2 - 4*d*f]*x))])/(Sqrt[e^2 - 4*d*f
]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e +
Sqrt[e^2 - 4*d*f]))]))/(2*d^2)
```

Maple [B] time = 0.028, size = 6765, normalized size = 9.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 + ex + d)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{x^2\,(d + ex + fx^2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)**(1/2)/x**2/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(x**2*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.114 $\int \frac{x^3}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=545

$$-\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}-\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{\left(2def-\left(e^2-df\right)\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{e\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2}+\frac{\sqrt{a+bx+cx^2}}{cf}$$

```
[Out] Sqrt[a + b*x + c*x^2]/(c*f) - (e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(Sqrt[c]*f^2) - (b*ArcTanh[(b + 2*c*x)/(2
*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*f) - ((2*d*e*f - (e^
2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^
2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*
d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqr
t[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*
f]]) + ((2*d*e*f - (e^2 - d*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(
4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*
d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^2
*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e
- b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 7.74521, antiderivative size = 545, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}-\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{\left(2def-\left(e^2-df\right)\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{e\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2}+\frac{\sqrt{a+bx+cx^2}}{cf}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^3/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/(c*f) - (e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(Sqrt[c]*f^2) - (b*ArcTanh[(b + 2*c*x)/(2
*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*f) - ((2*d*e*f - (e^
2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^
```

```
2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*
d*f]]*Sqrt[a + b*x + c*x^2]])]/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqr
t[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*
f]]) + ((2*d*e*f - (e^2 - d*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(
4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*
d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]])]/(Sqrt[2]*f^2
*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e
- b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.05813, size = 918, normalized size = 1.68

$$\frac{2\sqrt{a+x(b+cx)}f}{c}+\frac{\sqrt{2}\left(-e^3+\sqrt{e^2-4df}e^2+3dfe-df\sqrt{e^2-4df}\right)\log\left(-e-2fx+\sqrt{e^2-4df}\right)}{\sqrt{e^2-4df}\sqrt{c\left(e^2-\sqrt{e^2-4df}e-2df\right)+f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)}}+\frac{\sqrt{2}\left(e^3+\sqrt{e^2-4df}e^2-3dfe-df\sqrt{e^2-4df}\right)\log\left(e+2fx+\sqrt{e^2-4df}\right)}{\sqrt{e^2-4df}\sqrt{c\left(e^2+\sqrt{e^2-4df}e-2df\right)+f\left(2af-b\left(e+\sqrt{e^2-4df}\right)\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*f*Sqrt[a + x*(b + c*x)])/c + (Sqrt[2]*(-e^3 + 3*d*e*f + e^2*S
qrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])*Log[-e + Sqrt[e^2 - 4*d
*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2
 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]) + (Sqrt[2]*
(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])*L
og[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2
 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*
d*f]))]) - ((2*c*e + b*f)*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b
 + c*x)]])/c^(3/2) - (Sqrt[2]*(-e^3 + 3*d*e*f + e^2*Sqrt[e^2 - 4*
d*f] - d*f*Sqrt[e^2 - 4*d*f])*Log[4*a*f*Sqrt[e^2 - 4*d*f] + 2*c*e
^2*x - 8*c*d*f*x - 2*c*e*Sqrt[e^2 - 4*d*f]*x + b*(e^2 - 4*d*f - e
*Sqrt[e^2 - 4*d*f] + 2*f*Sqrt[e^2 - 4*d*f]*x) + 2*Sqrt[2]*Sqrt[e^
2 - 4*d*f]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2
 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)]])/(Sqrt[e^
2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f
+ b*(-e + Sqrt[e^2 - 4*d*f]))]) - (Sqrt[2]*(e^3 - 3*d*e*f + e^2*S
qrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])*Log[4*a*f*Sqrt[e^2 - 4*
d*f] - 2*c*e^2*x + 8*c*d*f*x - 2*c*e*Sqrt[e^2 - 4*d*f]*x + 2*Sqrt
[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
+ f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)] -
b*(e^2 + e*Sqrt[e^2 - 4*d*f] - 2*f*(2*d + Sqrt[e^2 - 4*d*f]*x))])
/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) +
f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]))/(2*f^2)
```

Maple [B] time = 0.026, size = 3131, normalized size = 5.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] (c*x^2+b*x+a)^(1/2)/c/f-1/2/f*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))-1/f^2*e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))/c^(1/2)+1/2/f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d
*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^
2*c)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(
-4*d*f+e^2)^(1/2))/f))*d-1/2/f^3*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*
f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*
ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f
-2*c*d*f+e^2*c)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f
-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f
+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2))/
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^2+3/2/f^2/(-4*d*f+e^2)^(1/2)*
2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2+1/f*(-c*(-4*d
*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/
2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f
-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*
c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b
*e*f-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
*d*e-1/2/f^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f
+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*l
n(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-
2*c*d*f+e^2*c)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-
c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2))/(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^3+1/2/f^2*2^(1/2)/(((-4*d*f+e^
2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/
f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-b*e*f-2*c*d*f+e^2*c)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x
-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*
b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2
)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2
)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1
/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^
(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*d-1/2/f^3*2^(1/2)/(((-4
*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+
e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*
e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)
^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^
2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^
2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+
e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c
)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^2-3/2/f^2/(-4*
d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(
1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2+
(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f
)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(
1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*
f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)
```

```
^(1/2))/f))*d*e+1/2/f^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)
^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^
2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-b*e*f-2*c*d*f+e^2*c)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*
f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*
(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+
b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1
/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a + bx + cx^2}\,(d + ex + fx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.115 $\int \frac{x^2}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=463

$$-\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

```
[Out] ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]*f
) - ((e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e -
Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sq
rt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e
^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 -
 4*d*f]]) - ((2*d*f - e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f -
 b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*
x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2
 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sq
rt[e^2 - 4*d*f]])
```

Rubi [A] time = 7.39372, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^2/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```


```
[Out] ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]*f
) - ((e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e -
Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sq
rt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e
^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 -
 4*d*f]]) - ((2*d*f - e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f -
 b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*
x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2
 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sq
rt[e^2 - 4*d*f]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.39008, size = 815, normalized size = 1.76

$$-\frac{\sqrt{2}\left(-e^2+\sqrt{e^2-4df}\,e+2df\right)\log\left(-e-2fx+\sqrt{e^2-4df}\right)}{\sqrt{e^2-4df}\sqrt{c\left(e^2-\sqrt{e^2-4df}\,e-2df\right)+f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)}}-\frac{\sqrt{2}\left(e^2+\sqrt{e^2-4df}\,e-2df\right)\log\left(e+2fx+\sqrt{e^2-4df}\right)}{\sqrt{e^2-4df}\sqrt{c\left(e^2+\sqrt{e^2-4df}\,e-2df\right)+f\left(2af-b\left(e+\sqrt{e^2-4df}\right)\right)}}+\frac{2\log\left(b+2cx+2\sqrt{c}\sqrt{a+x\cdots}\right.}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (-((Sqrt[2]*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*Log[-e + Sqrt[e^
2 - 4*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*S
qrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]])) - (
Sqrt[2]*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*Log[e + Sqrt[e^2 - 4*
d*f] + 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^
2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]]) + (2*Log[b
+ 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/Sqrt[c] + (Sqrt[2]*(-
e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*Log[4*a*f*Sqrt[e^2 - 4*d*f] +
2*c*e^2*x - 8*c*d*f*x - 2*c*e*Sqrt[e^2 - 4*d*f]*x + b*(e^2 - 4*d*
f - e*Sqrt[e^2 - 4*d*f] + 2*f*Sqrt[e^2 - 4*d*f]*x) + 2*Sqrt[2]*Sq
rt[e^2 - 4*d*f]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c
*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)]])/(Sq
rt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2
*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]]) + (Sqrt[2]*(e^2 - 2*d*f + e*
Sqrt[e^2 - 4*d*f])*Log[4*a*f*Sqrt[e^2 - 4*d*f] - 2*c*e^2*x + 8*c*
d*f*x - 2*c*e*Sqrt[e^2 - 4*d*f]*x + 2*Sqrt[2]*Sqrt[e^2 - 4*d*f]*S
qrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqr
t[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)] - b*(e^2 + e*Sqrt[e^2 - 4
*d*f] - 2*f*(2*d + Sqrt[e^2 - 4*d*f]*x))]])/(Sqrt[e^2 - 4*d*f]*Sqr
t[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[
e^2 - 4*d*f]))]]))/(2*f)
```

Maple [B] time = 0.023, size = 2321, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] 1/f*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+1/2/f^2*2
^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*
e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2+(c*(-4*d*f+e^2)^
(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((
(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d
*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(
```

```
c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2
*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e+1/
f/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+
e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c
)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1
/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+
e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*
f+e^2)^(1/2))/f))*d-1/2/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+
e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c
)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1
/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1
/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^
(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2
)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^2+1/2/f^2*2^(1/2)/(
(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c
*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2+1/f*(-c*(-4*d*f+e^2)^(
1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4
*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+
e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c
*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(
-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*
d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e-1/f/(-
4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^
2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/
f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f))*d+1/2/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f
+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*
c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-
c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)
^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^
2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+
e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+
e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c
)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.116 $\int \frac{x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=402

$$\frac{\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

```
[Out] ((e - Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f
]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2
 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqr
t[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((e +
Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2
*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a +
b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f -
 b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 2.55464, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$

$$\frac{\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```


```
[Out] ((e - Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f
]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2
 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqr
t[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((e +
Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2
*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a +
b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f -
 b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi in Sympy [A] time = 127.151, size = 384, normalized size = 0.96

$$\frac{\sqrt{2}\left(e-\sqrt{-4df+e^2}\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(4af-be+b\sqrt{-4df+e^2}+x\left(2bf-2ce+2c\sqrt{-4df+e^2}\right)\right)}{4\sqrt{a+bx+cx^2}\sqrt{2af^2-bef-2cdf+ce^2+(bf-ce)\sqrt{-4df+e^2}}}\right)}{2\sqrt{-4df+e^2}\sqrt{2af^2-bef-2cdf+ce^2+(bf-ce)\sqrt{-4df+e^2}}} - \frac{\sqrt{2}\left(e+\sqrt{-4df+e^2}\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(4af-b\left(e+\sqrt{-4df+e^2}\right)+x\left(2bf-2c\left(e+\sqrt{-4df+e^2}\right)\right)\right)}{4\sqrt{a+bx+cx^2}\sqrt{2af^2-bef-2cdf+ce^2-(bf-ce)\sqrt{-4df+e^2}}}\right)}{2\sqrt{-4df+e^2}\sqrt{2af^2-bef-2cdf+ce^2-(bf-ce)\sqrt{-4df+e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] sqrt(2)*(e - sqrt(-4*d*f + e**2))*atanh(sqrt(2)*(4*a*f - b*e + b*
sqrt(-4*d*f + e**2) + x*(2*b*f - 2*c*e + 2*c*sqrt(-4*d*f + e**2))
)/(4*sqrt(a + b*x + c*x**2)*sqrt(2*a*f**2 - b*e*f - 2*c*d*f + c*e
**2 + (b*f - c*e)*sqrt(-4*d*f + e**2))))/(2*sqrt(-4*d*f + e**2)*s
qrt(2*a*f**2 - b*e*f - 2*c*d*f + c*e**2 + (b*f - c*e)*sqrt(-4*d*f
 + e**2))) - sqrt(2)*(e + sqrt(-4*d*f + e**2))*atanh(sqrt(2)*(4*a
*f - b*(e + sqrt(-4*d*f + e**2)) + x*(2*b*f - 2*c*(e + sqrt(-4*d*
f + e**2))))/(4*sqrt(a + b*x + c*x**2)*sqrt(2*a*f**2 - b*e*f - 2*
c*d*f + c*e**2 - (b*f - c*e)*sqrt(-4*d*f + e**2))))/(2*sqrt(-4*d*
f + e**2)*sqrt(2*a*f**2 - b*e*f - 2*c*d*f + c*e**2 - (b*f - c*e)*
sqrt(-4*d*f + e**2)))
```

Mathematica [B] time = 6.17901, size = 874, normalized size = 2.17

$$\frac{\left(\sqrt{e^2-4df}-e\right)\sqrt{cx^2+bx+a}\log\left(-e-2fx+\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{ce^2-bfe-c\sqrt{e^2-4df}e+2af^2-2cdf+bf\sqrt{e^2-4df}}\sqrt{a+x(b+cx)}}+\frac{\left(e+\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}\log\left(e+2fx+\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{ce^2-bfe+c\sqrt{e^2-4df}e+2af^2-2cdf-bf\sqrt{e^2-4df}}\sqrt{a+x(b+cx)}}-\frac{\left(e+\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}\log\left(-be^2-2cxe^2-2c\sqrt{e^2-4df}xe-b\sqrt{e^2-4df}e+4bdf+8cdfx+2bf\sqrt{e^2-4df}x\cdots\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{ce^2-bfe+c\sqrt{e^2-4df}e+2af^2\cdots}}-\frac{\left(\sqrt{e^2-4df}-e\right)\sqrt{cx^2+bx+a}\log\left(be^2+2cxe^2-2c\sqrt{e^2-4df}xe-b\sqrt{e^2-4df}e-4bdf-8cdfx+2bf\sqrt{e^2-4df}x+\cdots\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{ce^2-bfe-c\sqrt{e^2-4df}e+2af^2\cdots}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((-e + Sqrt[e^2 - 4*d*f])*Sqrt[a + b*x + c*x^2]*Log[-e + Sqrt[e^2
 - 4*d*f] - 2*f*x])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d
*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d
*f]]*Sqrt[a + x*(b + c*x)]) + ((e + Sqrt[e^2 - 4*d*f])*Sqrt[a + b
*x + c*x^2]*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[2]*Sqrt[e^2
 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 -
 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + x*(b + c*x)]) - ((e + S
qrt[e^2 - 4*d*f])*Sqrt[a + b*x + c*x^2]*Log[-(b*e^2) + 4*b*d*f -
b*e*Sqrt[e^2 - 4*d*f] + 4*a*f*Sqrt[e^2 - 4*d*f] - 2*c*e^2*x + 8*c
*d*f*x - 2*c*e*Sqrt[e^2 - 4*d*f]*x + 2*b*f*Sqrt[e^2 - 4*d*f]*x +
2*Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^
2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x +
 c*x^2]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
```

```
 + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[
a + x*(b + c*x)]) - ((-e + Sqrt[e^2 - 4*d*f])*Sqrt[a + b*x + c*x^
2]*Log[b*e^2 - 4*b*d*f - b*e*Sqrt[e^2 - 4*d*f] + 4*a*f*Sqrt[e^2 -
 4*d*f] + 2*c*e^2*x - 8*c*d*f*x - 2*c*e*Sqrt[e^2 - 4*d*f]*x + 2*b
*f*Sqrt[e^2 - 4*d*f]*x + 2*Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 -
 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2
 - 4*d*f]]*Sqrt[a + b*x + c*x^2]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqr
t[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f
*Sqrt[e^2 - 4*d*f]]*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.02, size = 1516, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] 1/2/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*
d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e
^2*c)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*
d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*
(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-
4*d*f+e^2)^(1/2))/f))*e-1/2/f*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((
((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*
d*f+e^2*c)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*
f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x
-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(
-e+(-4*d*f+e^2)^(1/2))/f))-1/2/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/((-(-
4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f
+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)
+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f
+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*
c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4
*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4
*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+
e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e-1/2/f*2^(1
/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2+1/f*(-c*(-4*d*f+e
^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(
(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c
*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/
f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f
-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.15734, size = 15270, normalized size = 37.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt((2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f + (c^2*d^2*e^
2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*
(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^
2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^
4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d
*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(
a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c +
 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)
*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^
3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*
b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^
3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*
a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e
^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4
*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b
^2 - 6*a*c)*d*e^2)*f))*log(-(2*b^2*d^3 - 4*a*b*d^2*e + 2*a^2*d*e^
2 + sqrt(2)*(b^2*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4 - 4*(b^2*d^3 - 2
*a*b*d^2*e + a^2*d*e^2)*f - (2*c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 - 2*
a*b*c*d*e^5 + a^2*c*e^6 + 8*a^3*d^2*f^4 + (b^2*c + 3*a*c^2)*d^2*e
^4 - 2*(2*a^2*b*d^2*e + 3*a^3*d*e^2 - 4*(a*b^2 - 3*a^2*c)*d^3)*f^
3 + (5*a^2*b*d*e^3 + a^3*e^4 - 8*(b^2*c - 3*a*c^2)*d^4 + 4*(b^3 -
 2*a*b*c)*d^3*e - 2*(5*a*b^2 - 11*a^2*c)*d^2*e^2)*f^2 - (8*c^3*d^
5 - 12*b*c^2*d^4*e + a^2*b*e^5 + 2*(b^2*c + 9*a*c^2)*d^3*e^2 + (b
^3 - 10*a*b*c)*d^2*e^3 - 2*(a*b^2 - 4*a^2*c)*d*e^4)*f)*sqrt((b^2*
d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b
*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*
e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2
*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^
2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a
*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^
2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c
)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^
2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c -
 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((2*c*d^2 - b*d
*e + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2
*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2
*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*
d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b
*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*
e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2
*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^
2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a
*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^
2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c
)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^
2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c -
 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^
2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^
2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) + (4*b*c
*d^3 + a*b*d*e^2 - (b^2 + 4*a*c)*d^2*e)*x - (2*a*c^2*d^3*e^2 - 2*
a*b*c*d^2*e^3 + 2*a^2*c*d*e^4 - 8*a^3*d^2*f^3 + 2*(4*a^2*b*d^2*e
+ a^3*d*e^2 - 4*(a*b^2 - 2*a^2*c)*d^3)*f^2 - 2*(4*a*c^2*d^4 - 4*a
*b*c*d^3*e + a^2*b*d*e^3 - (a*b^2 - 6*a^2*c)*d^2*e^2)*f + (b*c^2*
d^3*e^2 - b^2*c*d^2*e^3 + a*b*c*d*e^4 - 4*a^2*b*d^2*f^3 + (4*a*b^
2*d^2*e + a^2*b*d*e^2 - 4*(b^3 - 2*a*b*c)*d^3)*f^2 - (4*b*c^2*d^4
 - 4*b^2*c*d^3*e + a*b^2*d*e^3 - (b^3 - 6*a*b*c)*d^2*e^2)*f)*x)*s
qrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^
3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*
```

```
c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2
)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a
*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*
c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*
c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2
+ 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5
+ (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(
a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((2*c*d^2 -
 b*d*e + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4
*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4
*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((
b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2
*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*
d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4
 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3
- a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 -
 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 2
2*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a
^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^
2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2
*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 -
4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (
4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log(
-(2*b^2*d^3 - 4*a*b*d^2*e + 2*a^2*d*e^2 - sqrt(2)*(b^2*d^2*e^2 -
2*a*b*d*e^3 + a^2*e^4 - 4*(b^2*d^3 - 2*a*b*d^2*e + a^2*d*e^2)*f -
 (2*c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 - 2*a*b*c*d*e^5 + a^2*c*e^6 + 8
*a^3*d^2*f^4 + (b^2*c + 3*a*c^2)*d^2*e^4 - 2*(2*a^2*b*d^2*e + 3*a
^3*d*e^2 - 4*(a*b^2 - 3*a^2*c)*d^3)*f^3 + (5*a^2*b*d*e^3 + a^3*e^
4 - 8*(b^2*c - 3*a*c^2)*d^4 + 4*(b^3 - 2*a*b*c)*d^3*e - 2*(5*a*b^
2 - 11*a^2*c)*d^2*e^2)*f^2 - (8*c^3*d^5 - 12*b*c^2*d^4*e + a^2*b*
e^5 + 2*(b^2*c + 9*a*c^2)*d^3*e^2 + (b^3 - 10*a*b*c)*d^2*e^3 - 2*
(a*b^2 - 4*a^2*c)*d*e^4)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/
(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 -
4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^
2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*
b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6
*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*
c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 -
 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 -
 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3
*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqr
t(c*x^2 + b*x + a)*sqrt((2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f + (c^2
*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e
^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^
3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/
(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 -
4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^
2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*
b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6
*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*
c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 -
 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 -
 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3
*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^
2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*
e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e
^3 - (b^2 - 6*a*c)*d*e^2)*f)) + (4*b*c*d^3 + a*b*d*e^2 - (b^2 + 4
*a*c)*d^2*e)*x - (2*a*c^2*d^3*e^2 - 2*a*b*c*d^2*e^3 + 2*a^2*c*d*e
^4 - 8*a^3*d^2*f^3 + 2*(4*a^2*b*d^2*e + a^3*d*e^2 - 4*(a*b^2 - 2*
a^2*c)*d^3)*f^2 - 2*(4*a*c^2*d^4 - 4*a*b*c*d^3*e + a^2*b*d*e^3 -
(a*b^2 - 6*a^2*c)*d^2*e^2)*f + (b*c^2*d^3*e^2 - b^2*c*d^2*e^3 + a
*b*c*d*e^4 - 4*a^2*b*d^2*f^3 + (4*a*b^2*d^2*e + a^2*b*d*e^2 - 4*(
b^3 - 2*a*b*c)*d^3)*f^2 - (4*b*c^2*d^4 - 4*b^2*c*d^3*e + a*b^2*d*
e^3 - (b^3 - 6*a*b*c)*d^2*e^2)*f)*x)*sqrt((b^2*d^2 - 2*a*b*d*e +
a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c
^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e
 + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b
^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^
2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3
*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2
*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*
c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e
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^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)
*f)))/x) + 1/4*sqrt(2)*sqrt((2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f -
(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a
^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*
b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e
^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^
6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^
4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 -
4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2
 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c -
a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b
^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d
^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 +
(b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))
/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e +
a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a
*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log(-(2*b^2*d^3 - 4*a*b*d^2*e +
 2*a^2*d*e^2 + sqrt(2)*(b^2*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4 - 4*(
b^2*d^3 - 2*a*b*d^2*e + a^2*d*e^2)*f + (2*c^3*d^4*e^2 - 3*b*c^2*d
^3*e^3 - 2*a*b*c*d*e^5 + a^2*c*e^6 + 8*a^3*d^2*f^4 + (b^2*c + 3*a
*c^2)*d^2*e^4 - 2*(2*a^2*b*d^2*e + 3*a^3*d*e^2 - 4*(a*b^2 - 3*a^2
*c)*d^3)*f^3 + (5*a^2*b*d*e^3 + a^3*e^4 - 8*(b^2*c - 3*a*c^2)*d^4
 + 4*(b^3 - 2*a*b*c)*d^3*e - 2*(5*a*b^2 - 11*a^2*c)*d^2*e^2)*f^2
- (8*c^3*d^5 - 12*b*c^2*d^4*e + a^2*b*e^5 + 2*(b^2*c + 9*a*c^2)*d
^3*e^2 + (b^3 - 10*a*b*c)*d^2*e^3 - 2*(a*b^2 - 4*a^2*c)*d*e^4)*f)
*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*
e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*
a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d
^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*
(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^
2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^
2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^
2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^
5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2
*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((2*
c*d^2 - b*d*e + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*
e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f
^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)
*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*
e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*
a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d
^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*
(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^
2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^
2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^
2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^
5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2
*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c
*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*
f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f
)) + (4*b*c*d^3 + a*b*d*e^2 - (b^2 + 4*a*c)*d^2*e)*x + (2*a*c^2*d
^3*e^2 - 2*a*b*c*d^2*e^3 + 2*a^2*c*d*e^4 - 8*a^3*d^2*f^3 + 2*(4*a
^2*b*d^2*e + a^3*d*e^2 - 4*(a*b^2 - 2*a^2*c)*d^3)*f^2 - 2*(4*a*c^
2*d^4 - 4*a*b*c*d^3*e + a^2*b*d*e^3 - (a*b^2 - 6*a^2*c)*d^2*e^2)*
f + (b*c^2*d^3*e^2 - b^2*c*d^2*e^3 + a*b*c*d*e^4 - 4*a^2*b*d^2*f^
3 + (4*a*b^2*d^2*e + a^2*b*d*e^2 - 4*(b^3 - 2*a*b*c)*d^3)*f^2 - (
4*b*c^2*d^4 - 4*b^2*c*d^3*e + a*b^2*d*e^3 - (b^3 - 6*a*b*c)*d^2*e
^2)*f)*x)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b
*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2
*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2
*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)
*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3
 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4
- 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3
- (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a
^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^
2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt
((2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 - b*c*d*e^3 +
a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^
2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2
)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*
d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2
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+ 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*
c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3
- 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8
*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*
a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^
2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*
c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3
 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 +
 a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d
^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^
2)*f))*log(-(2*b^2*d^3 - 4*a*b*d^2*e + 2*a^2*d*e^2 - sqrt(2)*(b^2
*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4 - 4*(b^2*d^3 - 2*a*b*d^2*e + a^2
*d*e^2)*f + (2*c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 - 2*a*b*c*d*e^5 + a^
2*c*e^6 + 8*a^3*d^2*f^4 + (b^2*c + 3*a*c^2)*d^2*e^4 - 2*(2*a^2*b*
d^2*e + 3*a^3*d*e^2 - 4*(a*b^2 - 3*a^2*c)*d^3)*f^3 + (5*a^2*b*d*e
^3 + a^3*e^4 - 8*(b^2*c - 3*a*c^2)*d^4 + 4*(b^3 - 2*a*b*c)*d^3*e
- 2*(5*a*b^2 - 11*a^2*c)*d^2*e^2)*f^2 - (8*c^3*d^5 - 12*b*c^2*d^4
*e + a^2*b*e^5 + 2*(b^2*c + 9*a*c^2)*d^3*e^2 + (b^3 - 10*a*b*c)*d
^2*e^3 - 2*(a*b^2 - 4*a^2*c)*d*e^4)*f)*sqrt((b^2*d^2 - 2*a*b*d*e
+ a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2
*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d
*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*
(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (
a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b
^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 +
 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(
2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3
*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^
4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((2*c*d^2 - b*d*e + a*e^2 - 2*a
*d*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*
d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2
*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2*a*b*d*e
+ a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2
*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d
*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*
(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (
a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b
^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 +
 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(
2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3
*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^
4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b
*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^
2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) + (4*b*c*d^3 + a*b*d*e^2
 - (b^2 + 4*a*c)*d^2*e)*x + (2*a*c^2*d^3*e^2 - 2*a*b*c*d^2*e^3 +
2*a^2*c*d*e^4 - 8*a^3*d^2*f^3 + 2*(4*a^2*b*d^2*e + a^3*d*e^2 - 4*
(a*b^2 - 2*a^2*c)*d^3)*f^2 - 2*(4*a*c^2*d^4 - 4*a*b*c*d^3*e + a^2
*b*d*e^3 - (a*b^2 - 6*a^2*c)*d^2*e^2)*f + (b*c^2*d^3*e^2 - b^2*c*
d^2*e^3 + a*b*c*d*e^4 - 4*a^2*b*d^2*f^3 + (4*a*b^2*d^2*e + a^2*b*
d*e^2 - 4*(b^3 - 2*a*b*c)*d^3)*f^2 - (4*b*c^2*d^4 - 4*b^2*c*d^3*e
 + a*b^2*d*e^3 - (b^3 - 6*a*b*c)*d^2*e^2)*f)*x)*sqrt((b^2*d^2 - 2
*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*
e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (
8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b
*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*
d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d
^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*
d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*
f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a
*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*
c^2)*d*e^4)*f)))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a + bx + cx^2}\,(d + ex + fx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.117 $\int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=374

$$\frac{\sqrt{2}f\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\sqrt{2}f\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

```
[Out] -((Sqrt[2]*f*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f
- c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f -
 b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x +
c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) + (Sqrt[2]*f*ArcTanh[(4*a*f
 - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f])
)*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b
*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 -
 4*d*f]])
```

Rubi [A] time = 0.791267, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{\sqrt{2}f\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\sqrt{2}f\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```


```
[Out] -((Sqrt[2]*f*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f
- c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f -
 b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x +
c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) + (Sqrt[2]*f*ArcTanh[(4*a*f
 - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f])
)*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b
*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 -
 4*d*f]])
```

Rubi in Sympy [A] time = 107.217, size = 357, normalized size = 0.95

$$-\frac{\sqrt{2} f \operatorname{atanh}\left(\frac{\sqrt{2}\left(4af-be+b\sqrt{-4df+e^2}+x\left(2bf-2ce+2c\sqrt{-4df+e^2}\right)\right)}{4\sqrt{a+bx+cx^2}\sqrt{2af^2-bef-2cdf+ce^2+(bf-ce)\sqrt{-4df+e^2}}}\right)}{\sqrt{-4df+e^2}\sqrt{2af^2-bef-2cdf+ce^2+(bf-ce)\sqrt{-4df+e^2}}} + \frac{\sqrt{2} f \operatorname{atanh}\left(\frac{\sqrt{2}\left(4af-b\left(e+\sqrt{-4df+e^2}\right)+x\left(2bf-2c\left(e+\sqrt{-4df+e^2}\right)\right)\right)}{4\sqrt{a+bx+cx^2}\sqrt{2af^2-bef-2cdf+ce^2-(bf-ce)\sqrt{-4df+e^2}}}\right)}{\sqrt{-4df+e^2}\sqrt{2af^2-bef-2cdf+ce^2-(bf-ce)\sqrt{-4df+e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] -sqrt(2)*f*atanh(sqrt(2)*(4*a*f - b*e + b*sqrt(-4*d*f + e**2) + x
*(2*b*f - 2*c*e + 2*c*sqrt(-4*d*f + e**2)))/(4*sqrt(a + b*x + c*x
**2)*sqrt(2*a*f**2 - b*e*f - 2*c*d*f + c*e**2 + (b*f - c*e)*sqrt(
-4*d*f + e**2))))/(sqrt(-4*d*f + e**2)*sqrt(2*a*f**2 - b*e*f - 2*
c*d*f + c*e**2 + (b*f - c*e)*sqrt(-4*d*f + e**2))) + sqrt(2)*f*at
anh(sqrt(2)*(4*a*f - b*(e + sqrt(-4*d*f + e**2)) + x*(2*b*f - 2*c
*(e + sqrt(-4*d*f + e**2))))/(4*sqrt(a + b*x + c*x**2)*sqrt(2*a*f
**2 - b*e*f - 2*c*d*f + c*e**2 - (b*f - c*e)*sqrt(-4*d*f + e**2))
))/(sqrt(-4*d*f + e**2)*sqrt(2*a*f**2 - b*e*f - 2*c*d*f + c*e**2
- (b*f - c*e)*sqrt(-4*d*f + e**2)))
```

Mathematica [A] time = 4.57193, size = 633, normalized size = 1.69

$$\sqrt{2} f\left(\frac{\log\left(\sqrt{e^2-4df}-e-2fx\right)}{\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\log\left(\sqrt{e^2-4df}+e+2fx\right)}{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\log\left(2\sqrt{2}\sqrt{e^2-4df}\sqrt{a+x(b+cx)}\sqrt{\ldots}\right.}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (Sqrt[2]*f*(Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x]/Sqrt[f*(-(b*e) +
2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*
f])] - Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x]/Sqrt[c*(e^2 - 2*d*f + e
*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))] - Lo
g[4*a*f*Sqrt[e^2 - 4*d*f] + 2*c*e^2*x - 8*c*d*f*x - 2*c*e*Sqrt[e^
2 - 4*d*f]*x + b*(e^2 - 4*d*f - e*Sqrt[e^2 - 4*d*f] + 2*f*Sqrt[e^
2 - 4*d*f]*x) + 2*Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[f*(-(b*e) + 2*a*
f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]
*Sqrt[a + x*(b + c*x)]]/Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d
*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])] + Log[4*a*f*Sqrt[e^
2 - 4*d*f] - 2*c*e^2*x + 8*c*d*f*x - 2*c*e*Sqrt[e^2 - 4*d*f]*x +
2*Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*
d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*
x)] - b*(e^2 + e*Sqrt[e^2 - 4*d*f] - 2*f*(2*d + Sqrt[e^2 - 4*d*f]
*x))]/Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(
e + Sqrt[e^2 - 4*d*f]))]))/Sqrt[e^2 - 4*d*f]
```

Maple [B] time = 0., size = 761, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] -1/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f
+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*
c)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(
1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d
*f+e^2)^(1/2))/f))+1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(
1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)
^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-b*e*f-2*c*d*f+e^2*c)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*
b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2
)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1
/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1
/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^
(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.7945, size = 15237, normalized size = 40.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^
2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*
(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^
2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^
4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d
*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(
a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c +
 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)
*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^
3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*
b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^
3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*
a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e
^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4
*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b
^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b*e)*f^2 + sqrt(2)*(c^2*
d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*
c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f - (c^3*d^3*e^3 - b*c^2*d^2*e^4
 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 +
 a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3
 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*e - 2*(b^3
- 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4
```

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*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a
*c^2)*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e
^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^
5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2
*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*
a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*
e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e
 - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c
)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d
^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b
*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 +
b*x + a)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 -
 b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^
2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 -
 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e
^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^
5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2
*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*
a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*
e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e
 - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c
)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d
^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b
*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2
- b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b
^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2
- 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e
)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x - (8*a^3*d*f^4 - 2*(4*a^2*b*d*
e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a
*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*
d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*
d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b
^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*
e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f +
b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c
^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e
 + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b
^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^
2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3
*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2
*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*
c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e
^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)
*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f +
(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a
^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*
b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f
^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^
6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^
4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 -
4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2
 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c -
a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b
^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d
^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 +
(b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))
/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e +
a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a
*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b*e)*f^2 - sq
rt(2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2
)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f - (c^3*d^3*e^3 - b*
c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^
2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)
*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*
e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 -
 (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*
b^2*c - 5*a*c^2)*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)
/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 -
 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e
^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a
*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 +
6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b
```

```
*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3
- 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5
- 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^
3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sq
rt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^
2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*
e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e
^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)
/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 -
 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e
^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a
*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 +
6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b
*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3
- 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5
- 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^
3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c
^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2
*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*
e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*
c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x - (8*a^3*d*f^4 - 2*
(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^
2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2
- 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4
- (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^
2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 -
(b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((c^2*e^2 - 2
*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*
e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (
8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b
*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*
d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d
^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*
d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*
f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a
*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*
c^2)*d*e^4)*f)))/x) + 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d
+ b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*
a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c
*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*
e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 +
 a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3
*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3
+ 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e
 + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 -
8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e
^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 -
 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)
*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*
d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4
*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*
c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b*
e)*f^2 + sqrt(2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*
e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f + (c^3*d
^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*
f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b
^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c -
a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d
*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*
c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f
 + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^
2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*
d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2
*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e +
(a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(
b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2
+ 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*
(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^
3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e
^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b
*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b
*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^
```

```
2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f
 + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^
2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*
d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2
*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e +
(a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(
b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2
 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*
(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^
3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e
^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*
b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d
^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)
*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3
*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3
 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*
d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a
^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^
3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*
e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((
c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2
*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*
d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4
 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3
 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 -
 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 2
2*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a
^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^
2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2
*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^
2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*
d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*
d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e
^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*
c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e
^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*
(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2
*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*
c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2
*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)
*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2
 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c -
2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2
*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2
*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log((2*(b
^2*d - a*b*e)*f^2 - sqrt(2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2
 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)
*f + (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2
 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3
 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*
(3*b^2*c - a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 -
 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d
*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f)*sqrt((c^2*e^2
 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2
*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4
+ (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^
3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*
c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3
)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^
2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^
4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 +
6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a
^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 -
 (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f
^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3
 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2
- 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2
*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4
+ (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^
3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*
c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3
)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^
```

```
2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^
4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 +
6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a
^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*
f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^
3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e
 - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)
*x + (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c
)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2
- 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^
4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b
*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 -
6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f
)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*
d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2
+ 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*
c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3
- 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8
*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*
a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^
2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*
c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3
 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.118 $\int \frac{1}{x\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=451

$$\frac{f\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{f\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d}$$

```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(Sqrt[a]
*d)) + (f*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^
2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*
d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[
c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]
]) - (f*(e - Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqrt[e^2
- 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqr
t[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*
f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*
e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 5.86835, antiderivative size = 451, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{f\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{f\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/(x*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```


```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(Sqrt[a]
*d)) + (f*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^
2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*
d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[
c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]
]) - (f*(e - Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqrt[e^2
- 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqr
t[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*
f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*
e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [B] time = 6.24593, size = 994, normalized size = 2.2

$$\frac{\sqrt{cx^2+bx+a}\log(x)}{\sqrt{a}d\sqrt{a+x(b+cx)}} - \frac{f\left(e+\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}\log\left(-e-2fx+\sqrt{e^2-4df}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{ce^2-bfe-c\sqrt{e^2-4df}e+2af^2-2cdf+bf\sqrt{e^2-4df}}\sqrt{a+x(b+cx)}} - \frac{f\left(\sqrt{e^2-4df}-e\right)\sqrt{cx^2+bx+a}\log\left(e+2fx+\sqrt{e^2-4df}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{ce^2-bfe+c\sqrt{e^2-4df}e+2af^2-2cdf-bf\sqrt{e^2-4df}}\sqrt{a+x(b+cx)}} - \frac{\sqrt{cx^2+bx+a}\log\left(2a+2\sqrt{cx^2+bx+a}\sqrt{a}+bx\right)}{\sqrt{a}d\sqrt{a+x(b+cx)}} + \frac{f\left(\sqrt{e^2-4df}-e\right)\sqrt{cx^2+bx+a}\log\left(-be^2-2cxe^2-2c\sqrt{e^2-4df}xe-b\sqrt{e^2-4df}e+4bdf+8cdfx+2bf\sqrt{e^2-4df}\cdots\right.}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{ce^2-bfe+c\sqrt{e^2-4df}e+2af^2\cdots}} + \frac{f\left(e+\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}\log\left(be^2+2cxe^2-2c\sqrt{e^2-4df}xe-b\sqrt{e^2-4df}e-4bdf-8cdfx+2bf\sqrt{e^2-4df}x\cdots\right.}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{ce^2-bfe-c\sqrt{e^2-4df}e+2af^2\cdots}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*Log[x])/(Sqrt[a]*d*Sqrt[a + x*(b + c*x)])
- (f*(e + Sqrt[e^2 - 4*d*f])*Sqrt[a + b*x + c*x^2]*Log[-e + Sqrt[
e^2 - 4*d*f] - 2*f*x])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 -
2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2
- 4*d*f]]*Sqrt[a + x*(b + c*x)]) - (f*(-e + Sqrt[e^2 - 4*d*f])*Sq
rt[a + b*x + c*x^2]*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[2]*
d*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*
Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + x*(b + c*x)])
 - (Sqrt[a + b*x + c*x^2]*Log[2*a + b*x + 2*Sqrt[a]*Sqrt[a + b*x
+ c*x^2]])/(Sqrt[a]*d*Sqrt[a + x*(b + c*x)]) + (f*(-e + Sqrt[e^2
- 4*d*f])*Sqrt[a + b*x + c*x^2]*Log[-(b*e^2) + 4*b*d*f - b*e*Sqrt
[e^2 - 4*d*f] + 4*a*f*Sqrt[e^2 - 4*d*f] - 2*c*e^2*x + 8*c*d*f*x -
 2*c*e*Sqrt[e^2 - 4*d*f]*x + 2*b*f*Sqrt[e^2 - 4*d*f]*x + 2*Sqrt[2
]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*
Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]]
)/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a
*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + x*
(b + c*x)]) + (f*(e + Sqrt[e^2 - 4*d*f])*Sqrt[a + b*x + c*x^2]*Lo
g[b*e^2 - 4*b*d*f - b*e*Sqrt[e^2 - 4*d*f] + 4*a*f*Sqrt[e^2 - 4*d*
f] + 2*c*e^2*x - 8*c*d*f*x - 2*c*e*Sqrt[e^2 - 4*d*f]*x + 2*b*f*Sq
rt[e^2 - 4*d*f]*x + 2*Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*
d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*
d*f]]*Sqrt[a + b*x + c*x^2]])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c
*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sq
rt[e^2 - 4*d*f]]*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.022, size = 859, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] 4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a^(1/2)*ln((2*
a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-2*f/(-e+(-4*d*f+e^2)^(1/2
))/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f
+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*
c)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(
1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d
*f+e^2)^(1/2))/f))-2*f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*
2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2+1/f*(-c*(-4*d
*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/
2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f
-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*
c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b
*e*f-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(fx^2 + ex + d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a + bx + cx^2}\,(d + ex + fx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

GIAC/XCAS [F(-2)]   time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.119 $\int \frac{1}{x^2\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=543

$$\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}-\frac{f\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2}-\frac{\sqrt{a+bx+cx^2}}{adx}$$

```
[Out] -(Sqrt[a + b*x + c*x^2]/(a*d*x)) + (b*ArcTanh[(2*a + b*x)/(2*Sqrt
[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)*d) + (e*ArcTanh[(2*a + b*
x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*d^2) - (f*(e^2 -
2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4
*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c
*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]
*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 -
 4*d*f]))]) + (f*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a
*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f
]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e -
 b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sq
rt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b
*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 10.059, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}-\frac{f\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2}-\frac{\sqrt{a+bx+cx^2}}{adx}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/(x^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(a*d*x)) + (b*ArcTanh[(2*a + b*x)/(2*Sqrt
[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)*d) + (e*ArcTanh[(2*a + b*
x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*d^2) - (f*(e^2 -
```

```
2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4
*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c
*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]
*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 -
 4*d*f]))]) + (f*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a
*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f
]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e -
 b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sq
rt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b
*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.87223, size = 866, normalized size = 1.59

$$-\frac{2\sqrt{a+x(b+cx)}d}{ax}-\frac{(bd+2ae)\log(x)}{a^{3/2}}+\frac{\sqrt{2}f\left(e^2+\sqrt{e^2-4df}e-2df\right)\log\left(-e-2fx+\sqrt{e^2-4df}\right)}{\sqrt{e^2-4df}\sqrt{c\left(e^2-\sqrt{e^2-4df}e-2df\right)+f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)}}+\frac{\sqrt{2}f\left(-e^2+\sqrt{e^2-4df}e+2df\right)\log\left(e+2fx+\sqrt{e^2\cdots}\right.}{\sqrt{e^2-4df}\sqrt{c\left(e^2+\sqrt{e^2-4df}e-2df\right)+f\left(2af-b\left(e+\cdots\right.\right.}}\cdots$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((-2*d*Sqrt[a + x*(b + c*x)])/(a*x) - ((b*d + 2*a*e)*Log[x])/a^(3
/2) + (Sqrt[2]*f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*Log[-e + Sqr
t[e^2 - 4*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f -
 e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))])
+ (Sqrt[2]*f*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*Log[e + Sqrt[e^
2 - 4*d*f] + 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*S
qrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]) + ((b
*d + 2*a*e)*Log[2*a + b*x + 2*Sqrt[a]*Sqrt[a + x*(b + c*x)]])/a^(
3/2) - (Sqrt[2]*f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*Log[4*a*f*S
qrt[e^2 - 4*d*f] + 2*c*e^2*x - 8*c*d*f*x - 2*c*e*Sqrt[e^2 - 4*d*f
]*x + b*(e^2 - 4*d*f - e*Sqrt[e^2 - 4*d*f] + 2*f*Sqrt[e^2 - 4*d*f
]*x) + 2*Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqr
t[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a +
 x*(b + c*x)]])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]) - (Sqrt[2
]*f*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*Log[4*a*f*Sqrt[e^2 - 4*d
*f] - 2*c*e^2*x + 8*c*d*f*x - 2*c*e*Sqrt[e^2 - 4*d*f]*x + 2*Sqrt[
2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) +
 f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)] - b
*(e^2 + e*Sqrt[e^2 - 4*d*f] - 2*f*(2*d + Sqrt[e^2 - 4*d*f]*x))])/
(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f
*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]))/(2*d^2)
```

Maple [B] time = 0.026, size = 983, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] 4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x*(c*x^2+b*x
+a)^(1/2)-2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*b/a^
(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-4*f^2/(-e+(-4
*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1
/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*
e*f-2*c*d*f+e^2*c)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-
e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x
-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c
*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2))/
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))+4*f^2/(e+(-4*d*f+e^2)^(1/2))^2
/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f
+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*
c)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f))+16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*
f+e^2)^(1/2))^2/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2)
)/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(fx^2 + ex + d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.120 $\int \frac{1}{x^3\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=679

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}-\frac{be\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{\left(e^2-df\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^3}+\frac{f\left(-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)-4def+2e^3\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{f\left(-\left(e^2-df\right)\left(\sqrt{e^2-4df}+e\right)-4def+2e^3\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{e\sqrt{a+bx+cx^2}}{ad^2x}-\frac{\sqrt{a+bx+cx^2}}{2adx^2}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(2*a*d*x^2) + (3*b*Sqrt[a + b*x + c*x^2])/
(4*a^2*d*x) + (e*Sqrt[a + b*x + c*x^2])/(a*d^2*x) - ((3*b^2 - 4*a
*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^
(5/2)*d) - (b*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x
^2])])/(2*a^(3/2)*d^2) - ((e^2 - d*f)*ArcTanh[(2*a + b*x)/(2*Sqrt
[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*d^3) + (f*(2*e^3 - 4*d*e*f
- (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e - Sq
rt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt
[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2
 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 -
 4*d*f]]) - (f*(2*e^3 - 4*d*e*f - (e^2 - d*f)*(e + Sqrt[e^2 - 4*d
*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e
+ Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
+ 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])
])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f +
2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 17.1875, antiderivative size = 679, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}-\frac{be\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{\left(e^2-df\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^3}+\frac{f\left(-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)-4def+2e^3\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{f\left(-\left(e^2-df\right)\left(\sqrt{e^2-4df}+e\right)-4def+2e^3\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{e\sqrt{a+bx+cx^2}}{ad^2x}-\frac{\sqrt{a+bx+cx^2}}{2adx^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/(x^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -Sqrt[a + b*x + c*x^2]/(2*a*d*x^2) + (3*b*Sqrt[a + b*x + c*x^2])/
(4*a^2*d*x) + (e*Sqrt[a + b*x + c*x^2])/(a*d^2*x) - ((3*b^2 - 4*a
*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^
(5/2)*d) - (b*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x
^2])])/(2*a^(3/2)*d^2) - ((e^2 - d*f)*ArcTanh[(2*a + b*x)/(2*Sqrt
[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*d^3) + (f*(2*e^3 - 4*d*e*f
- (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e - Sq
rt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt
[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2
 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 -
 4*d*f]]) - (f*(2*e^3 - 4*d*e*f - (e^2 - d*f)*(e + Sqrt[e^2 - 4*d
*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e
+ Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
+ 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])
])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f +
2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 3.36924, size = 1008, normalized size = 1.48

$$\frac{\frac{2d\sqrt{a+x(b+cx)}(-2ad+3bxd+4aex)}{a^2x^2}+\frac{\left(3b^2d^2+4abed-4a\left(cd^2+2afd-2ae^2\right)\right)\log(x)}{a^{5/2}}-\frac{4\sqrt{2}f\left(e^3+\sqrt{e^2-4df}e^2-3dfe-df\sqrt{e^2-4df}\right)\log\left(-e-2fx+\sqrt{e^2-4df}\right.}{\sqrt{e^2-4df}\sqrt{c\left(e^2-\sqrt{e^2-4df}e-2df\right)+f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*d*(-2*a*d + 3*b*d*x + 4*a*e*x)*Sqrt[a + x*(b + c*x)])/(a^2*x^
2) + ((3*b^2*d^2 + 4*a*b*d*e - 4*a*(c*d^2 - 2*a*e^2 + 2*a*d*f))*L
og[x])/a^(5/2) - (4*Sqrt[2]*f*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d
*f] - d*f*Sqrt[e^2 - 4*d*f])*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x])
/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) +
f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]) + (4*Sqrt[2]*f*(e^3 - 3*
d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f])*Log[e + Sq
rt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f
+ e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])
+ ((-3*b^2*d^2 - 4*a*b*d*e + 4*a*(c*d^2 - 2*a*e^2 + 2*a*d*f))*Log
[2*a + b*x + 2*Sqrt[a]*Sqrt[a + x*(b + c*x)]])/a^(5/2) + (4*Sqrt[
2]*f*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*
f])*Log[4*a*f*Sqrt[e^2 - 4*d*f] + 2*c*e^2*x - 8*c*d*f*x - 2*c*e*S
qrt[e^2 - 4*d*f]*x + b*(e^2 - 4*d*f - e*Sqrt[e^2 - 4*d*f] + 2*f*S
qrt[e^2 - 4*d*f]*x) + 2*Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[f*(-(b*e)
+ 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*
d*f])]*Sqrt[a + x*(b + c*x)]])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2
*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f
]))]) - (4*Sqrt[2]*f*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f
*Sqrt[e^2 - 4*d*f])*Log[4*a*f*Sqrt[e^2 - 4*d*f] - 2*c*e^2*x + 8*c
```

```
*d*f*x - 2*c*e*Sqrt[e^2 - 4*d*f]*x + 2*Sqrt[2]*Sqrt[e^2 - 4*d*f]*
Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sq
rt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)] - b*(e^2 + e*Sqrt[e^2 -
4*d*f] - 2*f*(2*d + Sqrt[e^2 - 4*d*f]*x))])/(Sqrt[e^2 - 4*d*f]*Sq
rt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt
[e^2 - 4*d*f]))]))/(8*d^3)
```

Maple [B] time = 0.029, size = 1296, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] 2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x^2*(c*x^2+b
*x+a)^(1/2)-3*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*b/
a^2/x*(c*x^2+b*x+a)^(1/2)+3/2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*
f+e^2)^(1/2))*b^2/a^(5/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/
2))/x)-2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c/a^(3/
2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-8*f^3/(-e+(-4*d*
f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b
*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)
*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f
-2*c*d*f+e^2*c)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(
-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)
/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-
1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-8*f^3/(e+(-4*d*f+e^2)^(1/2))^3/(-
4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^
2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/
f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f))+16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e
^2)^(1/2))^2/a/x*(c*x^2+b*x+a)^(1/2)-8*f^2*e/(-e+(-4*d*f+e^2)^(1/
2))^2/(e+(-4*d*f+e^2)^(1/2))^2*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c
*x^2+b*x+a)^(1/2))/x)-64*f^4/(-e+(-4*d*f+e^2)^(1/2))^3/(e+(-4*d*f
+e^2)^(1/2))^3/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))
/x)*d+64*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3/a
^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(fx^2 + ex + d)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^3),x, algorithm="giac")
```

```
[Out] Timed out
```

3.121 $\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=779

$$\frac{2\left(cx\left(\left(e^2-df\right)(abf-2ace+bcd)-de\left(-c(2af+be)+b^2f+2c^2d\right)\right)-\left(adf-ae^2+bde\right)\left(-c(2af+be)+b^2f+2c^2d\right)\right.}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$
$$+\frac{2e(b+2cx)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{2(2a+bx)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$
$$+\frac{\left(\left(e-\sqrt{e^2-4df}\right)\left(a\left(e^2-df\right)-bde+cd^2\right)+2df(bd-ae)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$
$$-\frac{\left(\left(\sqrt{e^2-4df}+e\right)\left(a\left(e^2-df\right)-bde+cd^2\right)+2df(bd-ae)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

```
[Out] (2*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) + (2*e*(b
 + 2*c*x))/((b^2 - 4*a*c)*f^2*Sqrt[a + b*x + c*x^2]) + (2*(c*d*e*
(b*c*d - 2*a*c*e + a*b*f) - (b*d*e - a*e^2 + a*d*f)*(2*c^2*d + b^
2*f - c*(b*e + 2*a*f)) + c*((b*c*d - 2*a*c*e + a*b*f)*(e^2 - d*f)
 - d*e*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*x))/((b^2 - 4*a*c)*f^
2*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x^2]
) + ((2*d*(b*d - a*e)*f + (e - Sqrt[e^2 - 4*d*f])*(c*d^2 - b*d*e
+ a*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*
(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*
d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b
*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d -
 a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e
- b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*d*(b*d - a*e)*f + (e + Sqrt[e^2
- 4*d*f])*(c*d^2 - b*d*e + a*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e
+ Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*
Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt
[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f
]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f
- b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 24.2664, antiderivative size = 779, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233

$$\frac{2\left(cx\left(\left(e^2-df\right)(abf-2ace+bcd)-de\left(-c(2af+be)+b^2f+2c^2d\right)\right)-\left(adf-ae^2+bde\right)\left(-c(2af+be)+b^2f+2c^2d\right)\right.}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$
$$+\frac{2e(b+2cx)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{2(2a+bx)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$
$$+\frac{\left(\left(e-\sqrt{e^2-4df}\right)\left(a\left(e^2-df\right)-bde+cd^2\right)+2df(bd-ae)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$
$$-\frac{\left(\left(\sqrt{e^2-4df}+e\right)\left(a\left(e^2-df\right)-bde+cd^2\right)+2df(bd-ae)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) + (2*e*(b
 + 2*c*x))/((b^2 - 4*a*c)*f^2*Sqrt[a + b*x + c*x^2]) + (2*(c*d*e*
```

```
(b*c*d - 2*a*c*e + a*b*f) - (b*d*e - a*e^2 + a*d*f)*(2*c^2*d + b^
2*f - c*(b*e + 2*a*f)) + c*((b*c*d - 2*a*c*e + a*b*f)*(e^2 - d*f)
 - d*e*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*x))/((b^2 - 4*a*c)*f^
2*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x^2]
) + ((2*d*(b*d - a*e)*f + (e - Sqrt[e^2 - 4*d*f])*(c*d^2 - b*d*e
+ a*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*
(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*
d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b
*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d -
 a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e
- b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*d*(b*d - a*e)*f + (e + Sqrt[e^2
- 4*d*f])*(c*d^2 - b*d*e + a*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e
+ Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*
Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt
[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f
]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f
- b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 5.96965, size = 1066, normalized size = 1.37

$$\frac{\frac{4(2fa^3+(-2cd-be+2cex+bfx)a^2+b(b(d-ex)-3cdx)a+b^3dx)}{(b^2-4ac)\sqrt{a+x(b+cx)}}+\frac{\sqrt{2}\left(c\left(\sqrt{e^2-4df}-e\right)d^2+b\left(e^2-\sqrt{e^2-4df}e-2df\right)d+a\left(-e^3+\sqrt{e^2-4df}e^2+3dfe-df\sqrt{e^2-4df}\right.\right.}{\sqrt{e^2-4df}\sqrt{c\left(e^2-\sqrt{e^2-4df}e-2df\right)+f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] ((4*(2*a^3*f + b^3*d*x + a^2*(-2*c*d - b*e + 2*c*e*x + b*f*x) + a
*b*(-3*c*d*x + b*(d - e*x))))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)
]) + (Sqrt[2]*(c*d^2*(-e + Sqrt[e^2 - 4*d*f]) + b*d*(e^2 - 2*d*f
- e*Sqrt[e^2 - 4*d*f]) + a*(-e^3 + 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f
] - d*f*Sqrt[e^2 - 4*d*f]))*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x])/
(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f
*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]) + (Sqrt[2]*(c*d^2*(e + Sq
rt[e^2 - 4*d*f]) - b*d*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + a*(e
^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Lo
g[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2
- 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d
*f]))]) - (Sqrt[2]*(c*d^2*(e + Sqrt[e^2 - 4*d*f]) - b*d*(e^2 - 2*
d*f + e*Sqrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*
d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[-4*a*f + 2*c*e*x + 2*c*Sqrt[e^
2 - 4*d*f]*x + b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x) - 2*Sqrt[2]*Sqrt
[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e
^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)]])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*
(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2
- 4*d*f]))]) - (Sqrt[2]*(c*d^2*(-e + Sqrt[e^2 - 4*d*f]) + b*d*(e^
2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + a*(-e^3 + 3*d*e*f + e^2*Sqrt[e
^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[b*(-e + Sqrt[e^2 - 4*d*
f] + 2*f*x) + 2*(2*a*f - c*e*x + c*Sqrt[e^2 - 4*d*f]*x + Sqrt[2]*
Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f -
```

```
e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])])/(Sqrt[e^2 - 4*d*f]
*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e +
Sqrt[e^2 - 4*d*f]))]))/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e +
 a^2*f) + a*c*(e^2 - 2*d*f)))
```

Maple [B] time = 0.034, size = 13951, normalized size = 17.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.122 $\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=609

$$-\frac{2\left(cx\left(-abe-2a(cd-af)+b^2d\right)+a(abf-2ace+bcd)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{f\left(2d(cd-af)-\left(e-\sqrt{e^2-4df}\right)(bd-ae)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{f\left(2d(cd-af)-\left(\sqrt{e^2-4df}+e\right)(bd-ae)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

```
[Out] (-2*(a*(b*c*d - 2*a*c*e + a*b*f) + c*(b^2*d - a*b*e - 2*a*(c*d -
a*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
)*Sqrt[a + b*x + c*x^2]) - (f*(2*d*(c*d - a*f) - (b*d - a*e)*(e -
 Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) +
 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2
*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a
+ b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c
*e - b*f)*Sqrt[e^2 - 4*d*f]]) + (f*(2*d*(c*d - a*f) - (b*d - a*e)
*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*
f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^
2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sq
rt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2
- (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 13.1909, antiderivative size = 609, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$-\frac{2\left(cx\left(-abe-2a(cd-af)+b^2d\right)+a(abf-2ace+bcd)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{f\left(2d(cd-af)-\left(e-\sqrt{e^2-4df}\right)(bd-ae)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{f\left(2d(cd-af)-\left(\sqrt{e^2-4df}+e\right)(bd-ae)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-2*(a*(b*c*d - 2*a*c*e + a*b*f) + c*(b^2*d - a*b*e - 2*a*(c*d -
a*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
)*Sqrt[a + b*x + c*x^2]) - (f*(2*d*(c*d - a*f) - (b*d - a*e)*(e -
 Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) +
 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2
*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a
+ b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c
*e - b*f)*Sqrt[e^2 - 4*d*f]]) + (f*(2*d*(c*d - a*f) - (b*d - a*e)
*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*
f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^
```

```
2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sq
rt[a + b*x + c*x^2]])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2
- (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 4.72377, size = 897, normalized size = 1.47

$$-\frac{4\left(\left(-2ce+bf+2cfx\right)a^2+c\left(b(d-ex)-2cdx\right)a+b^2cdx\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}}-\frac{\sqrt{2}f\left(-2cd^2+b\left(e-\sqrt{e^2-4df}\right)d+a\left(-e^2+\sqrt{e^2-4df}\,e+2df\right)\right)\log\left(-e-2fx+\sqrt{e^2-4df}\right)}{\sqrt{e^2-4df}\sqrt{c\left(e^2-\sqrt{e^2-4df}\,e-2df\right)+f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)}}+\frac{\sqrt{2}f\left(-2cd^2\cdots\right.}{\sqrt{e\cdots}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] ((-4*(b^2*c*d*x + a^2*(-2*c*e + b*f + 2*c*f*x) + a*c*(-2*c*d*x +
b*(d - e*x))))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (Sqrt[2]*f
*(-2*c*d^2 + b*d*(e - Sqrt[e^2 - 4*d*f]) + a*(-e^2 + 2*d*f + e*Sq
rt[e^2 - 4*d*f]))*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x])/(Sqrt[e^2
- 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f +
b*(-e + Sqrt[e^2 - 4*d*f]))]) + (Sqrt[2]*f*(-2*c*d^2 + b*d*(e + S
qrt[e^2 - 4*d*f]) - a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Log[e
+ Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*
d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])
)]) + (Sqrt[2]*f*(2*c*d^2 - b*d*(e + Sqrt[e^2 - 4*d*f]) + a*(e^2
- 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Log[-4*a*f + 2*c*e*x + 2*c*Sqrt[e
^2 - 4*d*f]*x + b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x) - 2*Sqrt[2]*Sqr
t[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[
e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)]])/(Sqrt[e^2 - 4*d*f]*Sqrt[c
*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2
 - 4*d*f]))]) + (Sqrt[2]*f*(-2*c*d^2 + b*d*(e - Sqrt[e^2 - 4*d*f]
) + a*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Log[b*(-e + Sqrt[e^2
- 4*d*f] + 2*f*x) + 2*(2*a*f - c*e*x + c*Sqrt[e^2 - 4*d*f]*x + Sq
rt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*
d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])])/(Sqrt[e^2 -
4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*
(-e + Sqrt[e^2 - 4*d*f])]))/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a
*b*e + a^2*f) + a*c*(e^2 - 2*d*f)))
```

Maple [B] time = 0.028, size = 10781, normalized size = 17.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.123 $\int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=609

$$\frac{2\left(a\left(-2acf+b^2f-bce+2c^2d\right)+cx(abf-2ace+bcd)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}+\frac{f\left(2d(ce-bf)-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{f\left(2d(ce-bf)-\left(\sqrt{e^2-4df}+e\right)(cd-af)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

```
[Out] (2*(a*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f) + c*(b*c*d - 2*a*c*e +
a*b*f)*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f
))*Sqrt[a + b*x + c*x^2]) + (f*(2*d*(c*e - b*f) - (c*d - a*f)*(e
- Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f])
+ 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 -
2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a
 + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b
*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (f*(2*d*(c*e - b*f) - (c*d - a*f
)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d
*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e
^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*S
qrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^
2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 12.7765, antiderivative size = 609, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{2\left(a\left(-c(2af+be)+b^2f+2c^2d\right)+cx(abf-2ace+bcd)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}+\frac{f\left(2d(ce-bf)-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{f\left(2d(ce-bf)-\left(\sqrt{e^2-4df}+e\right)(cd-af)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(a*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(b*c*d - 2*a*c*e +
a*b*f)*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f
))*Sqrt[a + b*x + c*x^2]) + (f*(2*d*(c*e - b*f) - (c*d - a*f)*(e
- Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f])
+ 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 -
2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a
 + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b
*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (f*(2*d*(c*e - b*f) - (c*d - a*f
)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d
*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e
```

```
^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*S
qrt[a + b*x + c*x^2]])]/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^
2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 3.99043, size = 868, normalized size = 1.43

$$\frac{4\left(-2cfa^2+\left(fb^2+c(fx-e)b+2c^2(d-ex)\right)a+bc^2dx\right)}{(b^2-4ac)\sqrt{a+x(b+cx)}}+\frac{\sqrt{2}f\left(2bdf+a\left(\sqrt{e^2-4df}-e\right)f-cd\left(e+\sqrt{e^2-4df}\right)\right)\log\left(-e-2fx+\sqrt{e^2-4df}\right)}{\sqrt{e^2-4df}\sqrt{c\left(e^2-\sqrt{e^2-4df}e-2df\right)+f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)}}+\frac{\sqrt{2}f\left(-2bdf+a\left(e+\sqrt{e^2-\right.\right.}{\sqrt{e^2-4df}\sqrt{c\left(e^2+\right.}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] ((4*(-2*a^2*c*f + b*c^2*d*x + a*(b^2*f + 2*c^2*(d - e*x) + b*c*(-
e + f*x))))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) + (Sqrt[2]*f*(2
*b*d*f + a*f*(-e + Sqrt[e^2 - 4*d*f]) - c*d*(e + Sqrt[e^2 - 4*d*f
]))*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e
^2 - 4*d*f]))]) + (Sqrt[2]*f*(-2*b*d*f + c*d*(e - Sqrt[e^2 - 4*d*
f]) + a*f*(e + Sqrt[e^2 - 4*d*f]))*Log[e + Sqrt[e^2 - 4*d*f] + 2*
f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f
]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]) - (Sqrt[2]*f*(-2*b*d
*f + c*d*(e - Sqrt[e^2 - 4*d*f]) + a*f*(e + Sqrt[e^2 - 4*d*f]))*L
og[-4*a*f + 2*c*e*x + 2*c*Sqrt[e^2 - 4*d*f]*x + b*(e + Sqrt[e^2 -
 4*d*f] - 2*f*x) - 2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4
*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c
*x)]])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*
f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]) - (Sqrt[2]*f*(2*b*d
*f + a*f*(-e + Sqrt[e^2 - 4*d*f]) - c*d*(e + Sqrt[e^2 - 4*d*f]))*
Log[b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x) + 2*(2*a*f - c*e*x + c*Sqr
t[e^2 - 4*d*f]*x + Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 -
4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b +
c*x)])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*
d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]))/(2*(c^2*d^2 - b
*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f)))
```

Maple [B] time = 0.024, size = 6813, normalized size = 11.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.124 $\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=666

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)} - \frac{f\left(f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{f\left(f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

```
[Out] (2*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f) - c*(2*c^2*d
- b*c*e + b^2*f - 2*a*c*f)*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b
*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x^2]) - (f*(c*(e^2 - 2*d*
f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))
*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqr
t[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a
*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(S
qrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
)*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e -
Sqrt[e^2 - 4*d*f]))]) + (f*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])
 + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e +
 Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*S
qrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[
e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]
*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])
```

Rubi [A] time = 4.16455, antiderivative size = 666, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)} - \frac{f\left(f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{f\left(f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f) - c*(2*c^2*d
- b*c*e + b^2*f - 2*a*c*f)*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b
*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x^2]) - (f*(c*(e^2 - 2*d*
f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))
*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqr
t[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a
*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(S
```

```
qrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
)*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e -
Sqrt[e^2 - 4*d*f]))]) + (f*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])
 + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e +
 Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*S
qrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[
e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]
*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 4.52761, size = 906, normalized size = 1.36

$$-\frac{4(fb^3+c(fx-e)b^2+c(c(d-ex)-3af)b+2c^2(cdx+a(e-fx)))}{(b^2-4ac)\sqrt{a+x(b+cx)}}+\frac{\sqrt{2}f\left(c\left(e^2+\sqrt{e^2-4df}e-2df\right)+f\left(2af-b\left(e+\sqrt{e^2-4df}\right)\right)\right)\log\left(-e-2fx+\sqrt{e^2-4df}\right)}{\sqrt{e^2-4df}\sqrt{c\left(e^2-\sqrt{e^2-4df}e-2df\right)+f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)}}+\sqrt{2}f\left(c\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] ((-4*(b^3*f + b^2*c*(-e + f*x) + b*c*(-3*a*f + c*(d - e*x)) + 2*c
^2*(c*d*x + a*(e - f*x))))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)])
+ (Sqrt[2]*f*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f -
b*(e + Sqrt[e^2 - 4*d*f])))*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x])/
(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f
*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]) + (Sqrt[2]*f*(c*(-e^2 + 2
*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(-2*a*f + b*(e - Sqrt[e^2 - 4*d*f
])))*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[
c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^
2 - 4*d*f]))]) + (Sqrt[2]*f*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]
) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f])))*Log[-4*a*f + 2*c*e*x
+ 2*c*Sqrt[e^2 - 4*d*f]*x + b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x) - 2
*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f -
b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)]])/(Sqrt[e^2 - 4
*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(
e + Sqrt[e^2 - 4*d*f]))]) - (Sqrt[2]*f*(c*(e^2 - 2*d*f + e*Sqrt[e
^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*Log[b*(-e +
 Sqrt[e^2 - 4*d*f] + 2*f*x) + 2*(2*a*f - c*e*x + c*Sqrt[e^2 - 4*d
*f]*x + Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c
*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])])/(S
qrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(
2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]))/(2*(c^2*d^2 - b*c*d*e + f*
(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f)))
```

Maple [B] time = 0., size = 3889, normalized size = 5.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] 2/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)*f^2/((x-1/2*(-e+(-4*d*f+e^2)^(1/2
))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)-4*f/((-4*d*f+e^2)^(1/2)*b*f
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/(4*a*c-4*c^2
/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+
e^2)^(1/2))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-
4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*x*c^2-4/(-4*d*f+e^
2)^(1/2)*f^2/((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-b*e*f-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*
f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(c*(-4*d*f
+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4
*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+
e^2*c)/f^2)^(1/2)*x*b*c+4/(-4*d*f+e^2)^(1/2)*f/((-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/(4*a*c-
4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4
*d*f+e^2)^(1/2))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(
-e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*x*c^2*e-2*f/(
(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d
*f+e^2*c)/(4*a*c-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(c*(
-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/
2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*
c*d*f+e^2*c)/f^2)^(1/2)*b*c-2/(-4*d*f+e^2)^(1/2)*f^2/((-4*d*f+e^2
)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/(
4*a*c-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(c*(-4*d*f+e^2)
^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+
e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c
)/f^2)^(1/2)*b^2+2/(-4*d*f+e^2)^(1/2)*f/((-4*d*f+e^2)^(1/2)*b*f-(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/(4*a*c-b^2)/((
x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*
e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*b*
f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*
b*c*e-2/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)*f^2*2^(1/2)/(((-4*d*f+e^2)^
(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2
)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-b*e*f-2*c*d*f+e^2*c)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*(
4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b
*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)
*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/
2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-2/(-4*d*f+e^2)^(1/2)/(-(-4
*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+
e^2*c)*f^2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+
e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*
f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2
*c)/f^2)^(1/2)-4*f/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2
*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*
(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+
1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f
-2*c*d*f+e^2*c)/f^2)^(1/2)*x*c^2+4/(-4*d*f+e^2)^(1/2)*f^2/(-(-4*d
*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^
2*c)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f
-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b
*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)
*x*b*c-4/(-4*d*f+e^2)^(1/2)*f/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f*d+c^2/f^
2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*x*c^2*e-2*f/(-(-4*d*f+e^2)^
```

```
(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/(4*
a*c-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^
2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+
e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c
)/f^2)^(1/2)*b*c+2/(-4*d*f+e^2)^(1/2)*f^2/(-(-4*d*f+e^2)^(1/2)*b*
f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/(4*a*c-b^2)
/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)
+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/
2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(
1/2)*b^2-2/(-4*d*f+e^2)^(1/2)*f/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/(4*a*c-b^2)/((x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*b*c*e
+2/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)*f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/
2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(
1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
b*e*f-2*c*d*f+e^2*c)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*
f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*
(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2
)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2
)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1
/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)),x, algorithm="giac")

[Out] Exception raised: RuntimeError
```

3.125 $\int \frac{1}{x(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=816

$$\frac{2\left(b^2+cxb-2ac\right)}{a\left(b^2-4ac\right)d\sqrt{cx^2+bx+a}}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)}{a^{3/2}d}$$
$$+\frac{f\left(\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c\left(e^2-df\right)\right)-2\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4\cdots}\right.}{2\sqrt{2}\sqrt{ce^2-bfe+2af^2-\cdots}}\right.}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{ce^2-bfe+2af^2-2cdf-(ce-bf)\sqrt{e^2-\cdots}}}$$
$$-\frac{f\left(\left(e+\sqrt{e^2-4df}\right)\left(f(be-af)-c\left(e^2-df\right)\right)-2\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e+\sqrt{e^2-\cdots}\right.}{2\sqrt{2}\sqrt{ce^2-bfe+2af^2-\cdots}}\right.}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{ce^2-bfe+2af^2-2cdf+(ce-bf)\sqrt{e^2-\cdots}}}$$
$$+\frac{2\left(ce(2ace-b(cd+af))+(be-af)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(2dec^2-b\left(e^2+df\right)c+bf(be-af)\right)x\right)}{\left(b^2-4ac\right)d\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{cx^2+bx+a}}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2
]) + (2*(c*e*(2*a*c*e - b*(c*d + a*f)) + (b*e - a*f)*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) + c*(2*c^2*d*e + b*f*(b*e - a*f) - b*c*(e
^2 + d*f))*x))/((b^2 - 4*a*c)*d*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*Sqrt[a + b*x + c*x^2]) - ArcTanh[(2*a + b*x)/(2*Sqrt[a]*
Sqrt[a + b*x + c*x^2])]/(a^(3/2)*d) + (f*((e - Sqrt[e^2 - 4*d*f])
*(f*(b*e - a*f) - c*(e^2 - d*f)) - 2*(f*(b*e^2 - b*d*f - a*e*f) -
 c*(e^3 - 2*d*e*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) +
 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2
*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a
+ b*x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 -
(c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (f*((e + Sqrt[e^2 - 4*d*f])*(f*
(b*e - a*f) - c*(e^2 - d*f)) - 2*(f*(b*e^2 - b*d*f - a*e*f) - c*(
e^3 - 2*d*e*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(
b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d
*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*
x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e
 - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 24.8498, antiderivative size = 814, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$

$$\frac{2\left(b^2+cxb-2ac\right)}{a\left(b^2-4ac\right)d\sqrt{cx^2+bx+a}}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)}{a^{3/2}d}$$
$$-\frac{f\left(2f\left(be^2-afe-bdf\right)-2c\left(e^3-2def\right)-\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4a\cdots}\right.}{2\sqrt{2}\sqrt{ce^2-bfe+2af^2-2\cdots}}\right.}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{ce^2-bfe+2af^2-2cdf-(ce-bf)\sqrt{e^2-\cdots}}}$$
$$+\frac{f\left(2f\left(be^2-afe-bdf\right)-2c\left(e^3-2def\right)-\left(e+\sqrt{e^2-4df}\right)\left(f(be-af)-c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e+\sqrt{e^2-4df\cdots}\right.}{2\sqrt{2}\sqrt{ce^2-bfe+2af^2-2c\cdots}}\right.}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{ce^2-bfe+2af^2-2cdf+(ce-bf)\sqrt{e^2-4\cdots}}}$$
$$+\frac{2\left(ce(2ace-b(cd+af))+(be-af)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(2dec^2-b\left(e^2+df\right)c+bf(be-af)\right)x\right)}{\left(b^2-4ac\right)d\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{cx^2+bx+a}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2
]) + (2*(c*e*(2*a*c*e - b*(c*d + a*f)) + (b*e - a*f)*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) + c*(2*c^2*d*e + b*f*(b*e - a*f) - b*c*(e
^2 + d*f))*x))/((b^2 - 4*a*c)*d*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*Sqrt[a + b*x + c*x^2]) - ArcTanh[(2*a + b*x)/(2*Sqrt[a]*
Sqrt[a + b*x + c*x^2])]/(a^(3/2)*d) - (f*(2*f*(b*e^2 - b*d*f - a*
e*f) - 2*c*(e^3 - 2*d*e*f) - (e - Sqrt[e^2 - 4*d*f])*(f*(b*e - a*
f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) +
 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2
*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a
+ b*x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 -
(c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + (f*(2*f*(b*e^2 - b*d*f - a*e*f)
 - 2*c*(e^3 - 2*d*e*f) - (e + Sqrt[e^2 - 4*d*f])*(f*(b*e - a*f) -
 c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(
b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d
*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*
x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e
 - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 5.87923, size = 1331, normalized size = 1.63

$$-\frac{2\left(fb^4+c(fx-e)b^3+c(c(d-ex)-4af)b^2+c^2(cdx+3a(e-fx))b+2ac^2(-cd+af+cex)\right)}{a\left(4ac-b^2\right)\left(c^2d^2-bced+f\left(fa^2-bea+b^2d\right)+ac\left(e^2-2df\right)\right)\sqrt{a+x(b+cx)}}+\frac{\log(x)}{a^{3/2}d}$$

$$-\frac{f\left(c\left(e^3+\sqrt{e^2-4df}e^2-3dfe-df\sqrt{e^2-4df}\right)+f\left(af\left(e+\sqrt{e^2-4df}\right)-b\left(e^2+\sqrt{e^2-4df}e-2df\right)\right)\right)\log\left(-e-2\right.}{\sqrt{2}d\sqrt{e^2-4df}\left(c^2d^2-bced+f\left(fa^2-bea+b^2d\right)+ac\left(e^2-2df\right)\right)\sqrt{c\left(e^2-\sqrt{e^2-4df}e-2df\right)+f\left(2af+b\left(\sqrt{e^2}\right.\right.}}$$

$$+\frac{f\left(c\left(e^3-\sqrt{e^2-4df}e^2-3dfe+df\sqrt{e^2-4df}\right)+f\left(af\left(e-\sqrt{e^2-4df}\right)+b\left(-e^2+\sqrt{e^2-4df}e+2df\right)\right)\right)\log\left(e+2f\right.}{\sqrt{2}d\sqrt{e^2-4df}\left(c^2d^2-bced+f\left(fa^2-bea+b^2d\right)+ac\left(e^2-2df\right)\right)\sqrt{c\left(e^2+\sqrt{e^2-4df}e-2df\right)+f\left(2af-b\left(e+\right.\right.}}$$

$$-\frac{\log\left(2a+2\sqrt{a+x(b+cx)}\sqrt{a}+bx\right)}{a^{3/2}d}$$

$$+\frac{f\left(c\left(-e^3+\sqrt{e^2-4df}e^2+3dfe-df\sqrt{e^2-4df}\right)+f\left(af\left(\sqrt{e^2-4df}-e\right)+b\left(e^2-\sqrt{e^2-4df}e-2df\right)\right)\right)\log\left(-4af\right.}{\sqrt{2}d\sqrt{e^2-4df}\left(c^2d^2-bced+f\left(fa^2-bea+b^2d\right)+ac\left(e\right.\right.}$$

$$+\frac{f\left(c\left(e^3+\sqrt{e^2-4df}e^2-3dfe-df\sqrt{e^2-4df}\right)+f\left(af\left(e+\sqrt{e^2-4df}\right)-b\left(e^2+\sqrt{e^2-4df}e-2df\right)\right)\right)\log\left(b\left(-e+\right.\right.}{\sqrt{2}d\sqrt{e^2-4df}\left(c^2d^2-bced+f\left(fa^2-bea+b^2d\right)+ac\left(e\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-2*(b^4*f + 2*a*c^2*(-(c*d) + a*f + c*e*x) + b^3*c*(-e + f*x) +
b^2*c*(-4*a*f + c*(d - e*x)) + b*c^2*(c*d*x + 3*a*(e - f*x))))/(a
*(-b^2 + 4*a*c)*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) +
a*c*(e^2 - 2*d*f))*Sqrt[a + x*(b + c*x)]) + Log[x]/(a^(3/2)*d) -
(f*(c*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d
*f]) + f*(a*f*(e + Sqrt[e^2 - 4*d*f]) - b*(e^2 - 2*d*f + e*Sqrt[e
^2 - 4*d*f])))*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x])/(Sqrt[2]*d*Sq
rt[e^2 - 4*d*f]*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) +
a*c*(e^2 - 2*d*f))*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f
*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]) + (f*(c*(e^3 - 3*d*e*f -
e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f]) + f*(a*f*(e - Sqrt
[e^2 - 4*d*f]) + b*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])))*Log[e +
 Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*(c^2*d^
2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))*Sqrt
[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e
^2 - 4*d*f]))]) - Log[2*a + b*x + 2*Sqrt[a]*Sqrt[a + x*(b + c*x)]
]/(a^(3/2)*d) + (f*(c*(-e^3 + 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d
*f*Sqrt[e^2 - 4*d*f]) + f*(a*f*(-e + Sqrt[e^2 - 4*d*f]) + b*(e^2
- 2*d*f - e*Sqrt[e^2 - 4*d*f])))*Log[-4*a*f + 2*c*e*x + 2*c*Sqrt[
e^2 - 4*d*f]*x + b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x) - 2*Sqrt[2]*Sq
rt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt
[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)]])/(Sqrt[2]*d*Sqrt[e^2 - 4*
d*f]*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 -
2*d*f))*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b
*(e + Sqrt[e^2 - 4*d*f]))]) + (f*(c*(e^3 - 3*d*e*f + e^2*Sqrt[e^2
 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]) + f*(a*f*(e + Sqrt[e^2 - 4*d*f
]) - b*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])))*Log[b*(-e + Sqrt[e^2
 - 4*d*f] + 2*f*x) + 2*(2*a*f - c*e*x + c*Sqrt[e^2 - 4*d*f]*x + S
qrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2
*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])])/(Sqrt[2]*d*
Sqrt[e^2 - 4*d*f]*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f)
+ a*c*(e^2 - 2*d*f))*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) +
 f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))])
```

Maple [B] time = 0.026, size = 4384, normalized size = 5.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] -4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/(c*x^2+b*x+
a)^(1/2)+8*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*b/a/(
4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*c*x+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e
+(-4*d*f+e^2)^(1/2))*b^2/a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+4*f/(-
e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a^(3/2)*ln((2*a+b*x+
2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+4*f^3/(-e+(-4*d*f+e^2)^(1/2))/(
-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-b*e*f-2*c*d*f+e^2*c)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2
*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b
*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)-8*f^2/(-e+(-4*d*f+e^2)^(1/2))/((-4
*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+
e^2*c)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/(
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c
*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*b
*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)
*x*c^2-8*f^3/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/((-4*d*f+
e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c
)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f
*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*x*b*
c+8*f^2/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^
(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/(4*
a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e
```

```
+(-4*d*f+e^2)^(1/2))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*x*c^2*e-4
*f^2/(-e+(-4*d*f+e^2)^(1/2))/((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/(4*a*c-b^2)/((x-1/2*(-e+(
-4*d*f+e^2)^(1/2))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2
*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*b*c-4*f^3/(
-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)*b*f
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/(4*a*c-b^2)/
((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-
c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*
b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2
)*b^2+4*f^2/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/((-4*d*f+e
^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)
/(4*a*c-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(c*(-4*d*f+e^
2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*
f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2
*c)/f^2)^(1/2)*b*c*e-4*f^3/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(
1/2)/((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f
-2*c*d*f+e^2*c)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*ln((((-4*d*f+e^2)^
(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2
+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/
f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^
(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d
*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f))+4*f^3/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(-(-
4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f
+e^2*c)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2
)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e
^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)
/f^2)^(1/2)+8*f^2/(e+(-4*d*f+e^2)^(1/2))/(-(-4*d*f+e^2)^(1/2)*b*f
+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/(4*a*c-4*c^2
/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(
-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*x*c^2-8*f^3/(e+(
-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*b*f+(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f*
d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*
d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*x*b*c+8*f^2/(e+(-4*
d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/(4*a*c-4*c^2/f*d+c
^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*x*c^2*e+4*f^2/(e+(-4*d
*f+e^2)^(1/2))/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-b*e*f-2*c*d*f+e^2*c)/(4*a*c-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*b*c-4*f^3/(e+(-4*d*f+e
^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/(4*a*c-b^2)/((x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*b^2+4*f
^2/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)
*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/(4*a*c-b
^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1
/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^
(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2
)^(1/2)*b*c*e-4*f^3/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(-(
-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*
f+e^2*c)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)
*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2+1/f*
```

```
(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+
1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/
2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-b*e*f-2*c*d*f+e^2*c)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx + a)^{\frac{3}{2}}(fx^2 + ex + d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)*x),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.126 $\int \frac{x^4}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

Optimal. Leaf size=140

$$-\frac{1}{4}\sqrt{-x^2-4x-3}x+\frac{5}{2}\sqrt{-x^2-4x-3}+\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)+\frac{11}{2}\sin^{-1}(x+2)$$

```
[Out] (5*Sqrt[-3 - 4*x - x^2])/2 - (x*Sqrt[-3 - 4*x - x^2])/4 + (11*Arc
Sin[2 + x])/2 + ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]
]/(2*Sqrt[2]) - ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]
]/(2*Sqrt[2]) - (5*ArcTanh[x/Sqrt[-3 - 4*x - x^2]])/4
```

Rubi [A] time = 1.03351, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467

$$-\frac{1}{4}\sqrt{-x^2-4x-3}x+\frac{5}{2}\sqrt{-x^2-4x-3}+\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)+\frac{11}{2}\sin^{-1}(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (5*Sqrt[-3 - 4*x - x^2])/2 - (x*Sqrt[-3 - 4*x - x^2])/4 + (11*Arc
Sin[2 + x])/2 + ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]
]/(2*Sqrt[2]) - ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]
]/(2*Sqrt[2]) - (5*ArcTanh[x/Sqrt[-3 - 4*x - x^2]])/4
```

Rubi in Sympy [A] time = 121.949, size = 153, normalized size = 1.09

$$-\frac{x\sqrt{-x^2-4x-3}}{4}+\frac{5\sqrt{-x^2-4x-3}}{2}-\frac{\sqrt{2}\,\text{atan}\left(\sqrt{2}\left(\frac{3\left(\frac{x}{3}+1\right)}{2\sqrt{-x^2-4x-3}}-\frac{1}{2}\right)\right)}{4}-\frac{\sqrt{2}\,\text{atan}\left(\sqrt{2}\left(\frac{3\left(\frac{x}{3}+1\right)}{2\sqrt{-x^2-4x-3}}+\frac{1}{2}\right)\right)}{4}+\frac{11\,\text{atan}\left(-\frac{-2x-4}{2\sqrt{-x^2-4x-3}}\right)}{2}-\frac{5\,\text{atanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] -x*sqrt(-x**2 - 4*x - 3)/4 + 5*sqrt(-x**2 - 4*x - 3)/2 - sqrt(2)*
atan(sqrt(2)*(3*(x/3 + 1)/(2*sqrt(-x**2 - 4*x - 3)) - 1/2))/4 - s
qrt(2)*atan(sqrt(2)*(3*(x/3 + 1)/(2*sqrt(-x**2 - 4*x - 3)) + 1/2)
)/4 + 11*atan(-(-2*x - 4)/(2*sqrt(-x**2 - 4*x - 3)))/2 - 5*atanh(
x/sqrt(-x**2 - 4*x - 3))/4
```

Mathematica [C] time = 6.29382, size = 1101, normalized size = 7.86

$$\sqrt{-x^2-4x-3}\left(\frac{5}{2}-\frac{x}{4}\right)+\frac{11}{2}\sin^{-1}(x+2)$$
$$+\frac{i\left(-7i+4\sqrt{2}\right)\tan^{-1}\left(\frac{78i\sqrt{2}x^4+224x^4+162i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x^3+20i\sqrt{2}x^3+1276x^3+648i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x^2-727i\sqrt{2}x^2+2236x^2+891i\sqrt{1-2i\sqrt{2}}}{128\sqrt{2}x^4+66ix^4+544\sqrt{2}x^3+208ix^3+514\sqrt{2}x^2+685ix^2+104\sqrt{2}x+1396i}\right.}{8\sqrt{1-2i\sqrt{2}}}$$
$$-\frac{\left(7i+4\sqrt{2}\right)\tanh^{-1}\left(\frac{78\sqrt{2}x^4+224ix^4+162\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}x^3+20\sqrt{2}x^3+1276ix^3+648\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}x^2-727\sqrt{2}x^2+2236ix^2+891\sqrt{1+2i\sqrt{2}}\sqrt{-x}}{128\sqrt{2}x^4-66ix^4+544\sqrt{2}x^3-208ix^3+514\sqrt{2}x^2-685ix^2+104\sqrt{2}x-1396ix+}\right.}{8\sqrt{1+2i\sqrt{2}}}$$
$$-\frac{\left(7i+4\sqrt{2}\right)\log\left(\left(-2ix+\sqrt{2}-2i\right)^2\left(2ix+\sqrt{2}+2i\right)^2\right)}{16\sqrt{1+2i\sqrt{2}}}$$
$$-\frac{\left(-7i+4\sqrt{2}\right)\log\left(\left(-2ix+\sqrt{2}-2i\right)^2\left(2ix+\sqrt{2}+2i\right)^2\right)}{16\sqrt{1-2i\sqrt{2}}}$$
$$+\frac{\left(-7i+4\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(2i\sqrt{2}x^2+2x^2-2\sqrt{2\left(1-2i\sqrt{2}\right)}\sqrt{-x^2-4x-3}x+8i\sqrt{2}x+4x-2\sqrt{2\left(1-2i\sqrt{2}\right)}\sqrt{-x}\right.\right.}{16\sqrt{1-2i\sqrt{2}}}$$
$$+\frac{\left(7i+4\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(-2i\sqrt{2}x^2+2x^2-2\sqrt{2\left(1+2i\sqrt{2}\right)}\sqrt{-x^2-4x-3}x-8i\sqrt{2}x+4x-2\sqrt{2\left(1+2i\sqrt{2}\right)}\sqrt{-x}\right.\right.}{16\sqrt{1+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (5/2 - x/4)*Sqrt[-3 - 4*x - x^2] + (11*ArcSin[2 + x])/2 + ((I/8)*
(-7*I + 4*Sqrt[2])*ArcTan[(132 - (471*I)*Sqrt[2] + 1316*x - (1168
*I)*Sqrt[2]*x + 2236*x^2 - (727*I)*Sqrt[2]*x^2 + 1276*x^3 + (20*I
)*Sqrt[2]*x^3 + 224*x^4 + (78*I)*Sqrt[2]*x^4 + (486*I)*Sqrt[1 - (
2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] + (891*I)*Sqrt[1 - (2*I)*Sqrt[
2]]*x*Sqrt[-3 - 4*x - x^2] + (648*I)*Sqrt[1 - (2*I)*Sqrt[2]]*x^2*
Sqrt[-3 - 4*x - x^2] + (162*I)*Sqrt[1 - (2*I)*Sqrt[2]]*x^3*Sqrt[-
3 - 4*x - x^2])/(885*I + 6*Sqrt[2] + (1396*I)*x + 104*Sqrt[2]*x +
 (685*I)*x^2 + 514*Sqrt[2]*x^2 + (208*I)*x^3 + 544*Sqrt[2]*x^3 +
(66*I)*x^4 + 128*Sqrt[2]*x^4)])/Sqrt[1 - (2*I)*Sqrt[2]] - ((7*I +
 4*Sqrt[2])*ArcTanh[(132*I - 471*Sqrt[2] + (1316*I)*x - 1168*Sqrt
[2]*x + (2236*I)*x^2 - 727*Sqrt[2]*x^2 + (1276*I)*x^3 + 20*Sqrt[2
]*x^3 + (224*I)*x^4 + 78*Sqrt[2]*x^4 + 486*Sqrt[1 + (2*I)*Sqrt[2]
]*Sqrt[-3 - 4*x - x^2] + 891*Sqrt[1 + (2*I)*Sqrt[2]]*x*Sqrt[-3 -
4*x - x^2] + 648*Sqrt[1 + (2*I)*Sqrt[2]]*x^2*Sqrt[-3 - 4*x - x^2]
 + 162*Sqrt[1 + (2*I)*Sqrt[2]]*x^3*Sqrt[-3 - 4*x - x^2])/(-885*I
+ 6*Sqrt[2] - (1396*I)*x + 104*Sqrt[2]*x - (685*I)*x^2 + 514*Sqrt
[2]*x^2 - (208*I)*x^3 + 544*Sqrt[2]*x^3 - (66*I)*x^4 + 128*Sqrt[2
]*x^4)])/(8*Sqrt[1 + (2*I)*Sqrt[2]]) - ((-7*I + 4*Sqrt[2])*Log[(-
2*I + Sqrt[2] - (2*I)*x)^2*(2*I + Sqrt[2] + (2*I)*x)^2])/(16*Sqrt
[1 - (2*I)*Sqrt[2]]) - ((7*I + 4*Sqrt[2])*Log[(-2*I + Sqrt[2] - (
2*I)*x)^2*(2*I + Sqrt[2] + (2*I)*x)^2])/(16*Sqrt[1 + (2*I)*Sqrt[2
]]) + ((-7*I + 4*Sqrt[2])*Log[(3 + 4*x + 2*x^2)*(3 + (6*I)*Sqrt[2
] + 4*x + (8*I)*Sqrt[2]*x + 2*x^2 + (2*I)*Sqrt[2]*x^2 - 2*Sqrt[2*
(1 - (2*I)*Sqrt[2])]*Sqrt[-3 - 4*x - x^2] - 2*Sqrt[2*(1 - (2*I)*S
qrt[2])]*x*Sqrt[-3 - 4*x - x^2])])/(16*Sqrt[1 - (2*I)*Sqrt[2]]) +
 ((7*I + 4*Sqrt[2])*Log[(3 + 4*x + 2*x^2)*(3 - (6*I)*Sqrt[2] + 4*
x - (8*I)*Sqrt[2]*x + 2*x^2 - (2*I)*Sqrt[2]*x^2 - 2*Sqrt[2*(1 + (
2*I)*Sqrt[2])]*Sqrt[-3 - 4*x - x^2] - 2*Sqrt[2*(1 + (2*I)*Sqrt[2]
)]*x*Sqrt[-3 - 4*x - x^2])])/(16*Sqrt[1 + (2*I)*Sqrt[2]])
```

Maple [A] time = 0.024, size = 159, normalized size = 1.1

$$\frac{11\ \arcsin(2+x)}{2}+\frac{5}{2}\sqrt{-x^2-4x-3}-\frac{x}{4}\sqrt{-x^2-4x-3}$$
$$+\frac{\sqrt{3}\sqrt{4}}{24}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\right)+5\,Artanh\left(3\frac{x}{-3/2-x}\frac{1}{\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}}\right)\right)\frac{}{\sqrt{1\left(x^2\left(\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] 11/2*arcsin(2+x)+5/2*(-x^2-4*x-3)^(1/2)-1/4*x*(-x^2-4*x-3)^(1/2)+
1/24*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(
1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+5*arctanh(3*x/(-3/2-x)/(
3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2
)^(1/2)/(1+x/(-3/2-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Fricas [A] time = 0.307625, size = 242, normalized size = 1.73

$$-\frac{1}{32}\sqrt{2}\left(4\sqrt{2}\sqrt{-x^2-4\,x-3}(x-10)-88\sqrt{2}\arctan\left(\frac{x+2}{\sqrt{-x^2-4\,x-3}}\right)-5\sqrt{2}\log\left(-\frac{2\sqrt{-x^2-4\,x-3}x+4\,x+3}{x^2}\right)+5\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="fricas")
```

```
[Out] -1/32*sqrt(2)*(4*sqrt(2)*sqrt(-x^2 - 4*x - 3)*(x - 10) - 88*sqrt(
2)*arctan((x + 2)/sqrt(-x^2 - 4*x - 3)) - 5*sqrt(2)*log(-(2*sqrt(
-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 5*sqrt(2)*log((2*sqrt(-x^2 -
4*x - 3)*x - 4*x - 3)/x^2) - 4*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*
sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 4*arctan(-1/2*(sqrt(2)*x - 3*s
qrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{-(x+1)(x+3)}\,(2x^2+4x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

GIAC/XCAS [A] time = 0.290451, size = 254, normalized size = 1.81

$$-\frac{1}{4}\sqrt{-x^2-4x-3}(x-10)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)$$
$$+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)+\frac{11}{2}\arcsin(x+2)$$
$$-\frac{5}{8}\ln\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)$$
$$+\frac{5}{8}\ln\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(-x^2 - 4*x - 3)*(x - 10) + 1/4*sqrt(2)*arctan(1/2*sqrt(
2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/4*sqrt(2)*arct
an(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 11/2*a
rcsin(x + 2) - 5/8*ln(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(s
qrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 5/8*ln(2*(sqrt(-x^2 -
 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 +
 3)
```

3.127 $\int \frac{x^3}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

Optimal. Leaf size=115

$$-\frac{1}{2}\sqrt{-x^2-4x-3}+\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}}+\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-2\sin^{-1}(x+2)$$

```
[Out] -Sqrt[-3 - 4*x - x^2]/2 - 2*ArcSin[2 + x] + ArcTan[(1 - (3 + x)/S
qrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + (3 + x)/S
qrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) + ArcTanh[x/Sqrt[-3 - 4
*x - x^2]]
```

Rubi [A] time = 0.886921, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433

$$-\frac{1}{2}\sqrt{-x^2-4x-3}+\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}}+\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-2\sin^{-1}(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] -Sqrt[-3 - 4*x - x^2]/2 - 2*ArcSin[2 + x] + ArcTan[(1 - (3 + x)/S
qrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + (3 + x)/S
qrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) + ArcTanh[x/Sqrt[-3 - 4
*x - x^2]]
```

Rubi in Sympy [A] time = 115.126, size = 129, normalized size = 1.12

$$-\frac{\sqrt{-x^2-4x-3}}{2}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3\left(\frac{x}{3}+1\right)}{2\sqrt{-x^2-4x-3}}-\frac{1}{2}\right)\right)}{4}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3\left(\frac{x}{3}+1\right)}{2\sqrt{-x^2-4x-3}}+\frac{1}{2}\right)\right)}{4}$$
$$-2\,\mathrm{atan}\left(-\frac{-2x-4}{2\sqrt{-x^2-4x-3}}\right)+\mathrm{atanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] -sqrt(-x**2 - 4*x - 3)/2 - sqrt(2)*atan(sqrt(2)*(3*(x/3 + 1)/(2*s
qrt(-x**2 - 4*x - 3)) - 1/2))/4 - sqrt(2)*atan(sqrt(2)*(3*(x/3 +
1)/(2*sqrt(-x**2 - 4*x - 3)) + 1/2))/4 - 2*atan(-(-2*x - 4)/(2*sq
rt(-x**2 - 4*x - 3))) + atanh(x/sqrt(-x**2 - 4*x - 3))
```

Mathematica [C] time = 6.28423, size = 1093, normalized size = 9.5

$$-2\sin^{-1}(x+2) - \frac{i\left(-2i+5\sqrt{2}\right)\tan^{-1}\left(\frac{66i\sqrt{2}x^4+40x^4+54i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x^3+316i\sqrt{2}x^3+332x^3+216i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x^2+469i\sqrt{2}x^2+920x^2+297i\sqrt{1-2i\sqrt{2}}\sqrt{-}}{100\sqrt{2}x^4+114ix^4+560\sqrt{2}x^3+392ix^3+1004\sqrt{2}x^2+455ix^2+736\sqrt{2}x+284ix+1}\right)}{8\sqrt{1-2i\sqrt{2}}} + \frac{\left(2i+5\sqrt{2}\right)\tanh^{-1}\left(\frac{66\sqrt{2}x^4+40ix^4+54\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}x^3+316\sqrt{2}x^3+332ix^3+216\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}x^2+469\sqrt{2}x^2+920ix^2+297\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x}}{100\sqrt{2}x^4-114ix^4+560\sqrt{2}x^3-392ix^3+1004\sqrt{2}x^2-455ix^2+736\sqrt{2}x-284ix+192\sqrt{2}}\right)}{8\sqrt{1+2i\sqrt{2}}} + \frac{\left(2i+5\sqrt{2}\right)\log\left(\left(-2ix+\sqrt{2}-2i\right)^2\left(2ix+\sqrt{2}+2i\right)^2\right)}{16\sqrt{1+2i\sqrt{2}}} + \frac{\left(-2i+5\sqrt{2}\right)\log\left(\left(-2ix+\sqrt{2}-2i\right)^2\left(2ix+\sqrt{2}+2i\right)^2\right)}{16\sqrt{1-2i\sqrt{2}}} - \frac{\left(-2i+5\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(2i\sqrt{2}x^2+2x^2-2\sqrt{2\left(1-2i\sqrt{2}\right)}\sqrt{-x^2-4x-3}x+8i\sqrt{2}x+4x-2\sqrt{2\left(1-2i\sqrt{2}\right)}\sqrt{-}\right.\right.}{16\sqrt{1-2i\sqrt{2}}} - \frac{\left(2i+5\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(-2i\sqrt{2}x^2+2x^2-2\sqrt{2\left(1+2i\sqrt{2}\right)}\sqrt{-x^2-4x-3}x-8i\sqrt{2}x+4x-2\sqrt{2\left(1+2i\sqrt{2}\right)}\sqrt{-}\right.\right.}{16\sqrt{1+2i\sqrt{2}}} - \frac{1}{2}\sqrt{-x^2-4x-3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] -Sqrt[-3 - 4*x - x^2]/2 - 2*ArcSin[2 + x] - ((I/8)*(-2*I + 5*Sqrt
[2])*ArcTan[(336 - (6*I)*Sqrt[2] + 964*x + (208*I)*Sqrt[2]*x + 92
0*x^2 + (469*I)*Sqrt[2]*x^2 + 332*x^3 + (316*I)*Sqrt[2]*x^3 + 40*
x^4 + (66*I)*Sqrt[2]*x^4 + (162*I)*Sqrt[1 - (2*I)*Sqrt[2]]*Sqrt[-
3 - 4*x - x^2] + (297*I)*Sqrt[1 - (2*I)*Sqrt[2]]*x*Sqrt[-3 - 4*x
- x^2] + (216*I)*Sqrt[1 - (2*I)*Sqrt[2]]*x^2*Sqrt[-3 - 4*x - x^2]
 + (54*I)*Sqrt[1 - (2*I)*Sqrt[2]]*x^3*Sqrt[-3 - 4*x - x^2])/(132*
I + 192*Sqrt[2] + (284*I)*x + 736*Sqrt[2]*x + (455*I)*x^2 + 1004*
Sqrt[2]*x^2 + (392*I)*x^3 + 560*Sqrt[2]*x^3 + (114*I)*x^4 + 100*S
qrt[2]*x^4)])/Sqrt[1 - (2*I)*Sqrt[2]] + ((2*I + 5*Sqrt[2])*ArcTan
h[(336*I - 6*Sqrt[2] + (964*I)*x + 208*Sqrt[2]*x + (920*I)*x^2 +
469*Sqrt[2]*x^2 + (332*I)*x^3 + 316*Sqrt[2]*x^3 + (40*I)*x^4 + 66
*Sqrt[2]*x^4 + 162*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] +
 297*Sqrt[1 + (2*I)*Sqrt[2]]*x*Sqrt[-3 - 4*x - x^2] + 216*Sqrt[1
+ (2*I)*Sqrt[2]]*x^2*Sqrt[-3 - 4*x - x^2] + 54*Sqrt[1 + (2*I)*Sqr
t[2]]*x^3*Sqrt[-3 - 4*x - x^2])/(-132*I + 192*Sqrt[2] - (284*I)*x
 + 736*Sqrt[2]*x - (455*I)*x^2 + 1004*Sqrt[2]*x^2 - (392*I)*x^3 +
 560*Sqrt[2]*x^3 - (114*I)*x^4 + 100*Sqrt[2]*x^4)])/(8*Sqrt[1 + (
2*I)*Sqrt[2]]) + ((-2*I + 5*Sqrt[2])*Log[(-2*I + Sqrt[2] - (2*I)*
x)^2*(2*I + Sqrt[2] + (2*I)*x)^2])/(16*Sqrt[1 - (2*I)*Sqrt[2]]) +
 ((2*I + 5*Sqrt[2])*Log[(-2*I + Sqrt[2] - (2*I)*x)^2*(2*I + Sqrt[
2] + (2*I)*x)^2])/(16*Sqrt[1 + (2*I)*Sqrt[2]]) - ((-2*I + 5*Sqrt[
2])*Log[(3 + 4*x + 2*x^2)*(3 + (6*I)*Sqrt[2] + 4*x + (8*I)*Sqrt[2
]*x + 2*x^2 + (2*I)*Sqrt[2]*x^2 - 2*Sqrt[2*(1 - (2*I)*Sqrt[2])]*S
qrt[-3 - 4*x - x^2] - 2*Sqrt[2*(1 - (2*I)*Sqrt[2])]*x*Sqrt[-3 - 4
*x - x^2])])/(16*Sqrt[1 - (2*I)*Sqrt[2]]) - ((2*I + 5*Sqrt[2])*Lo
g[(3 + 4*x + 2*x^2)*(3 - (6*I)*Sqrt[2] + 4*x - (8*I)*Sqrt[2]*x +
2*x^2 - (2*I)*Sqrt[2]*x^2 - 2*Sqrt[2*(1 + (2*I)*Sqrt[2])]*Sqrt[-3
 - 4*x - x^2] - 2*Sqrt[2*(1 + (2*I)*Sqrt[2])]*x*Sqrt[-3 - 4*x - x
^2])])/(16*Sqrt[1 + (2*I)*Sqrt[2]])
```

Maple [A] time = 0.019, size = 144, normalized size = 1.3

$$-2\arcsin(2+x)-\frac{1}{2}\sqrt{-x^2-4x-3}$$
$$+\frac{\sqrt{3}\sqrt{4}}{24}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\right)-4\,Artanh\left(3\frac{x}{-3/2-x}\frac{1}{\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}}\right)\right)\frac{}{\sqrt{1\left(x^2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] -2*arcsin(2+x)-1/2*(-x^2-4*x-3)^(1/2)+1/24*3^(1/2)*4^(1/2)*(3*x^2
/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(
1/2)*2^(1/2))-4*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2))
)/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{(2x^2+4x+3)\sqrt{-x^2-4x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Fricas [A] time = 0.299971, size = 236, normalized size = 2.05

$$-\frac{1}{8}\sqrt{2}\left(8\sqrt{2}\arctan\left(\frac{x+2}{\sqrt{-x^2-4x-3}}\right)+\sqrt{2}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right)-\sqrt{2}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(2)*(8*sqrt(2)*arctan((x + 2)/sqrt(-x^2 - 4*x - 3)) + sq
rt(2)*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - sqrt(2)*lo
g((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2) + 2*sqrt(2)*sqrt(-x^2
 - 4*x - 3) - arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x -
 3))/(2*x + 3)) - arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 -
4*x - 3))/(2*x + 3)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\sqrt{-(x+1)(x+3)}\,(2x^2+4x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

GIAC/XCAS [A] time = 0.275037, size = 250, normalized size = 2.17

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)$$
$$-\frac{1}{2}\sqrt{-x^2-4x-3}-2\arcsin(x+2)+\frac{1}{2}\ln\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)$$
$$-\frac{1}{2}\ln\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x +
 2) + 1)) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3)
 - 1)/(x + 2) + 1)) - 1/2*sqrt(-x^2 - 4*x - 3) - 2*arcsin(x + 2)
+ 1/2*ln(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*
x - 3) - 1)^2/(x + 2)^2 + 1) - 1/2*ln(2*(sqrt(-x^2 - 4*x - 3) - 1
)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

3.128 $\int \frac{x^2}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

Optimal. Leaf size=98

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)+\frac{1}{2}\sin^{-1}(x+2)$$

```
[Out] ArcSin[2 + x]/2 - ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[
2]]/Sqrt[2] + ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/
Sqrt[2] - ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/2
```

Rubi [A] time = 0.483978, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)+\frac{1}{2}\sin^{-1}(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ArcSin[2 + x]/2 - ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[
2]]/Sqrt[2] + ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/
Sqrt[2] - ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/2
```

Rubi in Sympy [A] time = 83.2454, size = 116, normalized size = 1.18

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3\left(\frac{x}{3}+1\right)}{2\sqrt{-x^2-4x-3}}-\frac{1}{2}\right)\right)}{2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3\left(\frac{x}{3}+1\right)}{2\sqrt{-x^2-4x-3}}+\frac{1}{2}\right)\right)}{2}+\frac{\mathrm{atan}\left(-\frac{-2x-4}{2\sqrt{-x^2-4x-3}}\right)}{2}-\frac{\mathrm{atanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*(3*(x/3 + 1)/(2*sqrt(-x**2 - 4*x - 3)) - 1/2
))/2 + sqrt(2)*atan(sqrt(2)*(3*(x/3 + 1)/(2*sqrt(-x**2 - 4*x - 3)
) + 1/2))/2 + atan(-(-2*x - 4)/(2*sqrt(-x**2 - 4*x - 3)))/2 - ata
nh(x/sqrt(-x**2 - 4*x - 3))/2
```

Mathematica [C] time = 6.2692, size = 1087, normalized size = 11.09

$$\frac{1}{2}\sin^{-1}(x+2)$$
$$+\frac{i\left(i+2\sqrt{2}\right)\tan^{-1}\left(\frac{6i\sqrt{2}x^4-16x^4+18i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x^3+68i\sqrt{2}x^3-68x^3+72i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x^2+185i\sqrt{2}x^2-44x^2+99i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x}{32\sqrt{2}x^4+66ix^4+208\sqrt{2}x^3+304ix^3+466\sqrt{2}x^2+493ix^2+440\sqrt{2}x+340ix+150\sqrt{2}+93i}\right.}{4\sqrt{1-2i\sqrt{2}}}$$
$$+\frac{i\left(-i+2\sqrt{2}\right)\tan^{-1}\left(\frac{6i\sqrt{2}x^4+16x^4+18i\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}x^3+68i\sqrt{2}x^3+68x^3+72i\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}x^2+185i\sqrt{2}x^2+44x^2+99i\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}}{32\sqrt{2}x^4-66ix^4+208\sqrt{2}x^3-304ix^3+466\sqrt{2}x^2-493ix^2+440\sqrt{2}x-340ix+150\sqrt{2}-9}\right.}{4\sqrt{1+2i\sqrt{2}}}$$
$$-\frac{\left(i+2\sqrt{2}\right)\log\left(\left(-2ix+\sqrt{2}-2i\right)^2\left(2ix+\sqrt{2}+2i\right)^2\right)}{8\sqrt{1-2i\sqrt{2}}}$$
$$-\frac{\left(-i+2\sqrt{2}\right)\log\left(\left(-2ix+\sqrt{2}-2i\right)^2\left(2ix+\sqrt{2}+2i\right)^2\right)}{8\sqrt{1+2i\sqrt{2}}}$$
$$+\frac{\left(i+2\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(2i\sqrt{2}x^2+2x^2-2\sqrt{2\left(1-2i\sqrt{2}\right)}\sqrt{-x^2-4x-3}x+8i\sqrt{2}x+4x-2\sqrt{2\left(1-2i\sqrt{2}\right)}\sqrt{-x^2-}\right.\right.}{8\sqrt{1-2i\sqrt{2}}}$$
$$+\frac{\left(-i+2\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(-2i\sqrt{2}x^2+2x^2-2\sqrt{2\left(1+2i\sqrt{2}\right)}\sqrt{-x^2-4x-3}x-8i\sqrt{2}x+4x-2\sqrt{2\left(1+2i\sqrt{2}\right)}\sqrt{-x}\right.\right.}{8\sqrt{1+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ArcSin[2 + x]/2 + ((I/4)*(I + 2*Sqrt[2])*ArcTan[(60 + (51*I)*Sqrt
[2] + 68*x + (176*I)*Sqrt[2]*x - 44*x^2 + (185*I)*Sqrt[2]*x^2 - 6
8*x^3 + (68*I)*Sqrt[2]*x^3 - 16*x^4 + (6*I)*Sqrt[2]*x^4 + (54*I)*
Sqrt[1 - (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] + (99*I)*Sqrt[1 - (2
*I)*Sqrt[2]]*x*Sqrt[-3 - 4*x - x^2] + (72*I)*Sqrt[1 - (2*I)*Sqrt[
2]]*x^2*Sqrt[-3 - 4*x - x^2] + (18*I)*Sqrt[1 - (2*I)*Sqrt[2]]*x^3
*Sqrt[-3 - 4*x - x^2]])/(93*I + 150*Sqrt[2] + (340*I)*x + 440*Sqrt
[2]*x + (493*I)*x^2 + 466*Sqrt[2]*x^2 + (304*I)*x^3 + 208*Sqrt[2]
*x^3 + (66*I)*x^4 + 32*Sqrt[2]*x^4)])/Sqrt[1 - (2*I)*Sqrt[2]] + (
(I/4)*(-I + 2*Sqrt[2])*ArcTan[(-60 + (51*I)*Sqrt[2] - 68*x + (176
*I)*Sqrt[2]*x + 44*x^2 + (185*I)*Sqrt[2]*x^2 + 68*x^3 + (68*I)*Sq
rt[2]*x^3 + 16*x^4 + (6*I)*Sqrt[2]*x^4 + (54*I)*Sqrt[1 + (2*I)*Sq
rt[2]]*Sqrt[-3 - 4*x - x^2] + (99*I)*Sqrt[1 + (2*I)*Sqrt[2]]*x*Sq
rt[-3 - 4*x - x^2] + (72*I)*Sqrt[1 + (2*I)*Sqrt[2]]*x^2*Sqrt[-3 -
 4*x - x^2] + (18*I)*Sqrt[1 + (2*I)*Sqrt[2]]*x^3*Sqrt[-3 - 4*x -
x^2])/(-93*I + 150*Sqrt[2] - (340*I)*x + 440*Sqrt[2]*x - (493*I)*
x^2 + 466*Sqrt[2]*x^2 - (304*I)*x^3 + 208*Sqrt[2]*x^3 - (66*I)*x^
4 + 32*Sqrt[2]*x^4)])/Sqrt[1 + (2*I)*Sqrt[2]] - ((-I + 2*Sqrt[2])
*Log[(-2*I + Sqrt[2] - (2*I)*x)^2*(2*I + Sqrt[2] + (2*I)*x)^2])/(
8*Sqrt[1 + (2*I)*Sqrt[2]]) - ((I + 2*Sqrt[2])*Log[(-2*I + Sqrt[2]
 - (2*I)*x)^2*(2*I + Sqrt[2] + (2*I)*x)^2])/(8*Sqrt[1 - (2*I)*Sqr
t[2]]) + ((I + 2*Sqrt[2])*Log[(3 + 4*x + 2*x^2)*(3 + (6*I)*Sqrt[2
] + 4*x + (8*I)*Sqrt[2]*x + 2*x^2 + (2*I)*Sqrt[2]*x^2 - 2*Sqrt[2*
(1 - (2*I)*Sqrt[2])]*Sqrt[-3 - 4*x - x^2] - 2*Sqrt[2*(1 - (2*I)*S
qrt[2])]*x*Sqrt[-3 - 4*x - x^2])])/(8*Sqrt[1 - (2*I)*Sqrt[2]]) +
((-I + 2*Sqrt[2])*Log[(3 + 4*x + 2*x^2)*(3 - (6*I)*Sqrt[2] + 4*x
- (8*I)*Sqrt[2]*x + 2*x^2 - (2*I)*Sqrt[2]*x^2 - 2*Sqrt[2*(1 + (2*
I)*Sqrt[2])]*Sqrt[-3 - 4*x - x^2] - 2*Sqrt[2*(1 + (2*I)*Sqrt[2])]
*x*Sqrt[-3 - 4*x - x^2])])/(8*Sqrt[1 + (2*I)*Sqrt[2]])
```

Maple [A] time = 0.009, size = 130, normalized size = 1.3

$$\frac{\arcsin(2+x)}{2} - \frac{\sqrt{3}\sqrt{4}}{12}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\right)-Artanh\left(3\frac{x}{-3/2-x}\frac{1}{\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}}\right)\right)\frac{1}{\sqrt{1\left(x^2\left(-\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] 1/2*arcsin(2+x)-1/12*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*
(2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(
3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1
+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Fricas [A] time = 0.304905, size = 213, normalized size = 2.17

$$\frac{1}{16}\sqrt{2}\left(4\sqrt{2}\arctan\left(\frac{x+2}{\sqrt{-x^2-4\,x-3}}\right)+\sqrt{2}\log\left(-\frac{2\sqrt{-x^2-4\,x-3}x+4\,x+3}{x^2}\right)-\sqrt{2}\log\left(\frac{2\sqrt{-x^2-4\,x-3}x-4\,x-3}{x^2}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(2)*(4*sqrt(2)*arctan((x + 2)/sqrt(-x^2 - 4*x - 3)) + sq
rt(2)*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - sqrt(2)*lo
g((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2) - 4*arctan(1/2*(sqrt(
2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 4*arctan(-1/2
*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(x+1)(x+3)}\,(2x^2+4x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

GIAC/XCAS [A] time = 0.275116, size = 231, normalized size = 2.36

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)+\frac{1}{2}\arcsin(x+2)-\frac{1}{4}\ln\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)+\frac{1}{4}\ln\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x
+ 2) + 1)) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3
) - 1)/(x + 2) + 1)) + 1/2*arcsin(x + 2) - 1/4*ln(2*(sqrt(-x^2 -
4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2
+ 1) + 1/4*ln(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 -
 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

3.129 $\int \frac{x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} \, dx$

Optimal. Leaf size=68

$$\frac{\tan^{-1}\left(\frac{\frac{3\sqrt{-x-1}}{\sqrt{x+3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[(1 - (3*Sqrt[-1 - x])/Sqrt[3 + x])/Sqrt[2]]/Sqrt[2]) + A
rcTan[(1 + (3*Sqrt[-1 - x])/Sqrt[3 + x])/Sqrt[2]]/Sqrt[2]
```

Rubi [A] time = 0.151485, antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcT
an[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2]
```

Rubi in Sympy [A] time = 25.0264, size = 71, normalized size = 1.04

$$-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3\left(\frac{x}{3}+1\right)}{2\sqrt{-x^2-4x-3}}-\frac{1}{2}\right)\right)}{2}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3\left(\frac{x}{3}+1\right)}{2\sqrt{-x^2-4x-3}}+\frac{1}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] -sqrt(2)*atan(sqrt(2)*(3*(x/3 + 1)/(2*sqrt(-x**2 - 4*x - 3)) - 1/
2))/2 - sqrt(2)*atan(sqrt(2)*(3*(x/3 + 1)/(2*sqrt(-x**2 - 4*x - 3
)) + 1/2))/2
```

Mathematica [C] time = 6.36198, size = 891, normalized size = 13.1

$$\frac{i\left(-2i+\sqrt{2}\right)\tan^{-1}\left(\frac{(x+2)\left(4\sqrt{2}x^2-18ix^2+16\sqrt{2}x-32ix+12\sqrt{2}-15i\right)}{-6i\sqrt{2}x^3+8x^3+6i\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}x^2-8i\sqrt{2}x^2+36x^2+12i\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}x+5i\sqrt{2}x+40x+9i\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}+6i\sqrt{2}+12}\right.}{4\sqrt{1+2i\sqrt{2}}}$$
$$+\frac{\left(2i+\sqrt{2}\right)\tanh^{-1}\left(\frac{(x+2)\left(4\sqrt{2}x^2+18ix^2+16\sqrt{2}x+32ix+12\sqrt{2}+15i\right)}{-6\sqrt{2}x^3+8ix^3+6\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x^2-8\sqrt{2}x^2+36ix^2+12\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x+5\sqrt{2}x+40ix+9\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}+6\sqrt{2}+12i}\right)}{4\sqrt{1-2i\sqrt{2}}}$$
$$+\frac{\left(2i+\sqrt{2}\right)\log\left(\left(-2ix+\sqrt{2}-2i\right)^2\left(2ix+\sqrt{2}+2i\right)^2\right)}{8\sqrt{1-2i\sqrt{2}}}$$
$$+\frac{\left(-2i+\sqrt{2}\right)\log\left(\left(-2ix+\sqrt{2}-2i\right)^2\left(2ix+\sqrt{2}+2i\right)^2\right)}{8\sqrt{1+2i\sqrt{2}}}$$
$$-\frac{\left(2i+\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(2i\sqrt{2}x^2+2x^2-2\sqrt{2\left(1-2i\sqrt{2}\right)}\sqrt{-x^2-4x-3}x+8i\sqrt{2}x+4x-2\sqrt{2\left(1-2i\sqrt{2}\right)}\sqrt{-x^2\cdot}\right.\right.}{8\sqrt{1-2i\sqrt{2}}}$$
$$-\frac{\left(-2i+\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(-2i\sqrt{2}x^2+2x^2-2\sqrt{2\left(1+2i\sqrt{2}\right)}\sqrt{-x^2-4x-3}x-8i\sqrt{2}x+4x-2\sqrt{2\left(1+2i\sqrt{2}\right)}\sqrt{-\cdot}\right.\right.}{8\sqrt{1+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ((I/4)*(-2*I + Sqrt[2])*ArcTan[((2 + x)*(-15*I + 12*Sqrt[2] - (32
*I)*x + 16*Sqrt[2]*x - (18*I)*x^2 + 4*Sqrt[2]*x^2))/(12 + (6*I)*S
qrt[2] + 40*x + (5*I)*Sqrt[2]*x + 36*x^2 - (8*I)*Sqrt[2]*x^2 + 8*
x^3 - (6*I)*Sqrt[2]*x^3 + (9*I)*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 -
 4*x - x^2] + (12*I)*Sqrt[1 + (2*I)*Sqrt[2]]*x*Sqrt[-3 - 4*x - x^
2] + (6*I)*Sqrt[1 + (2*I)*Sqrt[2]]*x^2*Sqrt[-3 - 4*x - x^2])])/Sq
rt[1 + (2*I)*Sqrt[2]] + ((2*I + Sqrt[2])*ArcTanh[((2 + x)*(15*I +
 12*Sqrt[2] + (32*I)*x + 16*Sqrt[2]*x + (18*I)*x^2 + 4*Sqrt[2]*x^
2))/(12*I + 6*Sqrt[2] + (40*I)*x + 5*Sqrt[2]*x + (36*I)*x^2 - 8*S
qrt[2]*x^2 + (8*I)*x^3 - 6*Sqrt[2]*x^3 + 9*Sqrt[1 - (2*I)*Sqrt[2]
]*Sqrt[-3 - 4*x - x^2] + 12*Sqrt[1 - (2*I)*Sqrt[2]]*x*Sqrt[-3 - 4
*x - x^2] + 6*Sqrt[1 - (2*I)*Sqrt[2]]*x^2*Sqrt[-3 - 4*x - x^2])])
/(4*Sqrt[1 - (2*I)*Sqrt[2]]) + ((-2*I + Sqrt[2])*Log[(-2*I + Sqrt
[2] - (2*I)*x)^2*(2*I + Sqrt[2] + (2*I)*x)^2])/(8*Sqrt[1 + (2*I)*
Sqrt[2]]) + ((2*I + Sqrt[2])*Log[(-2*I + Sqrt[2] - (2*I)*x)^2*(2*
I + Sqrt[2] + (2*I)*x)^2])/(8*Sqrt[1 - (2*I)*Sqrt[2]]) - ((2*I +
Sqrt[2])*Log[(3 + 4*x + 2*x^2)*(3 + (6*I)*Sqrt[2] + 4*x + (8*I)*S
qrt[2]*x + 2*x^2 + (2*I)*Sqrt[2]*x^2 - 2*Sqrt[2*(1 - (2*I)*Sqrt[2
])]*Sqrt[-3 - 4*x - x^2] - 2*Sqrt[2*(1 - (2*I)*Sqrt[2])]*x*Sqrt[-
3 - 4*x - x^2])])/(8*Sqrt[1 - (2*I)*Sqrt[2]]) - ((-2*I + Sqrt[2])
*Log[(3 + 4*x + 2*x^2)*(3 - (6*I)*Sqrt[2] + 4*x - (8*I)*Sqrt[2]*x
 + 2*x^2 - (2*I)*Sqrt[2]*x^2 - 2*Sqrt[2*(1 + (2*I)*Sqrt[2])]*Sqrt
[-3 - 4*x - x^2] - 2*Sqrt[2*(1 + (2*I)*Sqrt[2])]*x*Sqrt[-3 - 4*x
- x^2])])/(8*Sqrt[1 + (2*I)*Sqrt[2]])
```

Maple [A] time = 0.011, size = 92, normalized size = 1.4

$$\frac{\sqrt{3}\sqrt{4}\sqrt{2}}{12}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\right)\frac{1}{\sqrt{1\left(x^2\left(-\frac{3}{2}-x\right)^{-2}-4\right)\left(1+x\left(-\frac{3}{2}-x\right)^{-1}\right)^{-2}}}\left(1+x\left(\cdot\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] 1/12*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/
(1+x/(-3/2-x))*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2)*arctan(1/6*(3*
x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="maxima")
```

```
[Out] integrate(x/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Fricas [A] time = 0.291994, size = 54, normalized size = 0.79

$$-\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(6\,x^2+20\,x+15\right)}{4\,\sqrt{-x^2-4\,x-3}(2\,x+3)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/4*sqrt(2)*(6*x^2 + 20*x + 15)/(sqrt(-x^2 -
4*x - 3)*(2*x + 3)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(x+1)(x+3)}\,(2x^2+4x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

GIAC/XCAS [A] time = 0.270911, size = 92, normalized size = 1.35

$$\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right)+\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x +
 2) + 1)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3)
 - 1)/(x + 2) + 1))
```

3.130 $\int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

Optimal. Leaf size=95

$$-\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)+\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

```
[Out] -(Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/3 +
 (Sqrt[2]*ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/3 +
 ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/3
```

Rubi [A] time = 0.259939, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296

$$-\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)+\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] -(Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/3 +
 (Sqrt[2]*ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/3 +
 ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/3
```

Rubi in Sympy [A] time = 52.6953, size = 88, normalized size = 0.93

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3\left(\frac{x}{3}+1\right)}{2\sqrt{-x^2-4x-3}}-\frac{1}{2}\right)\right)}{3}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3\left(\frac{x}{3}+1\right)}{2\sqrt{-x^2-4x-3}}+\frac{1}{2}\right)\right)}{3}+\frac{\mathrm{atanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*(3*(x/3 + 1)/(2*sqrt(-x**2 - 4*x - 3)) - 1/2
))/3 + sqrt(2)*atan(sqrt(2)*(3*(x/3 + 1)/(2*sqrt(-x**2 - 4*x - 3)
) + 1/2))/3 + atanh(x/sqrt(-x**2 - 4*x - 3))/3
```

Mathematica [C] time = 4.35626, size = 800, normalized size = 8.42

$$\frac{1}{12}\left(-2\sqrt{1-2i\sqrt{2}}\tan^{-1}\left(\frac{\left(x^2+4x+3\right)\left(\right.}{2\sqrt{2}x^4+2\left(-\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}+6\sqrt{2}+2i\right)x^3+\left(-8\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}+23\sqrt{2}+\right.}\right.\right.$$

$$+2i\sqrt{1+2i\sqrt{2}}\tanh^{-1}\left(\frac{\left(2ix^2+8ix+2\sqrt{2}\right.}{2\sqrt{2}x^4+2\left(-\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}+6\sqrt{2}-2i\right)x^3+\left(-8\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}+23\sqrt{2}-20\right.}\right.$$

$$+i\left(\left(\sqrt{1-2i\sqrt{2}}-\sqrt{1+2i\sqrt{2}}\right)\log\left(4\left(2x^2+4x+3\right)^2\right)\right.$$

$$+\sqrt{1+2i\sqrt{2}}\log\left(\left(2x^2+4x+3\right)\left(\left(2+2i\sqrt{2}\right)x^2+\left(-2\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}+8i\sqrt{2}+4\right)x-2\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}+6i\right.\right.$$

$$-\sqrt{1-2i\sqrt{2}}\log\left(\left(2x^2+4x+3\right)\left(\left(2-2i\sqrt{2}\right)x^2-2\left(\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}+4i\sqrt{2}-2\right)x-2\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}-6i\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (-2*Sqrt[1 - (2*I)*Sqrt[2]]*ArcTan[((3 + 4*x + x^2)*(7 + (2*I)*Sq
rt[2] + 8*x + 2*x^2))/(2*Sqrt[2]*x^4 + x*(28*I + 16*Sqrt[2] - 11*
Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]) + x^2*(20*I + 23*Sq
rt[2] - 8*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]) + 3*(4*I
+ Sqrt[2] - 2*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]) + 2*x
^3*(2*I + 6*Sqrt[2] - Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2
]))] + (2*I)*Sqrt[1 + (2*I)*Sqrt[2]]*ArcTanh[((7*I + 2*Sqrt[2] +
(8*I)*x + (2*I)*x^2)*(3 + 4*x + x^2))/(2*Sqrt[2]*x^4 + x*(-28*I +
 16*Sqrt[2] - 11*Sqrt[1 - (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]) +
x^2*(-20*I + 23*Sqrt[2] - 8*Sqrt[1 - (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x
 - x^2]) + 3*(-4*I + Sqrt[2] - 2*Sqrt[1 - (2*I)*Sqrt[2]]*Sqrt[-3
- 4*x - x^2]) + 2*x^3*(-2*I + 6*Sqrt[2] - Sqrt[1 - (2*I)*Sqrt[2]]
*Sqrt[-3 - 4*x - x^2]))] + I*((Sqrt[1 - (2*I)*Sqrt[2]] - Sqrt[1 +
 (2*I)*Sqrt[2]])*Log[4*(3 + 4*x + 2*x^2)^2] + Sqrt[1 + (2*I)*Sqrt
[2]]*Log[(3 + 4*x + 2*x^2)*(3 + (6*I)*Sqrt[2] + (2 + (2*I)*Sqrt[2
])*x^2 - 2*Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] + x*(4 +
(8*I)*Sqrt[2] - 2*Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]))]
 - Sqrt[1 - (2*I)*Sqrt[2]]*Log[(3 + 4*x + 2*x^2)*(3 - (6*I)*Sqrt[
2] + (2 - (2*I)*Sqrt[2])*x^2 - 2*Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3
- 4*x - x^2] - 2*x*(-2 + (4*I)*Sqrt[2] + Sqrt[2 + (4*I)*Sqrt[2]]*
Sqrt[-3 - 4*x - x^2])]))/12
```

Maple [A] time = 0.009, size = 121, normalized size = 1.3

$$-\frac{\sqrt{3}\sqrt{4}}{18}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\right)+Artanh\left(3\frac{x}{-3/2-x}\frac{1}{\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}}\right)\right)\frac{}{\sqrt{1\left(x^2\left(-\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] -1/18*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan
(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+arctanh(3*x/(-3/2-x)/(3
*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)
^(1/2)/(1+x/(-3/2-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2+4x+3)\sqrt{-x^2-4x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Fricas [A] time = 0.293823, size = 167, normalized size = 1.76

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x+3\sqrt{-x^2-4x-3}\right)}{2(2x+3)}\right)-\frac{1}{6}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(x-3\sqrt{-x^2-4x-3}\right)}{2(2x+3)}\right)-\frac{1}{12}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right)+\frac{1}{12}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(2)*arctan(1/2*sqrt(2)*(x + 3*sqrt(-x^2 - 4*x - 3))/(2*x
 + 3)) - 1/6*sqrt(2)*arctan(-1/2*sqrt(2)*(x - 3*sqrt(-x^2 - 4*x -
 3))/(2*x + 3)) - 1/12*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/
x^2) + 1/12*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x+1)(x+3)}\,(2x^2+4x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

GIAC/XCAS [A] time = 0.272092, size = 223, normalized size = 2.35

$$-\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)-\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)+\frac{1}{6}\ln\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)-\frac{1}{6}\ln\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x
+ 2) + 1)) - 1/3*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3
) - 1)/(x + 2) + 1)) + 1/6*ln(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2
) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/6*ln(2*(sqr
t(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x
+ 2)^2 + 3)
```

3.131 $\int \frac{1}{x\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

Optimal. Leaf size=130

$$-\frac{\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}}+\frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)-\frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

```
[Out] -ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x^2])]/(3*Sqrt[3]) + (
Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/9 - (
Sqrt[2]*ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/9 - (
4*ArcTanh[x/Sqrt[-3 - 4*x - x^2]])/9
```

Rubi [A] time = 0.865418, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367

$$-\frac{\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}}+\frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)-\frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] -ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x^2])]/(3*Sqrt[3]) + (
Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/9 - (
Sqrt[2]*ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/9 - (
4*ArcTanh[x/Sqrt[-3 - 4*x - x^2]])/9
```

Rubi in Sympy [A] time = 124.364, size = 129, normalized size = 0.99

$$-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3\left(\frac{x}{3}+1\right)}{2\sqrt{-x^2-4x-3}}-\frac{1}{2}\right)\right)}{9}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3\left(\frac{x}{3}+1\right)}{2\sqrt{-x^2-4x-3}}+\frac{1}{2}\right)\right)}{9}-\frac{\sqrt{3}\,\mathrm{atan}\left(-\frac{\sqrt{3}(-4x-6)}{6\sqrt{-x^2-4x-3}}\right)}{9}-\frac{4\,\mathrm{atanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] -sqrt(2)*atan(sqrt(2)*(3*(x/3 + 1)/(2*sqrt(-x**2 - 4*x - 3)) - 1/
2))/9 - sqrt(2)*atan(sqrt(2)*(3*(x/3 + 1)/(2*sqrt(-x**2 - 4*x - 3
)) + 1/2))/9 - sqrt(3)*atan(-sqrt(3)*(-4*x - 6)/(6*sqrt(-x**2 - 4
*x - 3)))/9 - 4*atanh(x/sqrt(-x**2 - 4*x - 3))/9
```

Mathematica [C] time = 6.24972, size = 1068, normalized size = 8.22

$$\frac{\tan^{-1}\left(\frac{(2x+3)\sqrt{-x^2-4x-3}}{\sqrt{3}(x^2+4x+3)}\right)}{3\sqrt{3}}+\frac{i\left(-2i+\sqrt{2}\right)\tan^{-1}\left(\frac{2i\sqrt{2}x^4+8x^4+6i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x^3-4i\sqrt{2}x^3+44x^3+24i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x^2-35i\sqrt{2}x^2+72x^2+33i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x-4}{4\sqrt{2}x^4+2ix^4+16\sqrt{2}x^3+8ix^3+12\sqrt{2}x^2+31ix^2+60ix+36i}\right.}{6\sqrt{1-2i\sqrt{2}}}-\frac{\left(2i+\sqrt{2}\right)\tanh^{-1}\left(\frac{2\sqrt{2}x^4+8ix^4+6\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}x^3-4\sqrt{2}x^3+44ix^3+24\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}x^2-35\sqrt{2}x^2+72ix^2+33\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}x-48\sqrt{2}}{4\sqrt{2}x^4-2ix^4+16\sqrt{2}x^3-8ix^3+12\sqrt{2}x^2-31ix^2-60ix-36i}\right.}{6\sqrt{1+2i\sqrt{2}}}-\frac{\left(2i+\sqrt{2}\right)\log\left(\left(-2ix+\sqrt{2}-2i\right)^2\left(2ix+\sqrt{2}+2i\right)^2\right)}{12\sqrt{1+2i\sqrt{2}}}-\frac{\left(-2i+\sqrt{2}\right)\log\left(\left(-2ix+\sqrt{2}-2i\right)^2\left(2ix+\sqrt{2}+2i\right)^2\right)}{12\sqrt{1-2i\sqrt{2}}}+\frac{\left(-2i+\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(2i\sqrt{2}x^2+2x^2-2\sqrt{2\left(1-2i\sqrt{2}\right)}\sqrt{-x^2-4x-3}x+8i\sqrt{2}x+4x-2\sqrt{2\left(1-2i\sqrt{2}\right)}\sqrt{-x^2}\right.\right.}{12\sqrt{1-2i\sqrt{2}}}+\frac{\left(2i+\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(-2i\sqrt{2}x^2+2x^2-2\sqrt{2\left(1+2i\sqrt{2}\right)}\sqrt{-x^2-4x-3}x-8i\sqrt{2}x+4x-2\sqrt{2\left(1+2i\sqrt{2}\right)}\sqrt{-x^2}\right.\right.}{12\sqrt{1+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ArcTan[((3 + 2*x)*Sqrt[-3 - 4*x - x^2])/(Sqrt[3]*(3 + 4*x + x^2))
]/(3*Sqrt[3]) + ((I/6)*(-2*I + Sqrt[2])*ArcTan[((-18*I)*Sqrt[2] +
 36*x - (48*I)*Sqrt[2]*x + 72*x^2 - (35*I)*Sqrt[2]*x^2 + 44*x^3 -
 (4*I)*Sqrt[2]*x^3 + 8*x^4 + (2*I)*Sqrt[2]*x^4 + (18*I)*Sqrt[1 -
(2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] + (33*I)*Sqrt[1 - (2*I)*Sqrt[
2]]*x*Sqrt[-3 - 4*x - x^2] + (24*I)*Sqrt[1 - (2*I)*Sqrt[2]]*x^2*S
qrt[-3 - 4*x - x^2] + (6*I)*Sqrt[1 - (2*I)*Sqrt[2]]*x^3*Sqrt[-3 -
 4*x - x^2])/(36*I + (60*I)*x + (31*I)*x^2 + 12*Sqrt[2]*x^2 + (8*
I)*x^3 + 16*Sqrt[2]*x^3 + (2*I)*x^4 + 4*Sqrt[2]*x^4)])/Sqrt[1 - (
2*I)*Sqrt[2]] - ((2*I + Sqrt[2])*ArcTanh[(-18*Sqrt[2] + (36*I)*x
- 48*Sqrt[2]*x + (72*I)*x^2 - 35*Sqrt[2]*x^2 + (44*I)*x^3 - 4*Sqr
t[2]*x^3 + (8*I)*x^4 + 2*Sqrt[2]*x^4 + 18*Sqrt[1 + (2*I)*Sqrt[2]]
*Sqrt[-3 - 4*x - x^2] + 33*Sqrt[1 + (2*I)*Sqrt[2]]*x*Sqrt[-3 - 4*
x - x^2] + 24*Sqrt[1 + (2*I)*Sqrt[2]]*x^2*Sqrt[-3 - 4*x - x^2] +
6*Sqrt[1 + (2*I)*Sqrt[2]]*x^3*Sqrt[-3 - 4*x - x^2])/(-36*I - (60*
I)*x - (31*I)*x^2 + 12*Sqrt[2]*x^2 - (8*I)*x^3 + 16*Sqrt[2]*x^3 -
 (2*I)*x^4 + 4*Sqrt[2]*x^4)])/(6*Sqrt[1 + (2*I)*Sqrt[2]]) - ((-2*
I + Sqrt[2])*Log[(-2*I + Sqrt[2] - (2*I)*x)^2*(2*I + Sqrt[2] + (2
*I)*x)^2])/(12*Sqrt[1 - (2*I)*Sqrt[2]]) - ((2*I + Sqrt[2])*Log[(-
2*I + Sqrt[2] - (2*I)*x)^2*(2*I + Sqrt[2] + (2*I)*x)^2])/(12*Sqrt
[1 + (2*I)*Sqrt[2]]) + ((-2*I + Sqrt[2])*Log[(3 + 4*x + 2*x^2)*(3
 + (6*I)*Sqrt[2] + 4*x + (8*I)*Sqrt[2]*x + 2*x^2 + (2*I)*Sqrt[2]*
x^2 - 2*Sqrt[2*(1 - (2*I)*Sqrt[2])]*Sqrt[-3 - 4*x - x^2] - 2*Sqrt
[2*(1 - (2*I)*Sqrt[2])]*x*Sqrt[-3 - 4*x - x^2])])/(12*Sqrt[1 - (2
*I)*Sqrt[2]]) + ((2*I + Sqrt[2])*Log[(3 + 4*x + 2*x^2)*(3 - (6*I)
*Sqrt[2] + 4*x - (8*I)*Sqrt[2]*x + 2*x^2 - (2*I)*Sqrt[2]*x^2 - 2*
Sqrt[2*(1 + (2*I)*Sqrt[2])]*Sqrt[-3 - 4*x - x^2] - 2*Sqrt[2*(1 +
(2*I)*Sqrt[2])]*x*Sqrt[-3 - 4*x - x^2])])/(12*Sqrt[1 + (2*I)*Sqrt
[2]])
```

Maple [A] time = 0.023, size = 152, normalized size = 1.2

$$\frac{\sqrt{3}}{9}\arctan\left(\frac{(-6-4x)\sqrt{3}}{6}\frac{1}{\sqrt{-x^2-4x-3}}\right)$$
$$+\frac{\sqrt{3}\sqrt{4}}{54}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\right)+4\,Artanh\left(3\frac{x}{-3/2-x}\frac{1}{\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}}\right)\right)\frac{}{\sqrt{1\left(x^2\left(\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] 1/9*3^(1/2)*arctan(1/6*(-6-4*x)*3^(1/2)/(-x^2-4*x-3)^(1/2))+1/54*
3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(
3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+4*arctanh(3*x/(-3/2-x)/(3*x^2
/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/
2)/(1+x/(-3/2-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x), x)
```

Fricas [A] time = 0.300165, size = 221, normalized size = 1.7

$$\frac{1}{54}\sqrt{3}\left(\sqrt{3}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x+3\sqrt{-x^2-4\,x-3}\right)}{2\,(2\,x+3)}\right)+\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(x-3\sqrt{-x^2-4\,x-3}\right)}{2\,(2\,x+3)}\right)+2\sqrt{3}\log\left(-\frac{2\sqrt{-x^2-4}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x),x, algorithm="fricas")
```

```
[Out] 1/54*sqrt(3)*(sqrt(3)*sqrt(2)*arctan(1/2*sqrt(2)*(x + 3*sqrt(-x^2
 - 4*x - 3))/(2*x + 3)) + sqrt(3)*sqrt(2)*arctan(-1/2*sqrt(2)*(x
- 3*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 2*sqrt(3)*log(-(2*sqrt(-x^
2 - 4*x - 3)*x + 4*x + 3)/x^2) - 2*sqrt(3)*log((2*sqrt(-x^2 - 4*x
 - 3)*x - 4*x - 3)/x^2) - 6*arctan(1/3*sqrt(3)*(2*x + 3)/sqrt(-x^
2 - 4*x - 3)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-(x+1)(x+3)}(2x^2+4x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

GIAC/XCAS [A] time = 0.272434, size = 269, normalized size = 2.07

$$\frac{1}{9}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{1}{9}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)-\frac{2}{9}\ln\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)+\frac{2}{9}\ln\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x +
 2) + 1)) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 - 4*x -
3) - 1)/(x + 2) + 1)) + 1/9*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^
2 - 4*x - 3) - 1)/(x + 2) + 1)) - 2/9*ln(2*(sqrt(-x^2 - 4*x - 3)
- 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 2/
9*ln(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3)
 - 1)^2/(x + 2)^2 + 3)
```

3.132 $\int \frac{1}{x^2\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{-x^2-4x-3}}{9x}+\frac{2\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}}+\frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\frac{10}{27}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

```
[Out] Sqrt[-3 - 4*x - x^2]/(9*x) + (2*ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3
 - 4*x - x^2])])/(3*Sqrt[3]) + (2*Sqrt[2]*ArcTan[(1 - (3 + x)/Sqr
t[-3 - 4*x - x^2])/Sqrt[2]])/27 - (2*Sqrt[2]*ArcTan[(1 + (3 + x)/
Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/27 + (10*ArcTanh[x/Sqrt[-3 - 4*x
- x^2]])/27
```

Rubi [A] time = 1.00088, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$\frac{\sqrt{-x^2-4x-3}}{9x}+\frac{2\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}}+\frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\frac{10}{27}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] Sqrt[-3 - 4*x - x^2]/(9*x) + (2*ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3
 - 4*x - x^2])])/(3*Sqrt[3]) + (2*Sqrt[2]*ArcTan[(1 - (3 + x)/Sqr
t[-3 - 4*x - x^2])/Sqrt[2]])/27 - (2*Sqrt[2]*ArcTan[(1 + (3 + x)/
Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/27 + (10*ArcTanh[x/Sqrt[-3 - 4*x
- x^2]])/27
```

Rubi in Sympy [A] time = 118.371, size = 150, normalized size = 0.99

$$-\frac{2\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3\left(\frac{x}{3}+1\right)}{2\sqrt{-x^2-4x-3}}-\frac{1}{2}\right)\right)}{27}-\frac{2\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3\left(\frac{x}{3}+1\right)}{2\sqrt{-x^2-4x-3}}+\frac{1}{2}\right)\right)}{27}+\frac{2\sqrt{3}\,\mathrm{atan}\left(-\frac{\sqrt{3}(-4x-6)}{6\sqrt{-x^2-4x-3}}\right)}{9}+\frac{10\,\mathrm{atanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)}{27}+\frac{\sqrt{-x^2-4x-3}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] -2*sqrt(2)*atan(sqrt(2)*(3*(x/3 + 1)/(2*sqrt(-x**2 - 4*x - 3)) -
1/2))/27 - 2*sqrt(2)*atan(sqrt(2)*(3*(x/3 + 1)/(2*sqrt(-x**2 - 4*
x - 3)) + 1/2))/27 + 2*sqrt(3)*atan(-sqrt(3)*(-4*x - 6)/(6*sqrt(-
x**2 - 4*x - 3)))/9 + 10*atanh(x/sqrt(-x**2 - 4*x - 3))/27 + sqrt
(-x**2 - 4*x - 3)/(9*x)
```

Mathematica [C] time = 6.29769, size = 1135, normalized size = 7.52

$$-\frac{2\tan^{-1}\left(\frac{(2x+3)\sqrt{-x^2-4x-3}}{\sqrt{3}(x^2+4x+3)}\right)}{3\sqrt{3}}$$
$$-\frac{i\left(-i+2\sqrt{2}\right)\tan^{-1}\left(\frac{22i\sqrt{2}x^4+16x^4+18i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x^3+100i\sqrt{2}x^3+124x^3+72i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x^2+137i\sqrt{2}x^2+324x^2+99i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-}}{32\sqrt{2}x^4+34ix^4+176\sqrt{2}x^3+112ix^3+306\sqrt{2}x^2+125ix^2+216\sqrt{2}x+84ix+54\sqrt{2}+}\right.}{9\sqrt{1-2i\sqrt{2}}}$$
$$+\frac{\left(i+2\sqrt{2}\right)\tanh^{-1}\left(\frac{22\sqrt{2}x^4+16ix^4+18\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}x^3+100\sqrt{2}x^3+124ix^3+72\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}x^2+137\sqrt{2}x^2+324ix^2+99\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}}{32\sqrt{2}x^4-34ix^4+176\sqrt{2}x^3-112ix^3+306\sqrt{2}x^2-125ix^2+216\sqrt{2}x-84ix+54\sqrt{2}-45i}\right.}{9\sqrt{1+2i\sqrt{2}}}$$
$$+\frac{\left(i+2\sqrt{2}\right)\log\left(\left(-2ix+\sqrt{2}-2i\right)^2\left(2ix+\sqrt{2}+2i\right)^2\right)}{18\sqrt{1+2i\sqrt{2}}}$$
$$+\frac{\left(-i+2\sqrt{2}\right)\log\left(\left(-2ix+\sqrt{2}-2i\right)^2\left(2ix+\sqrt{2}+2i\right)^2\right)}{18\sqrt{1-2i\sqrt{2}}}$$
$$-\frac{\left(-i+2\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(2i\sqrt{2}x^2+2x^2-2\sqrt{2\left(1-2i\sqrt{2}\right)}\sqrt{-x^2-4x-3}x+8i\sqrt{2}x+4x-2\sqrt{2\left(1-2i\sqrt{2}\right)}\sqrt{-x^2}\right.\right.}{18\sqrt{1-2i\sqrt{2}}}$$
$$-\frac{\left(i+2\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(-2i\sqrt{2}x^2+2x^2-2\sqrt{2\left(1+2i\sqrt{2}\right)}\sqrt{-x^2-4x-3}x-8i\sqrt{2}x+4x-2\sqrt{2\left(1+2i\sqrt{2}\right)}\sqrt{-x^2}\right.\right.}{18\sqrt{1+2i\sqrt{2}}}$$
$$+\frac{\sqrt{-x^2-4x-3}}{9x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] Sqrt[-3 - 4*x - x^2]/(9*x) - (2*ArcTan[((3 + 2*x)*Sqrt[-3 - 4*x -
 x^2])/(Sqrt[3]*(3 + 4*x + x^2))])/(3*Sqrt[3]) - ((I/9)*(-I + 2*S
qrt[2])*ArcTan[(108 - (9*I)*Sqrt[2] + 324*x + (48*I)*Sqrt[2]*x +
324*x^2 + (137*I)*Sqrt[2]*x^2 + 124*x^3 + (100*I)*Sqrt[2]*x^3 + 1
6*x^4 + (22*I)*Sqrt[2]*x^4 + (54*I)*Sqrt[1 - (2*I)*Sqrt[2]]*Sqrt[
-3 - 4*x - x^2] + (99*I)*Sqrt[1 - (2*I)*Sqrt[2]]*x*Sqrt[-3 - 4*x
- x^2] + (72*I)*Sqrt[1 - (2*I)*Sqrt[2]]*x^2*Sqrt[-3 - 4*x - x^2]
+ (18*I)*Sqrt[1 - (2*I)*Sqrt[2]]*x^3*Sqrt[-3 - 4*x - x^2])/(45*I
+ 54*Sqrt[2] + (84*I)*x + 216*Sqrt[2]*x + (125*I)*x^2 + 306*Sqrt[
2]*x^2 + (112*I)*x^3 + 176*Sqrt[2]*x^3 + (34*I)*x^4 + 32*Sqrt[2]*
x^4)])/Sqrt[1 - (2*I)*Sqrt[2]] + ((I + 2*Sqrt[2])*ArcTanh[(108*I
- 9*Sqrt[2] + (324*I)*x + 48*Sqrt[2]*x + (324*I)*x^2 + 137*Sqrt[2
]*x^2 + (124*I)*x^3 + 100*Sqrt[2]*x^3 + (16*I)*x^4 + 22*Sqrt[2]*x
^4 + 54*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] + 99*Sqrt[1
+ (2*I)*Sqrt[2]]*x*Sqrt[-3 - 4*x - x^2] + 72*Sqrt[1 + (2*I)*Sqrt[
2]]*x^2*Sqrt[-3 - 4*x - x^2] + 18*Sqrt[1 + (2*I)*Sqrt[2]]*x^3*Sqr
t[-3 - 4*x - x^2])/(-45*I + 54*Sqrt[2] - (84*I)*x + 216*Sqrt[2]*x
 - (125*I)*x^2 + 306*Sqrt[2]*x^2 - (112*I)*x^3 + 176*Sqrt[2]*x^3
- (34*I)*x^4 + 32*Sqrt[2]*x^4)])/(9*Sqrt[1 + (2*I)*Sqrt[2]]) + ((
-I + 2*Sqrt[2])*Log[(-2*I + Sqrt[2] - (2*I)*x)^2*(2*I + Sqrt[2] +
 (2*I)*x)^2])/(18*Sqrt[1 - (2*I)*Sqrt[2]]) + ((I + 2*Sqrt[2])*Log
[(-2*I + Sqrt[2] - (2*I)*x)^2*(2*I + Sqrt[2] + (2*I)*x)^2])/(18*S
qrt[1 + (2*I)*Sqrt[2]]) - ((-I + 2*Sqrt[2])*Log[(3 + 4*x + 2*x^2)
*(3 + (6*I)*Sqrt[2] + 4*x + (8*I)*Sqrt[2]*x + 2*x^2 + (2*I)*Sqrt[
2]*x^2 - 2*Sqrt[2*(1 - (2*I)*Sqrt[2])]*Sqrt[-3 - 4*x - x^2] - 2*S
qrt[2*(1 - (2*I)*Sqrt[2])]*x*Sqrt[-3 - 4*x - x^2])])/(18*Sqrt[1 -
 (2*I)*Sqrt[2]]) - ((I + 2*Sqrt[2])*Log[(3 + 4*x + 2*x^2)*(3 - (6
*I)*Sqrt[2] + 4*x - (8*I)*Sqrt[2]*x + 2*x^2 - (2*I)*Sqrt[2]*x^2 -
 2*Sqrt[2*(1 + (2*I)*Sqrt[2])]*Sqrt[-3 - 4*x - x^2] - 2*Sqrt[2*(1
 + (2*I)*Sqrt[2])]*x*Sqrt[-3 - 4*x - x^2])])/(18*Sqrt[1 + (2*I)*S
qrt[2]])
```

Maple [A] time = 0.023, size = 169, normalized size = 1.1

$$\frac{1}{9x}\sqrt{-x^2-4x-3}-\frac{2\sqrt{3}}{9}\arctan\left(\frac{(-6-4x)\sqrt{3}}{6}\frac{1}{\sqrt{-x^2-4x-3}}\right)$$
$$+\frac{\sqrt{3}\sqrt{4}}{81}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\right)-5\,Artanh\left(3\frac{x}{-3/2-x}\frac{1}{\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}}\right)\right)\frac{1}{\sqrt{1\left(x^2\left(\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] 1/9*(-x^2-4*x-3)^(1/2)/x-2/9*3^(1/2)*arctan(1/6*(-6-4*x)*3^(1/2)/
(-x^2-4*x-3)^(1/2))+1/81*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1
/2)*(2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-5*ar
ctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2
-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2+4x+3)\sqrt{-x^2-4x-3}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x^2), x)
```

Fricas [A] time = 0.305584, size = 258, normalized size = 1.71

$$\frac{\sqrt{3}\left(2\sqrt{3}\sqrt{2}x\arctan\left(\frac{\sqrt{2}\left(x+3\sqrt{-x^2-4x-3}\right)}{2(2x+3)}\right)+2\sqrt{3}\sqrt{2}x\arctan\left(-\frac{\sqrt{2}\left(x-3\sqrt{-x^2-4x-3}\right)}{2(2x+3)}\right)-5\sqrt{3}x\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right)+5\right.}{162x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x^2),x, algorithm="fricas")
```

```
[Out] 1/162*sqrt(3)*(2*sqrt(3)*sqrt(2)*x*arctan(1/2*sqrt(2)*(x + 3*sqrt
(-x^2 - 4*x - 3))/(2*x + 3)) + 2*sqrt(3)*sqrt(2)*x*arctan(-1/2*sq
rt(2)*(x - 3*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 5*sqrt(3)*x*log(-
(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 5*sqrt(3)*x*log((2*sq
rt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2) + 36*x*arctan(1/3*sqrt(3)*(2
*x + 3)/sqrt(-x^2 - 4*x - 3)) + 6*sqrt(3)*sqrt(-x^2 - 4*x - 3))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{-(x+1)(x+3)}(2x^2+4x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

GIAC/XCAS [A] time = 0.273651, size = 363, normalized size = 2.4

$$\frac{2}{27}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right) - \frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right) + \frac{2}{27}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right) - \frac{\frac{\sqrt{-x^2-4x-3}-1}{x+2}+2}{18\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)} + \frac{5}{27}\ln\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right) - \frac{5}{27}\ln\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x^2),x, algorithm="giac")
```

```
[Out] 2/27*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x
+ 2) + 1)) - 4/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 - 4*x -
 3) - 1)/(x + 2) + 1)) + 2/27*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-
x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/18*((sqrt(-x^2 - 4*x - 3) -
 1)/(x + 2) + 2)/((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2
 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 5/27*ln(2*(sqrt(-x^2 - 4*x -
3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) -
 5/27*ln(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x
- 3) - 1)^2/(x + 2)^2 + 3)
```

3.133 $\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\,dx$

Optimal. Leaf size=149

$$-\frac{1}{32}(10-3x)\left(12x^2+17x+6\right)^{7/2}-\frac{873\left(12x^2+17x+6\right)^{7/2}}{1792}+\frac{25091(24x+17)\left(12x^2+17x+6\right)^{5/2}}{24576}-\frac{125455(24x+17)\left(12x^2+17x+6\right)^{3/2}}{4718592}+\frac{125455(24x+17)\sqrt{12x^2+17x+6}}{150994944}-\frac{125455\tanh^{-1}\left(\frac{24x+17}{4\sqrt{3}\sqrt{12x^2+17x+6}}\right)}{603979776\sqrt{3}}$$

```
[Out] (125455*(17 + 24*x)*Sqrt[6 + 17*x + 12*x^2])/150994944 - (125455*
(17 + 24*x)*(6 + 17*x + 12*x^2)^(3/2))/4718592 + (25091*(17 + 24*
x)*(6 + 17*x + 12*x^2)^(5/2))/24576 - (873*(6 + 17*x + 12*x^2)^(7
/2))/1792 - ((10 - 3*x)*(6 + 17*x + 12*x^2)^(7/2))/32 - (125455*A
rcTanh[(17 + 24*x)/(4*Sqrt[3]*Sqrt[6 + 17*x + 12*x^2])])/(6039797
76*Sqrt[3])
```

Rubi [A] time = 0.232327, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$-\frac{1}{32}(10-3x)\left(12x^2+17x+6\right)^{7/2}-\frac{873\left(12x^2+17x+6\right)^{7/2}}{1792}+\frac{25091(24x+17)\left(12x^2+17x+6\right)^{5/2}}{24576}-\frac{125455(24x+17)\left(12x^2+17x+6\right)^{3/2}}{4718592}+\frac{125455(24x+17)\sqrt{12x^2+17x+6}}{150994944}-\frac{125455\tanh^{-1}\left(\frac{24x+17}{4\sqrt{3}\sqrt{12x^2+17x+6}}\right)}{603979776\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x)^2*(30 + 31*x - 12*x^2)^2*Sqrt[6 + 17*x + 12*x^2],x]
```

```
[Out] (125455*(17 + 24*x)*Sqrt[6 + 17*x + 12*x^2])/150994944 - (125455*
(17 + 24*x)*(6 + 17*x + 12*x^2)^(3/2))/4718592 + (25091*(17 + 24*
x)*(6 + 17*x + 12*x^2)^(5/2))/24576 - (873*(6 + 17*x + 12*x^2)^(7
/2))/1792 - ((10 - 3*x)*(6 + 17*x + 12*x^2)^(7/2))/32 - (125455*A
rcTanh[(17 + 24*x)/(4*Sqrt[3]*Sqrt[6 + 17*x + 12*x^2])])/(6039797
76*Sqrt[3])
```

Rubi in Sympy [A] time = 28.563, size = 138, normalized size = 0.93

$$-\frac{(-9x+30)\left(12x^2+17x+6\right)^{\frac{7}{2}}}{96}+\frac{25091\,(24x+17)\left(12x^2+17x+6\right)^{\frac{5}{2}}}{24576}-\frac{125455\,(24x+17)\left(12x^2+17x+6\right)^{\frac{3}{2}}}{4718592}+\frac{125455\,(24x+17)\sqrt{12x^2+17x+6}}{150994944}-\frac{873\left(12x^2+17x+6\right)^{\frac{7}{2}}}{1792}-\frac{125455\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}(24x+17)}{12\sqrt{12x^2+17x+6}}\right)}{1811939328}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2+3*x)**2*(-12*x**2+31*x+30)**2*(12*x**2+17*x+6)**(1/2),x)
```

```
[Out] -(-9*x + 30)*(12*x**2 + 17*x + 6)**(7/2)/96 + 25091*(24*x + 17)*(
12*x**2 + 17*x + 6)**(5/2)/24576 - 125455*(24*x + 17)*(12*x**2 +
```

```
17*x + 6)**(3/2)/4718592 + 125455*(24*x + 17)*sqrt(12*x**2 + 17*x
 + 6)/150994944 - 873*(12*x**2 + 17*x + 6)**(7/2)/1792 - 125455*s
qrt(3)*atanh(sqrt(3)*(24*x + 17)/(12*sqrt(12*x**2 + 17*x + 6)))/1
811939328
```

Mathematica [A] time = 0.157314, size = 111, normalized size = 0.74

$$\frac{\sqrt{12x^2+17x+6}\left(6\sqrt{3x+2}\sqrt{4x+3}\left(171228266496x^7-732816211968x^6-1190083166208x^5+3438453030912x^4+897484\cdots\right.\right.}{6341787648\sqrt{3x+2}\sqrt{4x\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^2*(30 + 31*x - 12*x^2)^2*Sqrt[6 + 17*x + 12*x^2],x]
```

```
[Out] (Sqrt[6 + 17*x + 12*x^2]*(6*Sqrt[2 + 3*x]*Sqrt[3 + 4*x]*(47499909
1769 + 3132157281976*x + 7899203409792*x^2 + 8974844476416*x^3 +
3438453030912*x^4 - 1190083166208*x^5 - 732816211968*x^6 + 171228
266496*x^7) - 878185*Sqrt[3]*ArcSinh[2*Sqrt[2 + 3*x]]))/(63417876
48*Sqrt[2 + 3*x]*Sqrt[3 + 4*x])
```

Maple [A] time = 0.022, size = 147, normalized size = 1.

$$\frac{2132735+3010920\,x}{150994944}\sqrt{12\,x^2+17\,x+6}-\frac{125455\,\sqrt{12}}{3623878656}\ln\left(\frac{\sqrt{12}}{12}\left(\frac{17}{2}+12\,x\right)+\sqrt{12\,x^2+17\,x+6}\right)+\frac{2473875847}{33030144}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}+\frac{129220757\,x}{458752}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}+\frac{4267751\,x^2}{14336}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}+\frac{14991\,x^3}{1792}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}-\frac{8613\,x^4}{112}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}+\frac{27\,x^5}{2}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x)
```

```
[Out] 125455/150994944*(17+24*x)*(12*x^2+17*x+6)^(1/2)-125455/362387865
6*ln(1/12*(17/2+12*x)*12^(1/2)+(12*x^2+17*x+6)^(1/2))*12^(1/2)+24
73875847/33030144*(12*x^2+17*x+6)^(3/2)+129220757/458752*x*(12*x^
2+17*x+6)^(3/2)+4267751/14336*x^2*(12*x^2+17*x+6)^(3/2)+14991/179
2*x^3*(12*x^2+17*x+6)^(3/2)-8613/112*x^4*(12*x^2+17*x+6)^(3/2)+27
/2*x^5*(12*x^2+17*x+6)^(3/2)
```

Maxima [A] time = 0.789177, size = 209, normalized size = 1.4

$$\frac{27}{2}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}x^5-\frac{8613}{112}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}x^4+\frac{14991}{1792}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}x^3+\frac{4267751}{14336}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}x^2+\frac{129220757}{458752}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}x+\frac{2473875847}{33030144}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}+\frac{125455}{6291456}\sqrt{12\,x^2+17\,x+6}x-\frac{125455}{1811939328}\sqrt{3}\log\left(4\,\sqrt{3}\sqrt{12\,x^2+17\,x+6}+24\,x+17\right)+\frac{2132735}{150994944}\sqrt{12\,x^2+17\,x+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(12*x^2 + 17*x + 6)*(12*x^2 - 31*x - 30)^2*(3*x + 2)^2,x, algorithm=
```

```
[Out] 27/2*(12*x^2 + 17*x + 6)^(3/2)*x^5 - 8613/112*(12*x^2 + 17*x + 6)
^(3/2)*x^4 + 14991/1792*(12*x^2 + 17*x + 6)^(3/2)*x^3 + 4267751/1
4336*(12*x^2 + 17*x + 6)^(3/2)*x^2 + 129220757/458752*(12*x^2 + 1
7*x + 6)^(3/2)*x + 2473875847/33030144*(12*x^2 + 17*x + 6)^(3/2)
+ 125455/6291456*sqrt(12*x^2 + 17*x + 6)*x - 125455/1811939328*sq
rt(3)*log(4*sqrt(3)*sqrt(12*x^2 + 17*x + 6) + 24*x + 17) + 213273
5/150994944*sqrt(12*x^2 + 17*x + 6)
```

Fricas [A] time = 0.287122, size = 128, normalized size = 0.86

$$\frac{1}{25367150592}\sqrt{3}\left(8\sqrt{3}\left(171228266496\,x^7 - 732816211968\,x^6 - 1190083166208\,x^5 + 3438453030912\,x^4 + 8974844476416\,x^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(12*x^2 + 17*x + 6)*(12*x^2 - 31*x - 30)^2*(3*x + 2)^2,x, algorithm=
```

```
[Out] 1/25367150592*sqrt(3)*(8*sqrt(3)*(171228266496*x^7 - 732816211968
*x^6 - 1190083166208*x^5 + 3438453030912*x^4 + 8974844476416*x^3
+ 7899203409792*x^2 + 3132157281976*x + 474999091769)*sqrt(12*x^2
 + 17*x + 6) + 878185*log(sqrt(3)*(1152*x^2 + 1632*x + 577) - 24*
sqrt(12*x^2 + 17*x + 6)*(24*x + 17)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(3x+2)(4x+3)}\,(3x-10)^2\,(3x+2)^2\,(4x+3)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**2*(-12*x**2+31*x+30)**2*(12*x**2+17*x+6)**(1/2),x)
```

```
[Out] Integral(sqrt((3*x + 2)*(4*x + 3))*(3*x - 10)**2*(3*x + 2)**2*(4*
x + 3)**2, x)
```

GIAC/XCAS [A] time = 0.271274, size = 115, normalized size = 0.77

$$\frac{1}{1056964608}\left(8\left(48\left(24\left(96\left(24\left(48\left(168\,x-719\right)x-56047\right)x+3886417\right)x+973832951\right)x+20570842213\right)x+391519660247\right)x$$
$$+\frac{125455}{1811939328}\sqrt{3}\ln\left(\left|-4\sqrt{3}\left(2\sqrt{3}x-\sqrt{12\,x^2+17\,x+6}\right)-17\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(12*x^2 + 17*x + 6)*(12*x^2 - 31*x - 30)^2*(3*x + 2)^2,x, algorithm=
```

```
[Out] 1/1056964608*(8*(48*(24*(96*(24*(48*(168*x - 719)*x - 56047)*x +
3886417)*x + 973832951)*x + 20570842213)*x + 391519660247)*x + 47
4999091769)*sqrt(12*x^2 + 17*x + 6) + 125455/1811939328*sqrt(3)*l
n(abs(-4*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6)) - 17))
```

3.134 $\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx$

Optimal. Leaf size=103

$$-\frac{1}{20}\left(12x^2+17x+6\right)^{5/2}+\frac{97}{768}(24x+17)\left(12x^2+17x+6\right)^{3/2}-\frac{97(24x+17)\sqrt{12x^2+17x+6}}{24576}+\frac{97\tanh^{-1}\left(\frac{24x+17}{4\sqrt{3}\sqrt{12x^2+17x+6}}\right)}{98304\sqrt{3}}$$

```
[Out] (-97*(17 + 24*x)*Sqrt[6 + 17*x + 12*x^2])/24576 + (97*(17 + 24*x)
*(6 + 17*x + 12*x^2)^(3/2))/768 - (6 + 17*x + 12*x^2)^(5/2)/20 +
(97*ArcTanh[(17 + 24*x)/(4*Sqrt[3]*Sqrt[6 + 17*x + 12*x^2])])/(98
304*Sqrt[3])
```

Rubi [A] time = 0.113357, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{1}{20}\left(12x^2+17x+6\right)^{5/2}+\frac{97}{768}(24x+17)\left(12x^2+17x+6\right)^{3/2}-\frac{97(24x+17)\sqrt{12x^2+17x+6}}{24576}+\frac{97\tanh^{-1}\left(\frac{24x+17}{4\sqrt{3}\sqrt{12x^2+17x+6}}\right)}{98304\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x)*(30 + 31*x - 12*x^2)*Sqrt[6 + 17*x + 12*x^2],x]
```

```
[Out] (-97*(17 + 24*x)*Sqrt[6 + 17*x + 12*x^2])/24576 + (97*(17 + 24*x)
*(6 + 17*x + 12*x^2)^(3/2))/768 - (6 + 17*x + 12*x^2)^(5/2)/20 +
(97*ArcTanh[(17 + 24*x)/(4*Sqrt[3]*Sqrt[6 + 17*x + 12*x^2])])/(98
304*Sqrt[3])
```

Rubi in Sympy [A] time = 17.7073, size = 94, normalized size = 0.91

$$\frac{97\left(24x+17\right)\left(12x^2+17x+6\right)^{\frac{3}{2}}}{768}-\frac{97\left(24x+17\right)\sqrt{12x^2+17x+6}}{24576}-\frac{\left(12x^2+17x+6\right)^{\frac{5}{2}}}{20}+\frac{97\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}(24x+17)}{12\sqrt{12x^2+17x+6}}\right)}{294912}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2+3*x)*(-12*x**2+31*x+30)*(12*x**2+17*x+6)**(1/2),x)
```

```
[Out] 97*(24*x + 17)*(12*x**2 + 17*x + 6)**(3/2)/768 - 97*(24*x + 17)*s
qrt(12*x**2 + 17*x + 6)/24576 - (12*x**2 + 17*x + 6)**(5/2)/20 +
97*sqrt(3)*atanh(sqrt(3)*(24*x + 17)/(12*sqrt(12*x**2 + 17*x + 6)
))/294912
```

Mathematica [A] time = 0.0840474, size = 70, normalized size = 0.68

$$\frac{485\sqrt{3}\log\left(4\sqrt{36x^2+51x+18}+24x+17\right)+12\sqrt{12x^2+17x+6}\left(-884736x^4+1963008x^3+6837888x^2+5455144x+1353\right.}{1474560}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x)*(30 + 31*x - 12*x^2)*Sqrt[6 + 17*x + 12*x^2],x]
```

```
[Out] (12*Sqrt[6 + 17*x + 12*x^2]*(1353611 + 5455144*x + 6837888*x^2 +
1963008*x^3 - 884736*x^4) + 485*Sqrt[3]*Log[17 + 24*x + 4*Sqrt[18
 + 51*x + 36*x^2]])/1474560
```

Maple [A] time = 0.009, size = 96, normalized size = 0.9

$$-\frac{1649+2328\,x}{24576}\sqrt{12\,x^2+17\,x+6}+\frac{97\,\sqrt{12}}{589824}\ln\left(\frac{\sqrt{12}}{12}\left(\frac{17}{2}+12\,x\right)+\sqrt{12\,x^2+17\,x+6}\right)$$
$$+\frac{7093}{3840}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}+\frac{349\,x}{160}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}-\frac{3\,x^2}{5}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x)
```

```
[Out] -97/24576*(17+24*x)*(12*x^2+17*x+6)^(1/2)+97/589824*ln(1/12*(17/2
+12*x)*12^(1/2)+(12*x^2+17*x+6)^(1/2))*12^(1/2)+7093/3840*(12*x^2
+17*x+6)^(3/2)+349/160*x*(12*x^2+17*x+6)^(3/2)-3/5*x^2*(12*x^2+17
*x+6)^(3/2)
```

Maxima [A] time = 0.779977, size = 140, normalized size = 1.36

$$-\frac{3}{5}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}x^2+\frac{349}{160}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}x$$
$$+\frac{7093}{3840}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}-\frac{97}{1024}\sqrt{12\,x^2+17\,x+6}x$$
$$+\frac{97}{294912}\sqrt{3}\log\left(4\sqrt{3}\sqrt{12\,x^2+17\,x+6}+24\,x+17\right)-\frac{1649}{24576}\sqrt{12\,x^2+17\,x+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(12*x^2 + 17*x + 6)*(12*x^2 - 31*x - 30)*(3*x + 2),x, algorithm="ma
```

```
[Out] -3/5*(12*x^2 + 17*x + 6)^(3/2)*x^2 + 349/160*(12*x^2 + 17*x + 6)^
(3/2)*x + 7093/3840*(12*x^2 + 17*x + 6)^(3/2) - 97/1024*sqrt(12*x
^2 + 17*x + 6)*x + 97/294912*sqrt(3)*log(4*sqrt(3)*sqrt(12*x^2 +
17*x + 6) + 24*x + 17) - 1649/24576*sqrt(12*x^2 + 17*x + 6)
```

Fricas [A] time = 0.282939, size = 108, normalized size = 1.05

$$-\frac{1}{2949120}\sqrt{3}\left(8\sqrt{3}\left(884736\,x^4-1963008\,x^3-6837888\,x^2-5455144\,x-1353611\right)\sqrt{12\,x^2+17\,x+6}-485\log\left(\sqrt{3}(1152\,x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(12*x^2 + 17*x + 6)*(12*x^2 - 31*x - 30)*(3*x + 2),x, algorithm="fr
```

```
[Out] -1/2949120*sqrt(3)*(8*sqrt(3)*(884736*x^4 - 1963008*x^3 - 6837888
*x^2 - 5455144*x - 1353611)*sqrt(12*x^2 + 17*x + 6) - 485*log(sqr
t(3)*(1152*x^2 + 1632*x + 577) + 24*sqrt(12*x^2 + 17*x + 6)*(24*x
 + 17)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\left(-152x\sqrt{12x^2+17x+6}\right)dx-\int\left(-69x^2\sqrt{12x^2+17x+6}\right)dx$$
$$-\int 36x^3\sqrt{12x^2+17x+6}\,dx-\int\left(-60\sqrt{12x^2+17x+6}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+3*x)*(-12*x**2+31*x+30)*(12*x**2+17*x+6)**(1/2),x)
```

```
[Out] -Integral(-152*x*sqrt(12*x**2 + 17*x + 6), x) - Integral(-69*x**2
*sqrt(12*x**2 + 17*x + 6), x) - Integral(36*x**3*sqrt(12*x**2 + 1
7*x + 6), x) - Integral(-60*sqrt(12*x**2 + 17*x + 6), x)
```

GIAC/XCAS [A] time = 0.267539, size = 95, normalized size = 0.92

$$-\frac{1}{122880}\,(8\,(48\,(72\,(32\,x-71)x-17807)x-681893)x-1353611)\sqrt{12\,x^2+17\,x+6}$$
$$-\frac{97}{294912}\,\sqrt{3}\ln\left(\left|-4\,\sqrt{3}\left(2\,\sqrt{3}x-\sqrt{12\,x^2+17\,x+6}\right)-17\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(12*x^2 + 17*x + 6)*(12*x^2 - 31*x - 30)*(3*x + 2),x, algorithm="gi
```

```
[Out] -1/122880*(8*(48*(72*(32*x - 71)*x - 17807)*x - 681893)*x - 13536
11)*sqrt(12*x^2 + 17*x + 6) - 97/294912*sqrt(3)*ln(abs(-4*sqrt(3)
*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6)) - 17))
```

3.135 $\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)(30+31x-12x^2)}\,dx$

Optimal. Leaf size=28

$$\frac{1}{42}\tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)$$

```
[Out] ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x + 12*x^2])]/42
```

Rubi [A] time = 0.117939, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$

$$\frac{1}{42}\tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)*(30 + 31*x - 12*x^2)),x]
```

```
[Out] ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x + 12*x^2])]/42
```

Rubi in Sympy [A] time = 23.1493, size = 26, normalized size = 0.93

$$-\frac{\operatorname{atanh}\left(\frac{-291x-206}{84\sqrt{12x^2+17x+6}}\right)}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)/(-12*x**2+31*x+30),x)
```

```
[Out] -atanh((-291*x - 206)/(84*sqrt(12*x**2 + 17*x + 6)))/42
```

Mathematica [A] time = 0.0269163, size = 37, normalized size = 1.32

$$\frac{1}{42}\log\left(84\sqrt{12x^2+17x+6}+291x+206\right)-\frac{1}{42}\log(10-3x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)*(30 + 31*x - 12*x^2)),x]
```

```
[Out] -Log[10 - 3*x]/42 + Log[206 + 291*x + 84*Sqrt[6 + 17*x + 12*x^2]]
/42
```

Maple [B] time = 0.021, size = 163, normalized size = 5.8

$$\frac{1}{12}\sqrt{12\,(x+2/3)^2+x+\frac{2}{3}}+\frac{\sqrt{12}}{288}\ln\left(\frac{\sqrt{12}}{12}\left(\frac{17}{2}+12\,x\right)+\sqrt{12\,(x+2/3)^2+x+\frac{2}{3}}\right)$$
$$-\frac{1}{588}\sqrt{12\,(x-10/3)^2+97\,x-\frac{382}{3}}$$
$$-\frac{97\sqrt{12}}{14112}\ln\left(\frac{\sqrt{12}}{12}\left(\frac{17}{2}+12\,x\right)+\sqrt{12\,(x-10/3)^2+97\,x-\frac{382}{3}}\right)$$
$$+\frac{1}{42}Artanh\left(\frac{1}{28}\left(\frac{206}{3}+97\,x\right)\frac{1}{\sqrt{12\,(x-10/3)^2+97\,x-\frac{382}{3}}}\right)$$
$$-\frac{4}{49}\sqrt{12\,(x+3/4)^2-x-\frac{3}{4}}+\frac{\sqrt{12}}{294}\ln\left(\frac{\sqrt{12}}{12}\left(\frac{17}{2}+12\,x\right)+\sqrt{12\,(x+3/4)^2-x-\frac{3}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x)
```

```
[Out] 1/12*(12*(x+2/3)^2+x+2/3)^(1/2)+1/288*ln(1/12*(17/2+12*x)*12^(1/2
)+(12*(x+2/3)^2+x+2/3)^(1/2))*12^(1/2)-1/588*(12*(x-10/3)^2+97*x-
382/3)^(1/2)-97/14112*ln(1/12*(17/2+12*x)*12^(1/2)+(12*(x-10/3)^2
+97*x-382/3)^(1/2))*12^(1/2)+1/42*arctanh(1/28*(206/3+97*x)/(12*(
x-10/3)^2+97*x-382/3)^(1/2))-4/49*(12*(x+3/4)^2-x-3/4)^(1/2)+1/29
4*ln(1/12*(17/2+12*x)*12^(1/2)+(12*(x+3/4)^2-x-3/4)^(1/2))*12^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{12\,x^2+17\,x+6}}{(12\,x^2-31\,x-30)(3\,x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)*(3*x + 2)),x, algorithm='
```

```
[Out] -integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)*(3*x + 2
)), x)
```

Fricas [A] time = 0.294773, size = 72, normalized size = 2.57

$$\frac{1}{84}\log\left(\frac{291\,x+84\sqrt{12\,x^2+17\,x+6}+206}{x}\right)-\frac{1}{84}\log\left(\frac{291\,x-84\sqrt{12\,x^2+17\,x+6}+206}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)*(3*x + 2)),x, algorithm='
```

```
[Out] 1/84*log((291*x + 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) - 1/84*log
((291*x - 84*sqrt(12*x^2 + 17*x + 6) + 206)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{12x^2+17x+6}}{36x^3-69x^2-152x-60}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)/(-12*x**2+31*x+30),x)
```

```
[Out] -Integral(sqrt(12*x**2 + 17*x + 6)/(36*x**3 - 69*x**2 - 152*x - 6
0), x)
```

GIAC/XCAS [A] time = 0.281468, size = 85, normalized size = 3.04

$$\frac{1}{42}\ln\left(\left|-6\sqrt{3}x+20\sqrt{3}+3\sqrt{12x^2+17x+6}+42\right|\right)-\frac{1}{42}\ln\left(\left|-6\sqrt{3}x+20\sqrt{3}+3\sqrt{12x^2+17x+6}-42\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)*(3*x + 2)),x, algorithm='
```

```
[Out] 1/42*ln(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6)
 + 42)) - 1/42*ln(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 +
 17*x + 6) - 42))
```

3.136 $\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2(30+31x-12x^2)^2}\,dx$

Optimal. Leaf size=84

$$-\frac{388x+275}{98(10-3x)\sqrt{12x^2+17x+6}}+\frac{3137\sqrt{12x^2+17x+6}}{38416(10-3x)}+\frac{97\tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{3226944}$$

```
[Out] -(275 + 388*x)/(98*(10 - 3*x)*Sqrt[6 + 17*x + 12*x^2]) + (3137*Sq
rt[6 + 17*x + 12*x^2])/(38416*(10 - 3*x)) + (97*ArcTanh[(206 + 29
1*x)/(84*Sqrt[6 + 17*x + 12*x^2])])/3226944
```

Rubi [A] time = 0.209075, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$

$$-\frac{388x+275}{98(10-3x)\sqrt{12x^2+17x+6}}+\frac{3137\sqrt{12x^2+17x+6}}{38416(10-3x)}+\frac{97\tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{3226944}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^2*(30 + 31*x - 12*x^2)^2),x]
```

```
[Out] -(275 + 388*x)/(98*(10 - 3*x)*Sqrt[6 + 17*x + 12*x^2]) + (3137*Sq
rt[6 + 17*x + 12*x^2])/(38416*(10 - 3*x)) + (97*ArcTanh[(206 + 29
1*x)/(84*Sqrt[6 + 17*x + 12*x^2])])/3226944
```

Rubi in Sympy [A] time = 32.0861, size = 71, normalized size = 0.85

$$-\frac{97\,\mathrm{atanh}\left(\frac{-291x-206}{84\sqrt{12x^2+17x+6}}\right)}{3226944}-\frac{3492x+2475}{882\,(-3x+10)\,\sqrt{12x^2+17x+6}}+\frac{3137\sqrt{12x^2+17x+6}}{38416\,(-3x+10)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)**2/(-12*x**2+31*x+30)**2,x)
```

```
[Out] -97*atanh((-291*x - 206)/(84*sqrt(12*x**2 + 17*x + 6)))/3226944 -
 (3492*x + 2475)/(882*(-3*x + 10)*sqrt(12*x**2 + 17*x + 6)) + 313
7*sqrt(12*x**2 + 17*x + 6)/(38416*(-3*x + 10))
```

Mathematica [A] time = 0.23998, size = 93, normalized size = 1.11

$$\frac{\sqrt{12x^2+17x+6}\left(\frac{97\tanh^{-1}\left(\frac{7\sqrt{3x+2}}{6\sqrt{4x+3}}\right)}{\sqrt{3x+2}\sqrt{4x+3}}-\frac{42(37644x^2-98767x-88978)}{(3x+2)(12x^2-31x-30)}\right)}{1613472}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^2*(30 + 31*x - 12*x^2)^2),x]
```

```
[Out] (Sqrt[6 + 17*x + 12*x^2]*((-42*(-88978 - 98767*x + 37644*x^2))/((
2 + 3*x)*(-30 - 31*x + 12*x^2)) + (97*ArcTanh[(7*Sqrt[2 + 3*x])/(
6*Sqrt[3 + 4*x])])/(Sqrt[2 + 3*x]*Sqrt[3 + 4*x])))/1613472
```

Maple [B] time = 0.027, size = 245, normalized size = 2.9

$$-\frac{1}{72}\left(12\,(x+2/3)^2+x+\frac{2}{3}\right)^{\frac{3}{2}}\left(x+\frac{2}{3}\right)^{-2}+\frac{1}{288}\sqrt{12\,(x+2/3)^2+x+\frac{2}{3}}+\frac{\sqrt{12}}{6912}\ln\left(\frac{\sqrt{12}}{12}\left(\frac{17}{2}+12\,x\right)+\sqrt{12\,(x+2/3)^2+x+\frac{2}{3}}\right)-\frac{1}{67765824}\left(12\,(x-10/3)^2+97\,x-\frac{382}{3}\right)^{\frac{3}{2}}\left(x-\frac{10}{3}\right)^{-1}-\frac{97}{45177216}\sqrt{12\,(x-10/3)^2+97\,x-\frac{382}{3}}-\frac{7057\sqrt{12}}{813189888}\ln\left(\frac{\sqrt{12}}{12}\left(\frac{17}{2}+12\,x\right)+\sqrt{12\,(x-10/3)^2+97\,x-\frac{382}{3}}\right)+\frac{97}{3226944}\mathit{Artanh}\left(\frac{1}{28}\left(\frac{206}{3}+97\,x\right)\frac{1}{\sqrt{12\,(x-10/3)^2+97\,x-\frac{382}{3}}}\right)+\frac{17+24\,x}{135531648}\sqrt{12\,(x-10/3)^2+97\,x-\frac{382}{3}}+\frac{32}{2401}\left(12\,(x+3/4)^2-x-\frac{3}{4}\right)^{\frac{3}{2}}\left(x+\frac{3}{4}\right)^{-2}+\frac{384}{117649}\sqrt{12\,(x+3/4)^2-x-\frac{3}{4}}-\frac{16\sqrt{12}}{117649}\ln\left(\frac{\sqrt{12}}{12}\left(\frac{17}{2}+12\,x\right)+\sqrt{12\,(x+3/4)^2-x-\frac{3}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x)
```

```
[Out] -1/72/(x+2/3)^2*(12*(x+2/3)^2+x+2/3)^(3/2)+1/288*(12*(x+2/3)^2+x+
2/3)^(1/2)+1/6912*ln(1/12*(17/2+12*x)*12^(1/2)+(12*(x+2/3)^2+x+2/
3)^(1/2))*12^(1/2)-1/67765824/(x-10/3)*(12*(x-10/3)^2+97*x-382/3)
^(3/2)-97/45177216*(12*(x-10/3)^2+97*x-382/3)^(1/2)-7057/81318988
8*ln(1/12*(17/2+12*x)*12^(1/2)+(12*(x-10/3)^2+97*x-382/3)^(1/2))*
12^(1/2)+97/3226944*arctanh(1/28*(206/3+97*x)/(12*(x-10/3)^2+97*x
-382/3)^(1/2))+1/135531648*(17+24*x)*(12*(x-10/3)^2+97*x-382/3)^(
1/2)+32/2401/(x+3/4)^2*(12*(x+3/4)^2-x-3/4)^(3/2)+384/117649*(12*
(x+3/4)^2-x-3/4)^(1/2)-16/117649*ln(1/12*(17/2+12*x)*12^(1/2)+(12
*(x+3/4)^2-x-3/4)^(1/2))*12^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{12\,x^2+17\,x+6}}{(12\,x^2-31\,x-30)^2(3\,x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^2*(3*x + 2)^2),x, algorith
```

```
[Out] integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^2*(3*x +
2)^2), x)
```

Fricas [A] time = 0.289709, size = 170, normalized size = 2.02

$$\frac{97\left(36\,x^3-69\,x^2-152\,x-60\right)\log\left(\frac{291\,x+84\sqrt{12\,x^2+17\,x+6}+206}{x}\right)-97\left(36\,x^3-69\,x^2-152\,x-60\right)\log\left(\frac{291\,x-84\sqrt{12\,x^2+17\,x+6}+206}{x}\right.}{6453888\left(36\,x^3-69\,x^2-152\,x-60\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^2*(3*x + 2)^2),x, algorith
```

```
[Out] 1/6453888*(97*(36*x^3 - 69*x^2 - 152*x - 60)*log((291*x + 84*sqrt
(12*x^2 + 17*x + 6) + 206)/x) - 97*(36*x^3 - 69*x^2 - 152*x - 60)
*log((291*x - 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) - 168*(37644*x
^2 - 98767*x - 88978)*sqrt(12*x^2 + 17*x + 6))/(36*x^3 - 69*x^2 -
 152*x - 60)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(3x+2)(4x+3)}}{(3x-10)^2(3x+2)^2(4x+3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)**2/(-12*x**2+31*x+30)**2,x)
```

```
[Out] Integral(sqrt((3*x + 2)*(4*x + 3))/((3*x - 10)**2*(3*x + 2)**2*(4
*x + 3)**2), x)
```

GIAC/XCAS [A] time = 0.289936, size = 215, normalized size = 2.56

$$\frac{1}{9680832}\sqrt{3}\left(\sqrt{3}\left(175672\sqrt{3}+97\ln\left(\frac{7\sqrt{3}-12}{7\sqrt{3}+12}\right)\right)\operatorname{sign}\left(\frac{1}{3x+2}\right)-\left(97\sqrt{3}\ln\left(\frac{\left|-28\sqrt{3}+24\sqrt{\frac{1}{3x+2}+4}\right|}{4\left(7\sqrt{3}+6\sqrt{\frac{1}{3x+2}+4}\right)}\right)+134456\sqrt{\frac{1}{3x+\cdots}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^2*(3*x + 2)^2),x, algorith
```

```
[Out] 1/9680832*sqrt(3)*(sqrt(3)*(175672*sqrt(3) + 97*ln((7*sqrt(3) - 1
2)/(7*sqrt(3) + 12)))*sign(1/(3*x + 2)) - (97*sqrt(3)*ln(1/4*abs(
-28*sqrt(3) + 24*sqrt(1/(3*x + 2) + 4))/(7*sqrt(3) + 6*sqrt(1/(3*
x + 2) + 4))) + 134456*sqrt(1/(3*x + 2) + 4) + 28*(221183/(3*x +
2) - 18436)/(12*(1/(3*x + 2) + 4)^(3/2) - 49*sqrt(1/(3*x + 2) + 4
)))*sign(1/(3*x + 2)))
```

3.137 $\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3(30+31x-12x^2)^3}\,dx$

Optimal. Leaf size=139

$$-\frac{388x+275}{294(10-3x)^2\left(12x^2+17x+6\right)^{3/2}}-\frac{1634466587\sqrt{12x^2+17x+6}}{7589772288(10-3x)}-\frac{50555899\sqrt{12x^2+17x+6}}{19361664(10-3x)^2}+\frac{1042556x+738029}{8232(10-3x)^2\sqrt{12x^2+17x+6}}+\frac{40325\tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{637540872192}$$

```
[Out] -(275 + 388*x)/(294*(10 - 3*x)^2*(6 + 17*x + 12*x^2)^(3/2)) + (73
8029 + 1042556*x)/(8232*(10 - 3*x)^2*Sqrt[6 + 17*x + 12*x^2]) - (
50555899*Sqrt[6 + 17*x + 12*x^2])/(19361664*(10 - 3*x)^2) - (1634
466587*Sqrt[6 + 17*x + 12*x^2])/(7589772288*(10 - 3*x)) + (40325*
ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x + 12*x^2])])/637540872192
```

Rubi [A] time = 0.357293, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206

$$-\frac{388x+275}{294(10-3x)^2\left(12x^2+17x+6\right)^{3/2}}-\frac{1634466587\sqrt{12x^2+17x+6}}{7589772288(10-3x)}-\frac{50555899\sqrt{12x^2+17x+6}}{19361664(10-3x)^2}+\frac{1042556x+738029}{8232(10-3x)^2\sqrt{12x^2+17x+6}}+\frac{40325\tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{637540872192}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^3*(30 + 31*x - 12*x^2)^3),x]
```

```
[Out] -(275 + 388*x)/(294*(10 - 3*x)^2*(6 + 17*x + 12*x^2)^(3/2)) + (73
8029 + 1042556*x)/(8232*(10 - 3*x)^2*Sqrt[6 + 17*x + 12*x^2]) - (
50555899*Sqrt[6 + 17*x + 12*x^2])/(19361664*(10 - 3*x)^2) - (1634
466587*Sqrt[6 + 17*x + 12*x^2])/(7589772288*(10 - 3*x)) + (40325*
ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x + 12*x^2])])/637540872192
```

Rubi in Sympy [A] time = 47.0506, size = 126, normalized size = 0.91

$$-\frac{40325\,\mathrm{atanh}\left(\frac{-291x-206}{84\sqrt{12x^2+17x+6}}\right)}{637540872192}-\frac{1634466587\sqrt{12x^2+17x+6}}{7589772288\left(-3x+10\right)}-\frac{3492x+2475}{2646\left(-3x+10\right)^2\left(12x^2+17x+6\right)^{\frac{3}{2}}}+\frac{295564626x+\frac{418462443}{2}}{2333772\left(-3x+10\right)^2\sqrt{12x^2+17x+6}}-\frac{50555899\sqrt{12x^2+17x+6}}{19361664\left(-3x+10\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)**3/(-12*x**2+31*x+30)**3,x)
```

```
[Out] -40325*atanh((-291*x - 206)/(84*sqrt(12*x**2 + 17*x + 6)))/637540
872192 - 1634466587*sqrt(12*x**2 + 17*x + 6)/(7589772288*(-3*x +
10)) - (3492*x + 2475)/(2646*(-3*x + 10)**2*(12*x**2 + 17*x + 6)*
*(3/2)) + (295564626*x + 418462443/2)/(2333772*(-3*x + 10)**2*sqr
t(12*x**2 + 17*x + 6)) - 50555899*sqrt(12*x**2 + 17*x + 6)/(19361
664*(-3*x + 10)**2)
```

Mathematica [A] time = 0.18735, size = 129, normalized size = 0.93

$$\frac{\sqrt{12x^2+17x+6}\left(40325(3x+2)^{3/2}\left(-12x^2+31x+30\right)^2\tanh^{-1}\left(\frac{7\sqrt{3x+2}}{6\sqrt{4x+3}}\right)+42\sqrt{4x+3}\left(706089565584x^5-3206824169544x\right.\right.}{318770436096(3x+2)^2\sqrt{4x+3}\left(-12x^2+31x\right.}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^3*(30 + 31*x - 12*x^2)^3),x]
```

```
[Out] (Sqrt[6 + 17*x + 12*x^2]*(42*Sqrt[3 + 4*x]*(2773753482408 + 10124
325497244*x + 9848047480070*x^2 - 1096520427663*x^3 - 32068241695
44*x^4 + 706089565584*x^5) + 40325*(2 + 3*x)^(3/2)*(30 + 31*x - 1
2*x^2)^2*ArcTanh[(7*Sqrt[2 + 3*x])/(6*Sqrt[3 + 4*x])]))/(31877043
6096*(2 + 3*x)^2*Sqrt[3 + 4*x]*(30 + 31*x - 12*x^2)^2)
```

Maple [B] time = 0.032, size = 306, normalized size = 2.2

$$-\frac{1}{2592}\left(12\left(x+2/3\right)^2+x+\frac{2}{3}\right)^{\frac{3}{2}}\left(x+\frac{2}{3}\right)^{-3}+\frac{47}{1152}\left(12\left(x+2/3\right)^2+x+\frac{2}{3}\right)^{\frac{3}{2}}\left(x+\frac{2}{3}\right)^{-2}-\frac{23}{4608}\sqrt{12\left(x+2/3\right)^2+x+\frac{2}{3}}$$
$$-\frac{23\sqrt{12}}{110592}\ln\left(\frac{\sqrt{12}}{12}\left(\frac{17}{2}+12x\right)+\sqrt{12\left(x+2/3\right)^2+x+\frac{2}{3}}\right)$$
$$+\frac{1}{79692609024}\left(12\left(x-10/3\right)^2+97x-\frac{382}{3}\right)^{\frac{3}{2}}\left(x-\frac{10}{3}\right)^{-2}$$
$$-\frac{1261}{31239502737408}\left(12\left(x-10/3\right)^2+97x-\frac{382}{3}\right)^{\frac{3}{2}}\left(x-\frac{10}{3}\right)^{-1}$$
$$-\frac{40325}{8925572210688}\sqrt{12\left(x-10/3\right)^2+97x-\frac{382}{3}}$$
$$+\frac{40325}{637540872192}\mathit{Artanh}\left(\frac{1}{28}\left(\frac{206}{3}+97x\right)\frac{1}{\sqrt{12\left(x-10/3\right)^2+97x-\frac{382}{3}}}\right)$$
$$+\frac{21437+30264x}{62479005474816}\sqrt{12\left(x-10/3\right)^2+97x-\frac{382}{3}}$$
$$-\frac{570457\sqrt{12}}{31239502737408}\ln\left(\frac{\sqrt{12}}{12}\left(\frac{17}{2}+12x\right)+\sqrt{12\left(x-10/3\right)^2+97x-\frac{382}{3}}\right)$$
$$-\frac{128}{352947}\left(12\left(x+3/4\right)^2-x-\frac{3}{4}\right)^{\frac{3}{2}}\left(x+\frac{3}{4}\right)^{-3}$$
$$-\frac{230400}{5764801}\left(12\left(x+3/4\right)^2-x-\frac{3}{4}\right)^{\frac{3}{2}}\left(x+\frac{3}{4}\right)^{-2}-\frac{1410048}{282475249}\sqrt{12\left(x+3/4\right)^2-x-\frac{3}{4}}$$
$$+\frac{58752\sqrt{12}}{282475249}\ln\left(\frac{\sqrt{12}}{12}\left(\frac{17}{2}+12x\right)+\sqrt{12\left(x+3/4\right)^2-x-\frac{3}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x)
```

```
[Out] -1/2592/(x+2/3)^3*(12*(x+2/3)^2+x+2/3)^(3/2)+47/1152/(x+2/3)^2*(1
2*(x+2/3)^2+x+2/3)^(3/2)-23/4608*(12*(x+2/3)^2+x+2/3)^(1/2)-23/11
0592*ln(1/12*(17/2+12*x)*12^(1/2)+(12*(x+2/3)^2+x+2/3)^(1/2))*12^
(1/2)+1/79692609024/(x-10/3)^2*(12*(x-10/3)^2+97*x-382/3)^(3/2)-1
261/31239502737408/(x-10/3)*(12*(x-10/3)^2+97*x-382/3)^(3/2)-4032
5/8925572210688*(12*(x-10/3)^2+97*x-382/3)^(1/2)+40325/6375408721
92*arctanh(1/28*(206/3+97*x)/(12*(x-10/3)^2+97*x-382/3)^(1/2))+12
61/62479005474816*(17+24*x)*(12*(x-10/3)^2+97*x-382/3)^(1/2)-5704
```

```
57/31239502737408*ln(1/12*(17/2+12*x)*12^(1/2)+(12*(x-10/3)^2+97*
x-382/3)^(1/2))*12^(1/2)-128/352947/(x+3/4)^3*(12*(x+3/4)^2-x-3/4
)^(3/2)-230400/5764801/(x+3/4)^2*(12*(x+3/4)^2-x-3/4)^(3/2)-14100
48/282475249*(12*(x+3/4)^2-x-3/4)^(1/2)+58752/282475249*ln(1/12*(
17/2+12*x)*12^(1/2)+(12*(x+3/4)^2-x-3/4)^(1/2))*12^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{12\,x^2+17\,x+6}}{\left(12\,x^2-31\,x-30\right)^3\left(3\,x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^3*(3*x + 2)^3),x, algorit
```

```
[Out] -integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^3*(3*x +
 2)^3), x)
```

Fricas [A] time = 0.294531, size = 251, normalized size = 1.81

$$\frac{40325\left(1296\,x^6-4968\,x^5-6183\,x^4+16656\,x^3+31384\,x^2+18240\,x+3600\right)\log\left(\frac{291\,x+84\,\sqrt{12\,x^2+17\,x+6}+206}{x}\right)-40325\left(1296\,x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^3*(3*x + 2)^3),x, algorit
```

```
[Out] 1/1275081744384*(40325*(1296*x^6 - 4968*x^5 - 6183*x^4 + 16656*x^
3 + 31384*x^2 + 18240*x + 3600)*log((291*x + 84*sqrt(12*x^2 + 17*
x + 6) + 206)/x) - 40325*(1296*x^6 - 4968*x^5 - 6183*x^4 + 16656*
x^3 + 31384*x^2 + 18240*x + 3600)*log((291*x - 84*sqrt(12*x^2 + 1
7*x + 6) + 206)/x) + 168*(706089565584*x^5 - 3206824169544*x^4 -
1096520427663*x^3 + 9848047480070*x^2 + 10124325497244*x + 277375
3482408)*sqrt(12*x^2 + 17*x + 6))/(1296*x^6 - 4968*x^5 - 6183*x^4
 + 16656*x^3 + 31384*x^2 + 18240*x + 3600)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{12x^2+17x+6}}{46656x^9-268272x^8-76788x^7+1703619x^6+1218456x^5-3669588x^4-6898688x^3-4903920x^2-1641600x-2160}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)**3/(-12*x**2+31*x+30)**3,x)
```

```
[Out] -Integral(sqrt(12*x**2 + 17*x + 6)/(46656*x**9 - 268272*x**8 - 76
788*x**7 + 1703619*x**6 + 1218456*x**5 - 3669588*x**4 - 6898688*x
**3 - 4903920*x**2 - 1641600*x - 216000), x)
```

GIAC/XCAS [A] time = 0.289471, size = 313, normalized size = 2.25

$$\frac{\sqrt{3}\left(282273\,\sqrt{3}\left(2\,\sqrt{3}x-\sqrt{12\,x^2+17\,x+6}\right)^3-11460924\left(2\,\sqrt{3}x-\sqrt{12\,x^2+17\,x+6}\right)^2-37551180\,\sqrt{3}\left(2\,\sqrt{3}x-\sqrt{12\,x^2+1\right.\right.}{159385218048\left(3\left(2\,\sqrt{3}x-\sqrt{12\,x^2+17\,x+6}\right)^2-40\,\sqrt{3}\left(2\,\sqrt{3}x-\sqrt{12\,x^2+17\,x+6}\right)-188\right)^2}$$
$$+\frac{(8\,(2860316794\,x+6078171227)x+34383350229)x+8090114146}{2213683584\,(12\,x^2+17\,x+6)^{\frac{3}{2}}}$$
$$+\frac{40325}{637540872192}\ln\left(\left|-6\,\sqrt{3}x+20\,\sqrt{3}+3\,\sqrt{12\,x^2+17\,x+6}+42\right|\right)$$
$$-\frac{40325}{637540872192}\ln\left(\left|-6\,\sqrt{3}x+20\,\sqrt{3}+3\,\sqrt{12\,x^2+17\,x+6}-42\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^3*(3*x + 2)^3),x, algorit
```

```
[Out] 1/159385218048*sqrt(3)*(282273*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2
 + 17*x + 6))^3 - 11460924*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6)
)^2 - 37551180*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6)) -
83365264)/(3*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6))^2 - 40*sqrt(
3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6)) - 188)^2 + 1/221368358
4*((8*(2860316794*x + 6078171227)*x + 34383350229)*x + 8090114146
)/(12*x^2 + 17*x + 6)^(3/2) + 40325/637540872192*ln(abs(-6*sqrt(3
)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) + 42)) - 40325/63754
0872192*ln(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x +
 6) - 42))
```

3.138 $\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx$

Optimal. Leaf size=15

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

```
[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rubi [A] time = 0.00849011, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In]  Int[(-3 + 2*x)*(-3*x + x^2)^(2/3),x]

[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rubi in Sympy [A] time = 2.59438, size = 12, normalized size = 0.8

$$\frac{3\left(x^2-3x\right)^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-3+2*x)*(x**2-3*x)**(2/3),x)

[Out] 3*(x**2 - 3*x)**(5/3)/5
```

Mathematica [A] time = 0.0165649, size = 13, normalized size = 0.87

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-3 + 2*x)*(-3*x + x^2)^(2/3),x]

[Out] (3*((-3 + x)*x)^(5/3))/5
```

Maple [A] time = 0.005, size = 16, normalized size = 1.1

$$\frac{(-9+3\,x)\,x}{5}\left(x^2-3\,x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-3+2*x)*(x^2-3*x)^(2/3),x)
```

```
[Out] 3/5*(-3+x)*x*(x^2-3*x)^(2/3)
```

Maxima [A] time = 0.715396, size = 15, normalized size = 1.

$$\frac{3}{5}\left(x^2 - 3x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 3*x)^(2/3)*(2*x - 3),x, algorithm="maxima")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

Fricas [A] time = 0.27612, size = 15, normalized size = 1.

$$\frac{3}{5}\left(x^2 - 3x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 3*x)^(2/3)*(2*x - 3),x, algorithm="fricas")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

Sympy [A] time = 0.406947, size = 31, normalized size = 2.07

$$\frac{3x^2\left(x^2 - 3x\right)^{\frac{2}{3}}}{5} - \frac{9x\left(x^2 - 3x\right)^{\frac{2}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3+2*x)*(x**2-3*x)**(2/3),x)
```

```
[Out] 3*x**2*(x**2 - 3*x)**(2/3)/5 - 9*x*(x**2 - 3*x)**(2/3)/5
```

GIAC/XCAS [A] time = 0.270857, size = 15, normalized size = 1.

$$\frac{3}{5}\left(x^2 - 3x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 3*x)^(2/3)*(2*x - 3),x, algorithm="giac")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

3.139 $\int ((-3+x)x)^{2/3}(-3+2x)\,dx$

Optimal. Leaf size=16

$$\frac{3}{5}(-(3-x)x)^{5/3}$$

```
[Out] (3*(-((3 - x)*x))^(5/3))/5
```

Rubi [A] time = 0.00818837, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{3}{5}(-(3-x)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + x)*x)^(2/3)*(-3 + 2*x),x]

[Out] (3*(-((3 - x)*x))^(5/3))/5
```

Rubi in Sympy [A] time = 1.90351, size = 10, normalized size = 0.62

$$\frac{3\,(x\,(x-3))^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(((-3+x)*x)**(2/3)*(-3+2*x),x)

[Out] 3*(x*(x - 3))**(5/3)/5
```

Mathematica [A] time = 0.00920367, size = 13, normalized size = 0.81

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + x)*x)^(2/3)*(-3 + 2*x),x]

[Out] (3*((-3 + x)*x)^(5/3))/5
```

Maple [A] time = 0.004, size = 14, normalized size = 0.9

$$\frac{(-9+3\,x)\,x}{5}\,((-3+x)\,x)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-3+x)*x)^(2/3)*(-3+2*x),x)
```

```
[Out] 3/5*(-3+x)*x*((-3+x)*x)^(2/3)
```

Maxima [A] time = 0.701904, size = 12, normalized size = 0.75

$$\frac{3}{5}\left((x-3)x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x - 3)*x)^(2/3)*(2*x - 3),x, algorithm="maxima")
```

```
[Out] 3/5*((x - 3)*x)^(5/3)
```

Fricas [A] time = 0.274239, size = 15, normalized size = 0.94

$$\frac{3}{5}\left(x^2-3x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x - 3)*x)^(2/3)*(2*x - 3),x, algorithm="fricas")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

Sympy [A] time = 15.1847, size = 10, normalized size = 0.62

$$\frac{3\left(x\left(x-3\right)\right)^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((-3+x)*x)**(2/3)*(-3+2*x),x)
```

```
[Out] 3*(x*(x - 3))**(5/3)/5
```

GIAC/XCAS [A] time = 0.270977, size = 15, normalized size = 0.94

$$\frac{3}{5}\left(x^2-3x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x - 3)*x)^(2/3)*(2*x - 3),x, algorithm="giac")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

3.140 $\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

```
[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rubi [A] time = 0.0448619, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(9 - 9*x + 2*x^2))/(-3*x + x^2)^(1/3),x]
```

```
[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rubi in Sympy [A] time = 9.68523, size = 12, normalized size = 0.8

$$\frac{3\left(x^2-3x\right)^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(2*x**2-9*x+9)/(x**2-3*x)**(1/3),x)
```

```
[Out] 3*(x**2 - 3*x)**(5/3)/5
```

Mathematica [A] time = 0.0135119, size = 13, normalized size = 0.87

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(9 - 9*x + 2*x^2))/(-3*x + x^2)^(1/3),x]
```

```
[Out] (3*((-3 + x)*x)^(5/3))/5
```

Maple [A] time = 0.005, size = 20, normalized size = 1.3

$$\frac{3\,(-3+x)^2\,x^2}{5}\frac{1}{\sqrt[3]{x^2-3\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x)
```

```
[Out] 3/5*(-3+x)^2*x^2/(x^2-3*x)^(1/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^2 - 9x + 9)x}{(x^2 - 3x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2 - 9*x + 9)*x/(x^2 - 3*x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - 9*x + 9)*x/(x^2 - 3*x)^(1/3), x)
```

Fricas [A] time = 0.270231, size = 15, normalized size = 1.

$$\frac{3}{5}\left(x^2 - 3x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2 - 9*x + 9)*x/(x^2 - 3*x)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x(x-3)(2x-3)}{\sqrt[3]{x(x-3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(2*x**2-9*x+9)/(x**2-3*x)**(1/3),x)
```

```
[Out] Integral(x*(x - 3)*(2*x - 3)/(x*(x - 3))**(1/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^2 - 9x + 9)x}{(x^2 - 3x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2 - 9*x + 9)*x/(x^2 - 3*x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((2*x^2 - 9*x + 9)*x/(x^2 - 3*x)^(1/3), x)
```

3.141 $\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}}\,dx$

Optimal. Leaf size=15

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

```
[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rubi [A] time = 0.0932254, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(9 - 9*x + 2*x^2))/((-3 + x)*x)^(1/3),x]

[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rubi in Sympy [A] time = 11.9661, size = 12, normalized size = 0.8

$$\frac{3\left(x^2-3x\right)^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(2*x**2-9*x+9)/((-3+x)*x)**(1/3),x)

[Out] 3*(x**2 - 3*x)**(5/3)/5
```

Mathematica [A] time = 0.0103182, size = 13, normalized size = 0.87

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(9 - 9*x + 2*x^2))/((-3 + x)*x)^(1/3),x]

[Out] (3*((-3 + x)*x)^(5/3))/5
```

Maple [A] time = 0.005, size = 18, normalized size = 1.2

$$\frac{3\,(-3+x)^2\,x^2}{5}\frac{1}{\sqrt[3]{(-3+x)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x)
```

```
[Out] 3/5*(-3+x)^2*x^2/((-3+x)*x)^(1/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^2 - 9x + 9)x}{((x-3)x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2 - 9*x + 9)*x/((x - 3)*x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - 9*x + 9)*x/((x - 3)*x)^(1/3), x)
```

Fricas [A] time = 0.278864, size = 15, normalized size = 1.

$$\frac{3}{5}\left(x^2 - 3x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2 - 9*x + 9)*x/((x - 3)*x)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x(x-3)(2x-3)}{\sqrt[3]{x(x-3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(2*x**2-9*x+9)/((-3+x)*x)**(1/3),x)
```

```
[Out] Integral(x*(x - 3)*(2*x - 3)/(x*(x - 3))**(1/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^2 - 9x + 9)x}{((x-3)x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 - 9*x + 9)*x/((x - 3)*x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((2*x^2 - 9*x + 9)*x/((x - 3)*x)^(1/3), x)
```

3.142 $\displaystyle\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\,(g^2+3h^2x^2)}\,dx$

Optimal. Leaf size=242

$$\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(g^2+3h^2x^2\right)}{6\ 2^{2/3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}-\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(\left(1-\frac{3hx}{g}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{\frac{3hx}{g}+1}\right)}{2\ 2^{2/3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}+\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3hx}{g}\right)^{2/3}}{\sqrt{3}\sqrt[3]{\frac{3hx}{g}+1}}\right)}{2^{2/3}\sqrt{3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}$$

```
[Out] ((1 - (9*h^2*x^2)/g^2)^(1/3)*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*
h*x)/g)^(2/3))/(Sqrt[3]*(1 + (3*h*x)/g)^(1/3))])/(2^(2/3)*Sqrt[3]
*h*(-((c*g^2)/h^2) + 9*c*x^2)^(1/3)) + ((1 - (9*h^2*x^2)/g^2)^(1/
3)*Log[g^2 + 3*h^2*x^2])/(6*2^(2/3)*h*(-((c*g^2)/h^2) + 9*c*x^2)^
(1/3)) - ((1 - (9*h^2*x^2)/g^2)^(1/3)*Log[(1 - (3*h*x)/g)^(2/3) +
 2^(1/3)*(1 + (3*h*x)/g)^(1/3)])/(2*2^(2/3)*h*(-((c*g^2)/h^2) + 9
*c*x^2)^(1/3))
```

Rubi [A] time = 0.344257, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(g^2+3h^2x^2\right)}{6\ 2^{2/3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}-\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(\left(1-\frac{3hx}{g}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{\frac{3hx}{g}+1}\right)}{2\ 2^{2/3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}+\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3hx}{g}\right)^{2/3}}{\sqrt{3}\sqrt[3]{\frac{3hx}{g}+1}}\right)}{2^{2/3}\sqrt{3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)/((-((c*g^2)/h^2) + 9*c*x^2)^(1/3)*(g^2 + 3*h^2*x^2)),x]
```


```
[Out] ((1 - (9*h^2*x^2)/g^2)^(1/3)*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*
h*x)/g)^(2/3))/(Sqrt[3]*(1 + (3*h*x)/g)^(1/3))])/(2^(2/3)*Sqrt[3]
*h*(-((c*g^2)/h^2) + 9*c*x^2)^(1/3)) + ((1 - (9*h^2*x^2)/g^2)^(1/
3)*Log[g^2 + 3*h^2*x^2])/(6*2^(2/3)*h*(-((c*g^2)/h^2) + 9*c*x^2)^
(1/3)) - ((1 - (9*h^2*x^2)/g^2)^(1/3)*Log[(1 - (3*h*x)/g)^(2/3) +
 2^(1/3)*(1 + (3*h*x)/g)^(1/3)])/(2*2^(2/3)*h*(-((c*g^2)/h^2) + 9
*c*x^2)^(1/3))
```

Rubi in Sympy [A] time = 29.7888, size = 224, normalized size = 0.93

$$\frac{\sqrt[3]{2}\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(g^2+3h^2x^2\right)}{12h\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}}-\frac{\sqrt[3]{2}\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(\left(1-\frac{3hx}{g}\right)^{\frac{2}{3}}+\sqrt[3]{2}\sqrt[3]{1+\frac{3hx}{g}}\right)}{4h\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}}-\frac{\sqrt[3]{2}\sqrt{3}\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\operatorname{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}\left(1-\frac{3hx}{g}\right)^{\frac{2}{3}}}{3\sqrt[3]{1+\frac{3hx}{g}}}-\frac{\sqrt{3}}{3}\right)}{6h\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x+g)/(-c*g**2/h**2+9*c*x**2)**(1/3)/(3*h**2*x**2+g**2),x)
```

```
[Out] 2**(1/3)*(1 - 9*h**2*x**2/g**2)**(1/3)*log(g**2 + 3*h**2*x**2)/(1
2*h*(-c*g**2/h**2 + 9*c*x**2)**(1/3)) - 2**(1/3)*(1 - 9*h**2*x**2
/g**2)**(1/3)*log((1 - 3*h*x/g)**(2/3) + 2**(1/3)*(1 + 3*h*x/g)**
(1/3))/(4*h*(-c*g**2/h**2 + 9*c*x**2)**(1/3)) - 2**(1/3)*sqrt(3)*
(1 - 9*h**2*x**2/g**2)**(1/3)*atan(2**(2/3)*sqrt(3)*(1 - 3*h*x/g)
**(2/3)/(3*(1 + 3*h*x/g)**(1/3)) - sqrt(3)/3)/(6*h*(-c*g**2/h**2
+ 9*c*x**2)**(1/3))
```

Mathematica [C] time = 1.0381, size = 331, normalized size = 1.37

$$\frac{g^2x\left(\frac{gF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)}{g^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)+2h^2x^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)\right)}-\frac{hxF_1\left(1;\frac{1}{3},1;2;\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)}{3h^2x^2\left(F_1\left(2;\frac{1}{3},2;3;\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)-F_1\left(2;\frac{4}{3},1;3;\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)\right)-2g^2F_1\left(1;\frac{1}{3},1;2;\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)}\right)}{\left(g^2+3h^2x^2\right)\sqrt[3]{c\left(9x^2-\frac{g^2}{h^2}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g + h*x)/((-((c*g^2)/h^2) + 9*c*x^2)^(1/3)*(g^2 + 3*h^2*x^2)),x]
```

```
[Out] (g^2*x*((g*AppellF1[1/2, 1/3, 1, 3/2, (9*h^2*x^2)/g^2, (-3*h^2*x^
2)/g^2])/(g^2*AppellF1[1/2, 1/3, 1, 3/2, (9*h^2*x^2)/g^2, (-3*h^2
*x^2)/g^2] + 2*h^2*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (9*h^2*x^2)/g
^2, (-3*h^2*x^2)/g^2] + AppellF1[3/2, 4/3, 1, 5/2, (9*h^2*x^2)/g^
2, (-3*h^2*x^2)/g^2])) - (h*x*AppellF1[1, 1/3, 1, 2, (9*h^2*x^2)/
g^2, (-3*h^2*x^2)/g^2])/(-2*g^2*AppellF1[1, 1/3, 1, 2, (9*h^2*x^2
)/g^2, (-3*h^2*x^2)/g^2] + 3*h^2*x^2*(AppellF1[2, 1/3, 2, 3, (9*h
^2*x^2)/g^2, (-3*h^2*x^2)/g^2] - AppellF1[2, 4/3, 1, 3, (9*h^2*x^
2)/g^2, (-3*h^2*x^2)/g^2]))))/((c*(-(g^2/h^2) + 9*x^2))^(1/3)*(g^
2 + 3*h^2*x^2))
```

Maple [F] time = 0.314, size = 0, normalized size = 0.

$$\int \frac{hx+g}{3\,h^2x^2+g^2}\frac{1}{\sqrt[3]{-\frac{cg^2}{h^2}+9\,cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x)
```

```
[Out] int((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{hx+g}{(3\,h^2x^2+g^2)\left(9\,cx^2-\frac{cg^2}{h^2}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x + g)/((3*h^2*x^2 + g^2)*(9*c*x^2 - c*g^2/h^2)^(1/3)),x, algorithm
```

```
[Out] integrate((h*x + g)/((3*h^2*x^2 + g^2)*(9*c*x^2 - c*g^2/h^2)^(1/3
)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x + g)/((3*h^2*x^2 + g^2)*(9*c*x^2 - c*g^2/h^2)^(1/3)),x, algorithm
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{g+hx}{\sqrt[3]{c\left(-\frac{g}{h}+3x\right)\left(\frac{g}{h}+3x\right)}\,(g^2+3h^2x^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x+g)/(-c*g**2/h**2+9*c*x**2)**(1/3)/(3*h**2*x**2+g**2),x)
```

```
[Out] Integral((g + h*x)/((c*(-g/h + 3*x)*(g/h + 3*x))**(1/3)*(g**2 + 3
*h**2*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{hx+g}{(3\,h^2x^2+g^2)\left(9\,cx^2-\frac{cg^2}{h^2}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x + g)/((3*h^2*x^2 + g^2)*(9*c*x^2 - c*g^2/h^2)^(1/3)),x, algorithm
```

```
[Out] integrate((h*x + g)/((3*h^2*x^2 + g^2)*(9*c*x^2 - c*g^2/h^2)^(1/3
)), x)
```

3.143 $$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx$$

Optimal. Leaf size=488

$$\frac{3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\frac{f(b^2h^2-bcgh+c^2g^2)}{3c^2h^2}+\frac{bfx}{c}+fx^2\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}-\frac{3\,3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\left(1-\frac{3h(b+2cx)}{2cg-bh}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{\frac{3h(b+2cx)}{2cg-bh}+1}\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}+\frac{3\sqrt[6]{3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3h(b+2cx)}{2cg-bh}\right)^{2/3}}{\sqrt{3}\sqrt[3]{\frac{3h(b+2cx)}{2cg-bh}+1}}\right)}{f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}$$

```
[Out] (3*3^(1/6)*h*((c*h^2*(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2) - 9*b*x
 - 9*c*x^2))/(2*c*g - b*h)^2)^(1/3)*ArcTan[1/Sqrt[3] - (2^(2/3)*(
1 - (3*h*(b + 2*c*x))/(2*c*g - b*h))^(2/3))/(Sqrt[3]*(1 + (3*h*(b
 + 2*c*x))/(2*c*g - b*h))^(1/3))])/(f*(-(((c*g - 2*b*h)*(c*g + b*
h))/(c*h^2)) + 9*b*x + 9*c*x^2)^(1/3)) + (3^(2/3)*h*((c*h^2*(((c*
g - 2*b*h)*(c*g + b*h))/(c*h^2) - 9*b*x - 9*c*x^2))/(2*c*g - b*h)
^2)^(1/3)*Log[(f*(c^2*g^2 - b*c*g*h + b^2*h^2))/(3*c^2*h^2) + (b*
f*x)/c + f*x^2])/(2*f*(-(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9
*b*x + 9*c*x^2)^(1/3)) - (3*3^(2/3)*h*((c*h^2*(((c*g - 2*b*h)*(c*
g + b*h))/(c*h^2) - 9*b*x - 9*c*x^2))/(2*c*g - b*h)^2)^(1/3)*Log[
(1 - (3*h*(b + 2*c*x))/(2*c*g - b*h))^(2/3) + 2^(1/3)*(1 + (3*h*(
b + 2*c*x))/(2*c*g - b*h))^(1/3)])/(2*f*(-(((c*g - 2*b*h)*(c*g +
b*h))/(c*h^2)) + 9*b*x + 9*c*x^2)^(1/3))
```

Rubi [A] time = 1.37775, antiderivative size = 488, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 104, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019

$$\frac{3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\frac{f(b^2h^2-bcgh+c^2g^2)}{3c^2h^2}+\frac{bfx}{c}+fx^2\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}-\frac{3\,3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\left(1-\frac{3h(b+2cx)}{2cg-bh}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{\frac{3h(b+2cx)}{2cg-bh}+1}\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}+\frac{3\sqrt[6]{3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3h(b+2cx)}{2cg-bh}\right)^{2/3}}{\sqrt{3}\sqrt[3]{\frac{3h(b+2cx)}{2cg-bh}+1}}\right)}{f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)/(((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x + c*x^2)^(1/3)
(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 + (b*f*x)/c + f*x^2)),x]
```

```
[Out] (3*3^(1/6)*h*((c*h^2*(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2) - 9*b*x
 - 9*c*x^2))/(2*c*g - b*h)^2)^(1/3)*ArcTan[1/Sqrt[3] - (2^(2/3)*(
1 - (3*h*(b + 2*c*x))/(2*c*g - b*h))^(2/3))/(Sqrt[3]*(1 + (3*h*(b
 + 2*c*x))/(2*c*g - b*h))^(1/3))])/(f*(-(((c*g - 2*b*h)*(c*g + b*
h))/(c*h^2)) + 9*b*x + 9*c*x^2)^(1/3)) + (3^(2/3)*h*((c*h^2*(((c*
g - 2*b*h)*(c*g + b*h))/(c*h^2) - 9*b*x - 9*c*x^2))/(2*c*g - b*h)
^2)^(1/3)*Log[(f*(c^2*g^2 - b*c*g*h + b^2*h^2))/(3*c^2*h^2) + (b*
f*x)/c + f*x^2])/(2*f*(-(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9
*b*x + 9*c*x^2)^(1/3)) - (3*3^(2/3)*h*((c*h^2*(((c*g - 2*b*h)*(c*
g + b*h))/(c*h^2) - 9*b*x - 9*c*x^2))/(2*c*g - b*h)^2)^(1/3)*Log[
(1 - (3*h*(b + 2*c*x))/(2*c*g - b*h))^(2/3) + 2^(1/3)*(1 + (3*h*(
b + 2*c*x))/(2*c*g - b*h))^(1/3)])/(2*f*(-(((c*g - 2*b*h)*(c*g +
b*h))/(c*h^2)) + 9*b*x + 9*c*x^2)^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x+g)/(1/9*(2*b**2*h**2+b*c*g*h-c**2*g**2)/c/h**2+b*x+c*x**2)
2*b**2*h**2-b*c*g*h+c**2*g**2)/h**2)/c**2+b*f*x/c+f*x**2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.930348, size = 0, normalized size = 0.

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)/(((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x + c*x^2)
(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 + (b*f*x)/c + f*x^2)),x]
```

```
[Out] Integrate[(g + h*x)/(((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2
) + b*x + c*x^2)^(1/3)*((f*(b^2 - (-(c^2*g^2) + b*c*g*h + 2*b^2*h
^2)/(3*h^2)))/c^2 + (b*f*x)/c + f*x^2)), x]
```

Maple [F] time = 0.668, size = 0, normalized size = 0.

$$\int (hx+g)\frac{1}{\sqrt[3]{\frac{2\,b^2h^2+bcgh-c^2g^2}{9\,ch^2}+bx+cx^2}}\left(\frac{f}{c^2}\left(b^2+\frac{-2\,b^2h^2-bcgh+c^2g^2}{3\,h^2}\right)+\frac{bfx}{c}+fx^2\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(f*(b^2+1/
2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x)
```

```
[Out] int((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/
3)/(f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^
2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$3\int \frac{hx+g}{\left(cx^2+bx-\frac{c^2g^2-bcgh-2\,b^2h^2}{9\,ch^2}\right)^{\frac{1}{3}}\left(3\,fx^2+\frac{3\,bfx}{c}+\frac{\left(3\,b^2+\frac{c^2g^2-bcgh-2\,b^2h^2}{h^2}\right)f}{c^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(h*x + g)/((c*x^2 + b*x - 1/9*(c^2*g^2 - b*c*g*h - 2*b^2*h^2)/(c*h^2)
```

```
[Out] 3*integrate((h*x + g)/((c*x^2 + b*x - 1/9*(c^2*g^2 - b*c*g*h - 2*
b^2*h^2)/(c*h^2))^(1/3)*(3*f*x^2 + 3*b*f*x/c + (3*b^2 + (c^2*g^2
- b*c*g*h - 2*b^2*h^2)/h^2)*f/c^2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(h*x + g)/((c*x^2 + b*x - 1/9*(c^2*g^2 - b*c*g*h - 2*b^2*h^2)/(c*h^2)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(1/9*(2*b**2*h**2+b*c*g*h-c**2*g**2)/c/h**2+b*x+c*x**2)**(1/
2*b**2*h**2-b*c*g*h+c**2*g**2)/h**2)/c**2+b*f*x/c+f*x**2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,(hx+g)}{\left(cx^2+bx-\frac{c^2g^2-bcgh-2\,b^2h^2}{9\,ch^2}\right)^{\frac{1}{3}}\left(3\,fx^2+\frac{3\,bfx}{c}+\frac{\left(3\,b^2+\frac{c^2g^2-bcgh-2\,b^2h^2}{h^2}\right)f}{c^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(h*x + g)/((c*x^2 + b*x - 1/9*(c^2*g^2 - b*c*g*h - 2*b^2*h^2)/(c*h^2)
```

```
[Out] integrate(3*(h*x + g)/((c*x^2 + b*x - 1/9*(c^2*g^2 - b*c*g*h - 2*
b^2*h^2)/(c*h^2))^(1/3)*(3*f*x^2 + 3*b*f*x/c + (3*b^2 + (c^2*g^2
- b*c*g*h - 2*b^2*h^2)/h^2)*f/c^2)), x)
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```